Emulation of the Halo Mass Function

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Introduction

- Gravitational evolution of dark matter structures in the Universe is highly non-linear.

- Collision-less N-body simulations have been highly successful in explaining large scale structure.

- Cosmic web structures:
  - Voids: Vast (mostly) empty spaces
  - Walls: sheet-like 2D structures
  - Filaments: Tubular
  - Halos: Spheroidal, dense
Dark matter halos

- Most dense regions in the dark matter Universe - believed to host most of galaxies and clusters.
- Compact – gravitationally bound, virialized
- Create gravitational potential well for baryons - host galaxies
- Mass ranges $10^{10} - 10^{16} M_{\text{sun}}$
- Statistics
  - halo density profile
  - concentration-Luminosity relation
  - halo occupation distribution
  - halo mass function
Halo mass function

- $n(M)$: Number density of dark matter halos in the Universe binned by mass
- hmf is currently not calculated via first principles

Computed via:
- Semi-analytic models
- Observations
  - Cluster counts, SNe
  - Simulations
Cosmological simulations

- N-body simulations:
  - Perturbed distribution of DM particles
  - Collision-less dynamics + gravity
  - Parameters from precision cosmology
  - Solved numerically at discrete time steps
## Cosmological parameters

- Total matter density (at $z=0$) $\Omega_m$
- Cosmological constant (at $z=0$) $\Omega_\Lambda$
- Baryon density (at $z=0$) $\Omega_b$
- Spectral index $n_s$
- Hubble parameter $h$
- Power spectrum normalization $\sigma_8$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Planck</th>
<th>WMAP9</th>
<th>WMAP7</th>
<th>WMAP5</th>
<th>GiggleZ</th>
<th>WMAP3</th>
<th>WMAP1</th>
<th>Millennium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_\Lambda$</td>
<td>0.6817</td>
<td>0.7181</td>
<td>0.728</td>
<td>0.723</td>
<td>0.726</td>
<td>0.732</td>
<td>0.710</td>
<td>0.750</td>
</tr>
<tr>
<td>$\Omega_c$</td>
<td>0.2678</td>
<td>0.236</td>
<td>0.226</td>
<td>0.231</td>
<td>0.228</td>
<td>0.224</td>
<td>0.243</td>
<td>0.205</td>
</tr>
<tr>
<td>$\Omega_b$</td>
<td>0.049</td>
<td>0.0461</td>
<td>0.0455</td>
<td>0.0459</td>
<td>0.0456</td>
<td>0.044</td>
<td>0.047</td>
<td>0.045</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.9619</td>
<td>0.9646</td>
<td>0.967</td>
<td>0.962</td>
<td>0.960</td>
<td>0.99</td>
<td>0.947</td>
<td>1</td>
</tr>
<tr>
<td>$H_0$</td>
<td>67.04</td>
<td>69.7</td>
<td>70.4</td>
<td>70.2</td>
<td>70.5</td>
<td>70.4</td>
<td>72.0</td>
<td>73.0</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>0.8347</td>
<td>0.817</td>
<td>0.81</td>
<td>0.817</td>
<td>0.812</td>
<td>0.776</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

http://hmf.icrar.org/hmf_parameters/
Simulation-specific parameters

- Periodic box size: \( L = 100-1000 \ h^{-1} \ Mpc \)
- Number of particles: \( N_p = 128^3 - 1024^3 \)
- Force resolution: \( \varepsilon = 1 - 20 \ h^{-1} \ Mpc \)
- Mass of each particle: \( m = 10^8 - 10^{10} \ M_{\odot} \)
- Starting redshift: \( z_{ini} = 100 - 50 \)
Run simulations

• Resource
  – Typical simulations are run on 100s of cores with highly parallelized numerical codes.

• Time
  – relatively low-res simulation of $128^3$ particles took about a week on 24 cores! High-res simulations run for several months on the largest supercomputers.

• Space
  – Each snapshot of particles (at a certain $z$) includes info about co-ordinates, velocity etc. - for each particle. Each file is in the order of TB.
Halo finders

- Usual approaches involve specifying either inter-particle distance thresholds or density thresholds.

- Majority of Halo finders depend free parameters.
  - Friends-of-Friends (FOF) linking length parameter, $b$
  - Over-density threshold, $\delta_c$
So many halo finders!

- 1974 SO: Press & Schechter
- 1985 FOF: Davis et al.
- 1992 DENMAX: Gelb & Bertschinger
- (1994 SO: Lacey & Cole)
- 1995 adaptive FOF: van Kampen et al.
- 1996 isoDen: Pfützner & Salmon
- 1997 BDM: Klypin & Holtzman
- 1998 HOP: Eisenstein & Hut
- 1999 hierarchical FOF: Gottloeber et al.
- 2001 SKID: Stadel
- 2001 enhanced BDM: Bullock et al.
- 2001 SUBFIND: Springel
- 2004 MHF: Gill, Knebe & Gibson
- 2004 AdaptaHOP: Aubert, Pichon & Colombi
- 2005 improved DENMAX: Weller et al.
- 2005 VOBOZ: Neyrinck et al.
- 2006 PSB: Kim & Park
- 2006 6DFOF: Driemond et al.
- 2007 subhalo finder: Shaw et al.
- 2007 Ntropy-fofsv: Gardner, Connolly & McBride
- 2009 HSF: Maciejewski et al.
- 2009 LANL finder: Habib et al.
- 2009 AHF: Knollmann & Knebe
- 2010 pHOP: Skory et al.
- 2010 ASOHF: Planelles & Quilis
- 2010 pSO: Sutter & Ricker
- 2010 pFOF: Rasera et al.
- 2010 ORIGAMI: Falck et al.
- 2010 HOT: Ascasibar
- 2010 Rockstar: Behroozi

arXiv:1104.0949
Finally, we can compute hmf $n(M)$

The Halo mass function (from a simulation) depends on:

