Statistical Challenges in Cosmology

Elisabeth Krause, Stanford

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Our Simple Universe

On large scales, the Universe can be modeled with remarkably few parameters:
- age of the Universe
- geometry of space
- density of atoms
- density of matter
- amplitude of fluctuations
- scale dependence of fluctuations

[of course, details often not quite as simple]
Our Puzzling Universe

Ordinary Matter

"Dark Matter"

5%

25%

"Dark Energy"

70%

- accelerates the expansion
- dominates the total energy density
- smoothly distributed

acceleration first measured by SN 1998

next frontier: understand
A Few Key Concepts

**Special Relativity**
- looking outwards is looking back in time
  - parameterized by redshift, $z$

**General Relativity**
- matter curves space
- curved space tells matter how to move
- space is relative

**We live in an expanding Universe**
- parameterized by scale factor, $a = 1/(1+z)$
- expansion is accelerating!
Cosmic History

- Afterglow Light Pattern 400,000 yrs.
- Dark Ages
- Development of Galaxies, Planets, etc.
- Dark Energy Accelerated Expansion
- Inflation
- Quantum Fluctuations
- 1st Stars about 400 million yrs.
- Big Bang Expansion 13.7 billion years

NASA/WMAP Science Team
Cosmic Acceleration

CMB + supernovae + large-scale structure:

homogeneity, isotropy, flatness + acceleration impossible with GR + matter only

observations require a repulsive force

- cosmological constant $\Lambda$: $w = p/\rho = -1$?
- dynamic scalar field, $w(a)$?
- breakdown of GR?

dominates dynamics of late-time Universe
Many new DE/modified gravity theories developed over last decade

Most can be categorized based on how they break GR:

The only local, second-order gravitational field equations that can be derived from a four-dimensional action that is constructed solely from the metric tensor, and admitting Bianchi identities, are GR + Λ.

Lovelock's theorem (1969)

Subject to viability conditions: ghosts? instabilities?
Many new DE/modified gravity theories developed over last decade

Most can be categorized based on how they break GR:

The only local, second-order gravitational field equations that can be derived from a four-dimensional action that is constructed solely from the metric tensor, and admit Bianchi identities, are GR + $\Lambda$.

Lovelock's theorem (1969)

Subject to viability conditions: ghosts? instabilities?

Theory Space: Breaking GR

No favored alternative theory, theory space hard to summarize succinctly

Need unifying frameworks + phenomenology to compare to data
Testing Cosmic Acceleration

size of $\Lambda$ difficult to explain

important to test GR over cosmological scales

Expansion history

$$H^2(a) = H_0^2 \left( \Omega_M a^{-3} + \Omega_{DE} a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)} \right)$$

- from supernovae, CMB peaks + baryonic acoustic oscillations (BAO)
- agreement with $\Lambda$CDM
- not much information on dark energy/gravity: at most $w_0$, $w_a$
Cosmic Structure Formation

- gravity drives the formation of cosmic structure
- much more information than expansion rate
- linear level: perturbed Einstein equation
- non-linear evolution: numerical simulations
- reliably predict dark matter distribution, for wCDM cosmologies + individual MG models

Springel+, 2006
From Photons to Parameter Constraints

reduce data → make maps & catalogs → remove foregrounds → measure galaxy shapes

The Forward Process.
Galaxies: Intrinsic galaxy shapes to measured image:

- Intrinsic galaxy (shape unknown) → Gravitational lensing causes a shear ($g$) → Atmosphere and telescope cause a convolution → Detectors measure a pixelated image → Image also contains noise

Stars: Point sources to star images:

- Intrinsic star (point source) → Atmosphere and telescope cause a convolution → Detectors measure a pixelated image → Image also contains noise

Fig. 2. Illustration of the forward problem. The upper panels show how the original galaxy image is sheared, blurred, pixelised and made noisy. The lower panels show the equivalent process for (point-like) stars. We only have access to the right hand images.

Stars are far enough away from us to appear point-like. They therefore provide noisy and pixelised images of the convolution kernel (lower panel of Figure 2). The convolution kernel is typically of a similar size to the galaxies.

