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QMC Opening Workshop
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Lecture: *Generating Random Fields the Circulant Way*

Speaker: Ian H. Sloan

Abstract:

The generation of Gaussian random fields over a physical domain is a challenging problem in computational mathematics, especially when the correlation length is short and the field is rough. The traditional approach is to make use of a truncated Karhunen-Loeve (KL) expansion, but the generation of even a single realisation of the field may then be effectively beyond reach (especially for 3-dimensional domains) if the need is to obtain an expected L2 error of say 5%, because of the potentially very slow convergence of the KL expansion. In this talk, based on joint work with Ivan Graham, Frances Kuo, Dirk Nuyens, and Rob Scheichl, a completely different approach is used, in which the field is initially generated at a regular grid on a 2- or 3-dimensional rectangle that contains the physical domain, and then possibly interpolated to obtain the field at other points. In that case there is no need for any truncation. Rather the main problem becomes the factorisation of a large dense matrix. For this we use circulant embedding and FFT ideas. Quasi-Monte Carlo integration is then used to evaluate the expected value of some functional of the finite-element solution of an elliptic PDE with a random field as input.