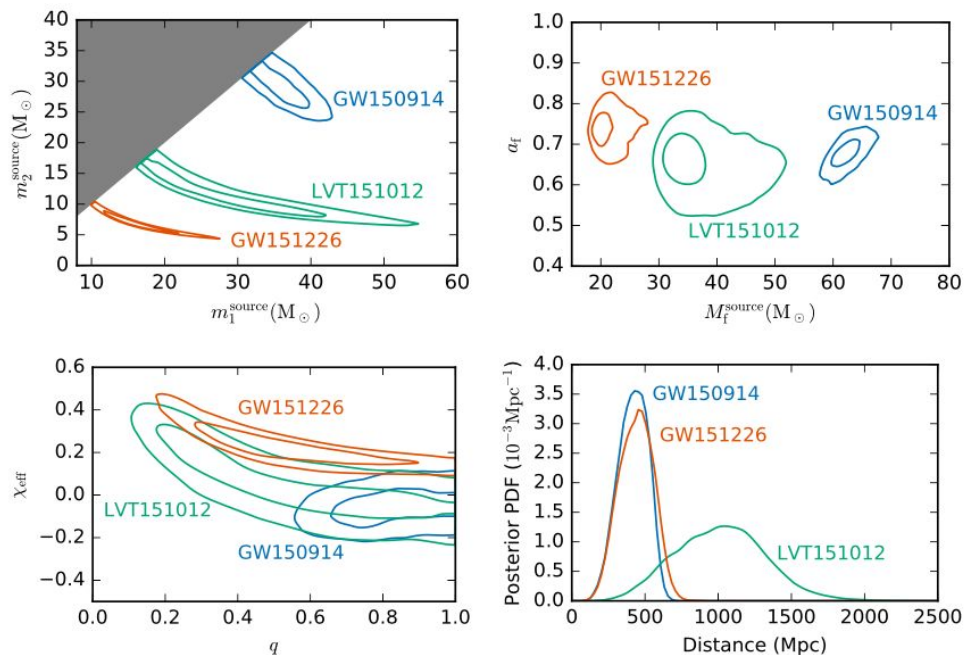


Probability density estimation for populations of gravitational wave sources

SAMSI April Workshop

Gravitational Wave Measurements of Compact Binary Coalescences

- Chirp mass
- Mass ratio
- Effective spin
- Luminosity distance



Phys. Rev. X 6, 041015 (2016) Fig. 4

Science Goal:

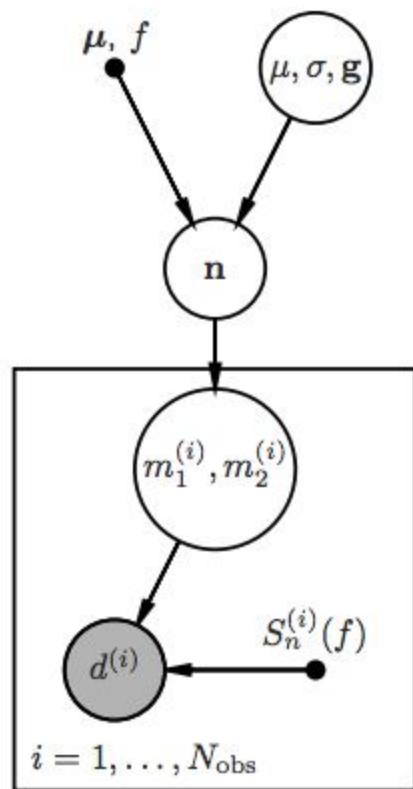
- Estimate the underlying rate density of compact binary coalescences as a function of the binaries' properties
 - Find trends: i.e. clusters indicative of different populations, evolution with redshift
 - Compare to population synthesis predictions
 - Constrain physical parameters
 - Model selection

Example: Joint density $f(m1, m2)$

Mandel et al. (2017): [arXiv:1608.08223](https://arxiv.org/abs/1608.08223)

Modeled on Foreman-Mackey et al. (2014): [arXiv:1406.3020](https://arxiv.org/abs/1406.3020)

- Bin the $\ln(m1)-\ln(m2)$ plane: the density $f(m1, m2)$ is parametrized by the height of each bin n_{ij}
- Hierarchical Bayesian model to account for measurement uncertainty
- The likelihood $L(n_{ij})$ is given by an inhomogeneous Poisson process
- The “prior” is set to a multivariate Gaussian with some “shape”
- Get posteriors for the model parameters: the means and covariance of the bin heights



Benefits of binning and applying a prior

- Prior enforces smoothness in the limit of few detections
- The shape of the prior can encode physics
 - If we parametrize the “prior” and the parameters have physical significance, we can constrain physics
- Straightforward to incorporate selection effects into the model, if these are known

Extensions?

- Higher-dimensional joint densities
 - 2 masses and 2 (or more) spins
 - 2 masses and 1 redshift (time evolution)
- Make more nonparametric?
- Make more parametric?

Alternatives?

- Sujit, Angie and Bo: Density estimation for exoplanets with series of Bernstein polynomials
- Daniel Wysocki and Richard O'Shaughnessy: Gaussian mixture models
 - Extend to Dirichlet processes?