Fig. 3. Illustration of the inverse problem. We begin on the right with a set of galaxy and star images. The full inverse problem would be to derive both the shears and the intrinsic galaxy shapes. However shear is the quantity of interest for cosmologists.
From Photons to Parameter Constraints

foreground removal
star, galaxy classification
galaxy shape estimation
galaxy cluster finding
SN detection + light curve fitting
...

reduce data
make maps & catalogs

SNe luminosity distance measurement
CMB angular diameter distance measurement and perturbations
BAO angular diameter distance measurement
Combination

Matter Density

Cosmological Constant, i.e. Dark Energy
reduce data make maps & catalogs

SN

BAO

CMB

No Big Bang

Union 2.1 SN is Compilation with SN Systematics
From Photons to Parameter Constraints

- foreground removal
- star, galaxy classification
- galaxy shape estimation
- galaxy cluster finding
- SN detection + light curve fitting

reduce data
make maps & catalogs

compare maps + catalogs with theory
How to compare with data?

physics + model parameters
generate initial conditions, evolve

Springel+, 2006
How to compare with data?

**physics** + **model parameters**

generate initial conditions, evolve

galaxy formation models

Springel+, 2006

dark matter

galaxies, light

Springel+, 2006

?
What to look for in the galaxy distribution?

- clusters (over densities),
- voids (under densities)
- two-point correlations
- three-point correlations,...

need redshift, understand galaxy bias

BAOs

lin. growth

non-lin. structure
LSS Probes of Dark Energy

Galaxy Clustering

- measure BAOs + shape of correlation function
- → growth of structure, expansion history
- Key systematic: galaxy bias

\[\alpha = 1.016 \pm 0.017 \]
\[\chi^2 = 30.53/39 \text{ dof}\]
Galaxy Clusters

measure number counts

\[ N(\hat{M}, z, \Delta z) = \frac{dn}{dM \, dz} \Delta V(z, \Delta z) \]

→ distribution of peaks, growth of structure, expansion history

but need to identify clusters + member galaxies, infer masses!
LSS Probes of Dark Energy

Weak Gravitational Lensing

credit: ESA
Weak Gravitational Lensing

- Light deflected by tidal field of LSS
- Coherent distortion of galaxy shapes ("shear")
- Shear related to (projected) matter distribution
- Key uncertainties
  - Shape measurements
  - Assume random intrinsic orientation, average over many galaxies
LSS Probes of Dark Energy

Weak Gravitational Lensing Ib

- light deflected by tidal field of LSS
- coherent distortion of galaxy shapes ("shear")
- remapping of CMB anisotropies

- CMB lensing affected by different systematics than shear estimates from galaxy distortions
- consistency check

credit: ESA
Weak Gravitational Lensing II

- lensing produces (almost) purely E-mode type shear
- observational B-modes >> cosmological B-modes
- measure shear correlation function/power spectrum
- probes total matter power spectrum (w/ broad projection kernel)
- measure average (tangential) shear around galaxies/clusters
- probes halo mass

LSS Probes of Dark Energy
The Power of Combining Probes

- Best constraints obtained by combining cosmological probes
  - independent probes: multiply likelihoods

- Combining LSS probes (from same survey) requires more advanced strategies
  - clustering, clusters and WL probe same underlying density field, are correlated
  - correlated systematic effects
  - requires joint analysis

The Observational Foundations of Dark Energy

- Weak-lensing is also complementary.
- SNe luminosity distance measurement (Nobel 2011)
- CMB angular diameter distance measurement
- BAO angular diameter distance measurement and perturbations

Combination

Matter Density

Olivier Doré AAS, WFIRST Science, Kissimmee, January 5th 2016
Joint Analysis Ingredients

Science Case
- parameters of interest
- which science?
- large data vector
- which probes + scales?

Likelihood Function
- number counts: Poisson
- 2PCF: ~ Gaussian (?)
- improvements needed for stage IV surveys

Model Data Vector
- consistent modeling of all observables
- including all cosmology + nuisance parameters

Joint Covariance
- large and complicated, non-(block) diagonal matrix
- use template + regularization

Cosmology Priors

Nuisance Parameters
- systematic effects
- \(|n| \gg |\pi|\)
- parameterize + prioritize!

