Regularized Optimization with Spatial Coupling

- Spatial coupling
  - Match resource (supply) and people (demand) over spatial area
  - May have sensitivity issue among optimal solutions due to global optimization
  - Result in unreliable decision making
- Regularized optimization
  - Suggest some form of regularization among the decision variables

$$
\begin{aligned}
(\mathrm{GP}) \quad \min_{\mathbf{x}_{\cdot\cdot}} \quad & F(\mathbf{x}_{\cdot\cdot}) + \sum_{i \in I} \sum_{k \in \delta(i)} f_{ik}(\phi_i(\mathbf{x}_{i\cdot}), \phi_k(\mathbf{x}_{k\cdot})) \\
\text{s.t.} \quad & H(\mathbf{x}_{\cdot\cdot}) \leq 0, \\
& g_i(\mathbf{x}_{i\cdot}) \leq 0 \text{ for } i \in I, \\
& h_j(\mathbf{x}_{\cdot j}) \leq 0 \text{ for } j \in J, \\
& \mathbf{x}_{\cdot\cdot} \geq 0,
\end{aligned}
$$

  - Application in transportation problems, evacuation planning, measure of access to service, etc.

Distributed Algorithms

- Dual Decomposition
  - Decompose the big problems into sub problems for each demand location *i*
  - Dualize the coupling constraints, and update the multipliers for violation
  - SLOW!

$$f^{(t)} - f^* \leq \frac{\mathbf{dist}(x^{(1)} - x^*) + G^2||\alpha||_2^2}{2\sum_{i=1}^{t} \alpha_i}$$

(Boyd, et al. 2003)

- Block Dual Decomposition
  - Decompose the big problems into blocks of sub problems
  - Exploit the spatial structure
    - *Interior supplier*: serves ***only*** the demand in the block
    - *Exterior supplier*: serves the demand in the block, but not interior supplier
  - Preserve some of the competition among demand locations