Regularized Optimization with Spatial Coupling

• Spatial coupling
  • Match resource (supply) and people (demand) over spatial area
  • May have sensitivity issue among optimal solutions due to global optimization
  • Result in unreliable decision making

• Regularized optimization
  • Suggest some form of regularization among the decision variables

\[
\text{(GP)} \quad \min_{x_{..}} \quad F(x_{..}) + \sum_{i \in I} \sum_{k \in \delta(i)} f_{ik}(\phi_i(x_{i..}), \phi_k(x_{k..}))
\]

\[
\begin{align*}
\text{s.t.} & \quad H(x_{..}) \leq 0, \\
& \quad g_i(x_{i..}) \leq 0 \text{ for } i \in I, \\
& \quad h_j(x_{j..}) \leq 0 \text{ for } j \in J, \\
& \quad x_{..} \geq 0,
\end{align*}
\]

• Application in transportation problems, evacuation planning, measure of access to service, etc.
Distributed Algorithms

• Dual Decomposition
  • Decompose the big problems into sub problems for each demand location $i$
  • Dualize the coupling constraints, and update the multipliers for violation
  • SLOW!

\[
  f(t) - f^* \leq \frac{\text{dist}(x^{(1)} - x^*) + G^2||\alpha||_2^2}{2 \sum_{i=1}^t \alpha_i}
\]

(Boyd, et al. 2003)

• Block Dual Decomposition
  • Decompose the big problems into blocks of sub problems
  • Exploit the spatial structure
    • Interior supplier: serves only the demand in the block
    • Exterior supplier: serves the demand in the block, but not interior supplier
  • Preserve some of the competition among demand locations