External Data Simulations

Priors

\[ p(\pi | \tilde{d}) \propto p(\pi) \int \mathcal{L}(\hat{d} | d(\pi, n), C) p(n) d^n n \]
LSST: The Experiment
- largest planned LSS survey
- map visible sky every 3 nights
- high priority in P5, decadal survey
- construction started 2015
- commissioning first light 2019
- survey duration 2022-2032

LSST: Science Collaborations
- Solar System
- Stars, Milky Way, Local Volume
- Transients
- Galaxies
- Active Galactic Nuclei
- Informatics and Statistics
- Dark Energy
Prepare for and carry out cosmology analyses with the LSST survey

- five key cosmology probes, organized in Working Groups (WG)
  - Galaxy Clustering, Galaxy Clusters, Strong Lensing, Supernovae, Weak Lensing; Theory & Joint Probes
- “Enabling Analyses” WGs: understand LSST system + systematics

lots of work until 2019, lots to learn from ongoing surveys!
Prepare for and carry out cosmology analyses with the LSST survey. Five key cosmology probes, organized in Working Groups (WG) - Galaxy Clustering, Galaxy Clusters, Strong Lensing, Supernovae, Weak Lensing; Theory & Joint Probes. “Enabling Analyses” WGs: understand LSST system + systematics. Lots of work until 2019, now is a good time for new ideas!

The LSST Dark Energy Science Collaboration
Introducing CosmoLike
EK & Eifler ’16 (arXiv:1601.05779)

- Likelihood analysis library for combined probes analyses
- Observables from three object types, and their cross-correlations
  - galaxies (positions), clusters (positions, N_{200}), sources (shapes, positions)
  - galaxy clustering, cluster abundance + cluster lensing (mass self-calibration),
    galaxy-galaxy lensing, cosmic shear, CMB cross-correlations
  - separate n(z) + specific nuisance parameters for each object type

- Consistent modeling across probes
  - including systematic effects

- Computes non-Gaussian (cross-)covariances
  - halo model + regularization from O(25) simulated realizations

- Optimized for high-dimensional likelihood analyses

- Improvements by trial and error on DES → lessons for LSST
CosmoLike Data Vector

**Cosmological parameters**
- CosmoLike
- Halo
- Growth factor
- Transfer function
- Distances
- Z-distribution

**Cosmo3d.c**
- Coyote U. Emulator
- P_{lin}(k,z)
- P_{nl}(k,z)
- T(k,z)
- D(k,z)

**Halo.c**
- Collapse density \( \delta_c(z) \)
- Peak height \( \nu(M,z) \)
- Halo properties
  - c(M,z)
  - b(M,z)
  - n(M,z)
- HOD, bias model

**Clusters.c**
- Scaling relation \( M_{obs}(M) \)
- Cluster selection function

**Redshift.c**
- Photo-z model

**Cosmo2d.c**
- Projection functions
- Limber approx.

**New physics enters here**
Combined Probes Forecasts: Covariance

- SN uncorrelated, hooray [for now].
- Analytic covariance for everything else:
  - Halo model bispectrum + trispectrum, sample variance
  - \( \text{Cov} (N, N) \): Poisson + power spectrum
  - \( \text{Cov} (<\delta\delta>, N) \): bispectrum, power spectrum
  - \( \text{Cov} (<\delta\delta>, <\delta\delta>) \), etc.: Covariance of 2pt statistics of (projected) density field

\[
\text{Cov}(P(k_1), P(k_2)) \approx \frac{2\delta_D (k_1 + k_2)}{N_{k_1}} P^2(k_1) + \frac{T(k_1, k_2)}{V_s} + \frac{\partial P(k_1)}{\partial \rho_L} \frac{\partial P(k_2)}{\partial \rho_L} \sigma^2(\rho_L)
\]

- LSST forecasts: > 7 million elements...
Combined Probes Forecasts: Theory Covariance

details: EK & Eifler’16
Combined Probes Forecasts: Theory Covariance

all is well in theory land...

for actual data analysis, need to combine theory & data

smart covariance estimators, e.g. shrinkage

details: EK & Eifler’16
The Power of Combining Probes
Zoom into $w_0$-$w_a$ plane

- Very non-linear gain in constraining power
- Most stringent requirements on numerical simulations, photo-z, shear calibration, etc. flow from Multi-Probe Statistical Limits
“Precision cosmology”: excellent statistics - systematics limited

(and man-power limited!)

Easy to come up with large list of systematics + nuisance parameters

- galaxies: LF, bias (e.g., 5 HOD parameters + $b_2$ per z-bin,type)
- cluster mass-observable relation: mean relation + scatter parameters
- shear calibration, photo-z uncertainties, intrinsic alignments,...

