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QMC Opening Workshop
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Lecture: *High Accuracy Algorithms for Interpolating and Integrating Multivariate Functions Defined by Sparse Samples in High Dimensions*

Speaker: James M. Hyman

Abstract:

We will describe and analyze accurate and efficient numerical algorithms to interpolate and approximate the integral of multivariate functions. The algorithms can be applied when we are given the function values at an arbitrary positioned, and usually small, existing sparse set of function values (samples), and additional samples are impossible, or difficult (e.g. expensive) to obtain. The methods are based on local, and global, tensor-product sparse quasi-interpolation methods that are exact for a class of sparse multivariate orthogonal polynomials.

Although the algorithms can be applied for general distributions of sample points, they are especially effective in improving the approximation of integrals defined on low-discrepancy (approximately uniformly distributed) sample. One advantage of this approach is that it continues to be effective on even high-discrepancy sampling distributions. This allows the algorithms to be used with adaptive sampling algorithms that concentrate sample points in regions with locally high gradients.

We will compare the convergence rate of the method on smooth and discontinuous functions defined on both low discrepancy quasi-Monte Carlo distributions of samples, as well as for sample distributions that are far from uniformly distributed. We observe that although the approach does not change the error convergence rate, the constant multiplicative factor in the error is often reduced by a factor of over 100 or more.

Joint research with Jeremy Dewar, Lin Li, and Mu Tian