- Cosmological parameters
  - $\Omega_m, \Omega_\Lambda, \Omega_b, n_s, h, \sigma_8$
- Simulation-specific parameters
  - $L, N_p, z_{ini}, m, \varepsilon$
- Free parameter of the halo finder
  - $b, \delta_c$
<table>
<thead>
<tr>
<th>REF.</th>
<th>FITTING FUNCTION $f(\sigma)$</th>
<th>MASS RANGE</th>
<th>REDSHIFT RANGE</th>
<th>COSMOLOGY Fitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Press and Schechter (1974)</td>
<td>$f_{PS}(\sigma) = \sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma} \exp \left[ -\frac{\delta_c^2}{2\sigma^2} \right]$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sheth et al. (2001)</td>
<td>$f_{ST}(\sigma) = A \sqrt{\frac{2a}{\pi}} \left[ 1 + \left( \frac{\sigma^2}{a\delta_c^2} \right)^p \right] \frac{\delta_c}{\sigma} \exp \left[ -\frac{a\delta_c^2}{2\sigma^2} \right]$, ( A = 0.3222, a = 0.707, p = 0.3. )</td>
<td>-</td>
<td>-</td>
<td>Einstein-de Sitter</td>
</tr>
<tr>
<td>Jenkins et al. (2001)</td>
<td>$f_1(\sigma) = 0.315 \exp \left[ (\ln \sigma^{-1} + 0.61)^{3.8} \right]$</td>
<td>$-1.2 &lt; \ln \sigma^{-1} &lt; 1.05$</td>
<td>0 - 5</td>
<td>$\tau$CDM, ACDM</td>
</tr>
<tr>
<td>Reed et al. (2003)</td>
<td>$f_{R03}(\sigma) = f_{ST}(\sigma) \exp \left[ -\frac{0.7}{\sigma \cosh(2\sigma)^6} \right]$</td>
<td>$-1.7 &lt; \ln \sigma^{-1} &lt; 0.9$</td>
<td>0 - 15</td>
<td>$\Omega_M = 0.3, \Omega_\Lambda = 0.7$</td>
</tr>
<tr>
<td>Warren et al. (2006)</td>
<td>$f_W(\sigma) = 0.7234 \left( \sigma^{-1.625} + 0.2538 \right) \exp \left[ -\frac{1.1982}{\sigma^2} \right]$</td>
<td>$10^{10} M_\odot &lt; M &lt; 10^{15} M_\odot$</td>
<td>0</td>
<td>ACDM: WMAP1</td>
</tr>
<tr>
<td>Reed et al. (2007)</td>
<td>$f_{R07}(\sigma) = \nu \exp \left[ -\frac{ca^2}{2} - \frac{0.03(\delta_c/\sigma)^9.6}{(n_{eff}+3)^2} \right]$</td>
<td>$-1.7 &lt; \ln \sigma^{-1} &lt; 0.9$</td>
<td>0 - 30</td>
<td>ACDM: WMAP1</td>
</tr>
<tr>
<td>Tinker and Kravtsov (2008)</td>
<td>$f_T(\sigma, z) = A \left( \left( \frac{\delta}{\sigma} \right)^a + 1 \right) \exp \left[ -\frac{\delta_c}{\sigma} \right]$, ( A = 0.186 (1+z)^{-0.14}, a = 1.47 (1+z)^{-0.06}, b = 2.57 (1+z)^{-\alpha_c}, c = 1.19, \alpha = \exp \left[ -\left( \frac{0.75}{\ln(\Delta_{vir}/75)} \right)^{1.2} \right] )</td>
<td>$-0.6 &lt; \ln \sigma^{-1} &lt; 0.4$</td>
<td>0 - 2.5</td>
<td>ACDM: WMAP1, WMAP3+</td>
</tr>
</tbody>
</table>

arXiv:1306.6721
Can we build an emulator for hmf?

- Change in hmf is smooth for parameter variation
- We can select a subset of parameter space as priors, say: $(\Omega_m, \Omega_\Lambda, \Omega_b, n_s, h, \sigma_8, \delta_c, b)$ for running a number of N-body simulations.

- Emulators for the dark matter power spectrum, concentration-mass relation etc are already constructed: [http://www.hep.anl.gov/cosmology/CosmicEmu/](http://www.hep.anl.gov/cosmology/CosmicEmu/)

- Along the same line, build an emulator for hmf, with more parameters.
- Variation in hmf at different redshifts can also be studied.
Plans during this workshop

- Simulations take a long time. Instead mock data for hmf can be generated from fitting functions:
  
  https://github.com/steven-murray/HMFcalc
  Check out:  http://hmf.icrar.org/

- Can change a few parameters (mostly cosmological)

- Build a GP emulator for a subset of parameter space.

- Decide on what parameters to specify for real simulations, range, number of simulations etc.
Mock data: varying $\Omega_m$
Mock data: varying $\sigma_8$
Mock data: varying $n_s$
Future

- Other halo population parameters:
  - Halo occupation distributions
  - Density profiles
  - Spin parameters etc.

- hmf evolution
  - With redshift
Thank you!
Planck Collaboration: Cosmological parameters

Planck 2015 results