$\Sigma$(poll among DES working groups) $\sim$ 500-1000 parameters

Self-calibration + marginalization

- can be costly (computationally, constraining power)
CosmoLike Data Vector

cosmological parameters

transfer function $T(k,z)$
growth factor $D(k,z)$
$P_{\text{lin}}(k,z)$
distances $P_{\text{nl}}(k,z)$

Coyote U. Emulator

$\delta_c(z)$ collapse density
$\nu(M,z)$ peak height

halo properties
$c(M,z)$
$b(M,z)$
$n(M,z)$

HOD, bias model

scaling relation $M_{\text{obs}}(M)$
cluster selection function

non-linear regime
baryons
cluster finding
galaxy formation
intrinsic alignments
non-Gaussian photo-zs
shear calibration

z-distr. $n(z)$
photo-z model

projection functions
Limber approx.

$N(M_{\text{obs}};z_i)$
$C^{XY}(l;Z_i,Z_i)$
$\chi^2$
Work Plan for Known Systematics

- What’s the dominant known systematic?
  
  *No one-fits-all answer, need to be more specific!*

- Specify data vector (probes + scales)

- Identify + model systematic effects
  
  - find suitable parameterization(s)
  - need to be consistent across probes

- Constrain parameterization + priors on nuisance parameters
  
  - independent observations
  - other observables from same data set
  - split data set
Joint Analysis Work Plan: Step 1

Model, Priors
- Theory
- Simulations

Likelihood Analysis

Precision
- Refine Systematics Model
- Forecasts Impact

Consistency

Accuracy

Parameter Constraints
The Trouble with Systematics

- A systematics free survey....
- Bias free parameter estimates with statistical uncertainty
The Trouble with Systematics

ignored systematic effect in analysis:

parameter bias
The Trouble with Systematics

marginalize systematic effect, correct parameterization
remove parameter bias, increase uncertainty
The Trouble with Systematics

marginalize systematic effect, correct parameterization
remove parameter bias, increase uncertainty

improve priors on nuisance parameters
The Trouble with Systematics

marginalize systematic effect, imperfect parameterization
residual parameter bias, increased uncertainty
Combined Probes Systematics

“Precision cosmology”: excellent statistics - systematics limited
Physics from Galaxies: photo-zs

- galaxy evolution: not as clean as the CMB
- galaxies come in all shapes, sizes, colors
- what do we need to understand for cosmology?
- estimate redshift/distance: measure galaxy colors (flux in different filters)
Physics from Galaxies: photo-zs

- galaxy evolution: not as clean as the CMB
  - galaxies come in all shape, sizes, colors
  - what do we need to understand for cosmology?
  - estimate redshift/distance: measure galaxy colors
    - ambiguous for some galaxy types + imperfect photometry
Physics from Galaxies: Bias

galaxy evolution: not as clean as the CMB
- what do we need to understand for cosmology?
- estimate redshift/distance: measure galaxy colors
- relation between a galaxy population and matter field, galaxy bias
  - on large scales, linear relation between galaxies and matter density
  - perturbative methods in quasi-linear regime large, active area of research
    - comes at the cost of extra parameters
  - on small scales, several galaxies within massive halos
    - approximate (halo) models, or expensive simulations + emulators
    - all of these models functions of redshift + galaxy type
Cut-off for Galaxy Bias Models?

LSST, WL + clustering
WL to $l < 5000$
clustering: vary cut-off scales
develop perturbative biasing up to $k \sim 0.6 \, h/\text{Mpc}$ - with well-constrained new parameters
understand non-linear regime

details: EK & Eifler ’16
Joint Analysis Work Plan

Data, Model, Priors
- Observations
- Theory
- Simulations

Likelihood Analysis

Parameter Constraints
- Precision
- Consistency
- Accuracy

Forecasts to Prioritize Systematics
Single Probe Analyses
Unknown Systematics? vs. New Physics?

multi-probe analysis, pass 1 - now what?
Unknown Systematics? vs. New Physics?

- scale dependence?
- dependence on galaxy selection?
- cross-calibrate with more accurate measurements
  - spectroscopic redshifts
  - galaxy shapes from space-based imaging
    [potentially expensive]
- correlate with different surveys
  - predict cross-correlations based on LSST analysis
  - constrain uncorrelated systematics
  - e.g., cross-correlation with CMB-S4 lensing
- invent optimized estimators
  [fun, but not a general solution]

![Shear bias 68% constraints graph](image)

**FIG. 5.**

**Left panel:**

CMB lensing will thus provide a valuable consistency check for building confidence in the results from LSST.

**Right panel:**

- Contribution to the shear calibration is present, but still within the 68% confidence region.
- The colored bands show the 68% confidence constraints, corresponding to the curves in the left panel.
- Intrinsic alignment impacts the dependence on galaxy selection?
- The lines show the bias in the self-calibrated value of the shear calibration bias.

**VI. SENSITIVITY TO PHOTOMETRIC REDSHIF uncertainties**

**Mean redshift**

- **LSST requirement**
- **Combi1:** $gg, g_i$
- **Combi2:** $gg, g_i; g, g_i, CMB$
- **LSST full:** $gg, g_i; CMB$
- **CMB S4 lensing**

**Eli Schaan, EK, + 2016**

**VIII. CONCLUSION**

**LSST WL x CMB-S4 lensing**

- calibrate shear calibration bias
- Schaan, EK, + 2016
Unknown Systematics? vs. New Physics?

multi-probe analysis, pass 1 - now what?
would comparison with Planck results change this plan?

Planck best fit
Experimenter Bias?

- Nuisance parameters will outnumber cosmological parameters by far.
- What models + priors to adapt? When is the analysis done?
- Don’t use (implicit) $w = -1$ prior to constrain galaxy properties.

A warning from particle physics.

Credit: A. Roodman, R. Kessler, Particle Data Group.
Why Blind Analyses?

- Experimenter’s bias
  - choice of data samples + selections
  - choice of priors + evaluation of systematics
  - decision to stop work + publish
- Blind Analysis: Method to prevent experimenter’s bias
  - hide the answer
  - must be customize for measurement
Blind Analysis Strategies

- **Encrypt** measurement
  - transform data, e.g. add non-changing random number
  - DO NOT blind how result changes due to changes in analysis
  - DO NOT blind calibration data

- **Hide** signal region

- **Inject** unknown fraction of simulated results

- **Define** checks to do after unblinding beforehand
Blind Analysis Strategies for Cosmology

- **Encrypt** measurement shear catalogs, mass calibration
  - transform data, e.g. multiply by non-changing random number
  - DO NOT blind how result changes due to changes in analysis
  - DO NOT blind calibration data

- **Hide** signal region only possible for BAO?

- **Inject** unknown fraction of simulated results
  - events (SN), offsets (all measurements)

- **Define** checks to do after unblinding beforehand
  - staged unblinding: non-cosmo parameters first
Blind Analysis Strategies in DES (preliminary)

Two-stage process

**measurement** (correlation & mass functions)

- shear catalog blinded; cluster calibration under debate;
- transform correlation functions

\[ \hat{w}(\theta) \rightarrow \hat{w}(\theta) + \frac{\partial w}{\partial \Omega_m} \Delta \Omega_m \]

- still defining null-test, ‘allowed’ plots for sample selection

**parameter estimation**

- off-set all parameter results by (constant) random numbers
- needed: decisions on models to run, model selection criteria

Criteria for publication (???)
Joint Analysis Work Plan

Data, Model, Priors
- Observations
- Theory
- Simulations

Likelihood Analysis
- Single Probe Analyses
- Combined Probes Analysis

Forecast to Prioritize Systematics

Precision

Consistency

Accuracy

Parameter Constraints

Blinding
A Second Cosmology Pie Chart

Cosmology Parameters

- 70%
- 25%
- 5%

Sample Cut Parameters

- 25%

“Systematics Parameters”

- observational systematics
- survey specific
- astrophysical systematics
- observable + survey specific
Cosmology Parameters

Sample Cut Parameters

“Systematics Parameters”
- observational systematics
- survey specific
- astrophysical systematics
- observable + survey specific

Sample cuts + systematics highly interconnected
⇒ 95% systematics…
Statistical Challenges in Cosmology

foreground removal
star, galaxy classification
galaxy shape estimation
galaxy cluster finding
SN detection + light curve fitting
...

reduce data
make maps & catalogs

choice of summary statistics
emulation of N-body simulations
estimators for large covariances
choice of likelihood
samplers & likelihood estimation
systematics marginalization
model selection
design of blind analyses
...

Olivier Doré, WFIRST Science, Kissimmee, January 5th 2016

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and many more - join the Cosmology Working Group!