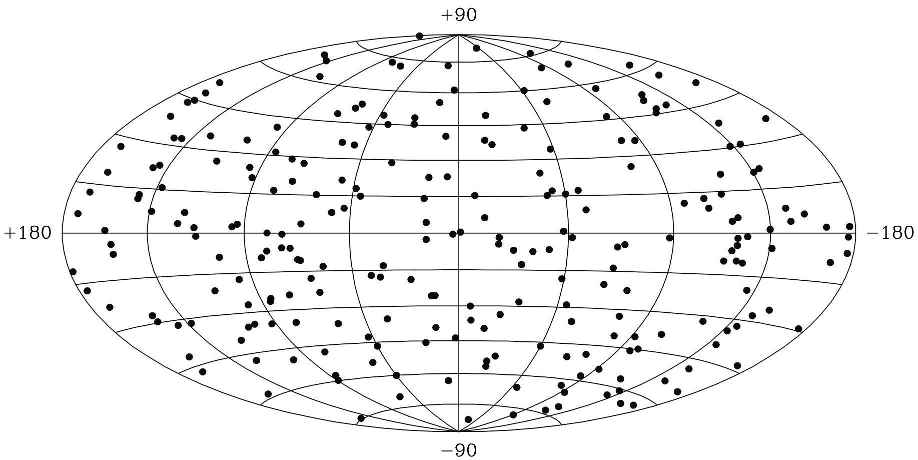
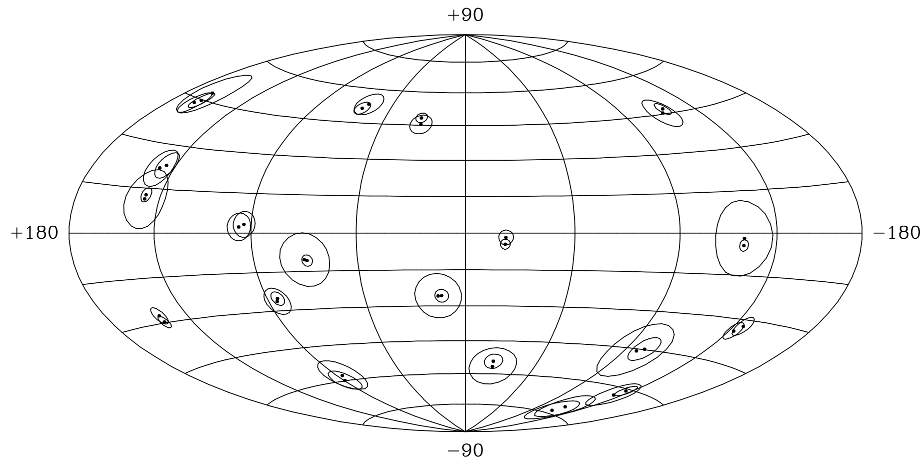
Spatio-Temporal Coincidences

Do GRB sources repeat?

250 GRB directions



Subset with neighbor within 3° (39)

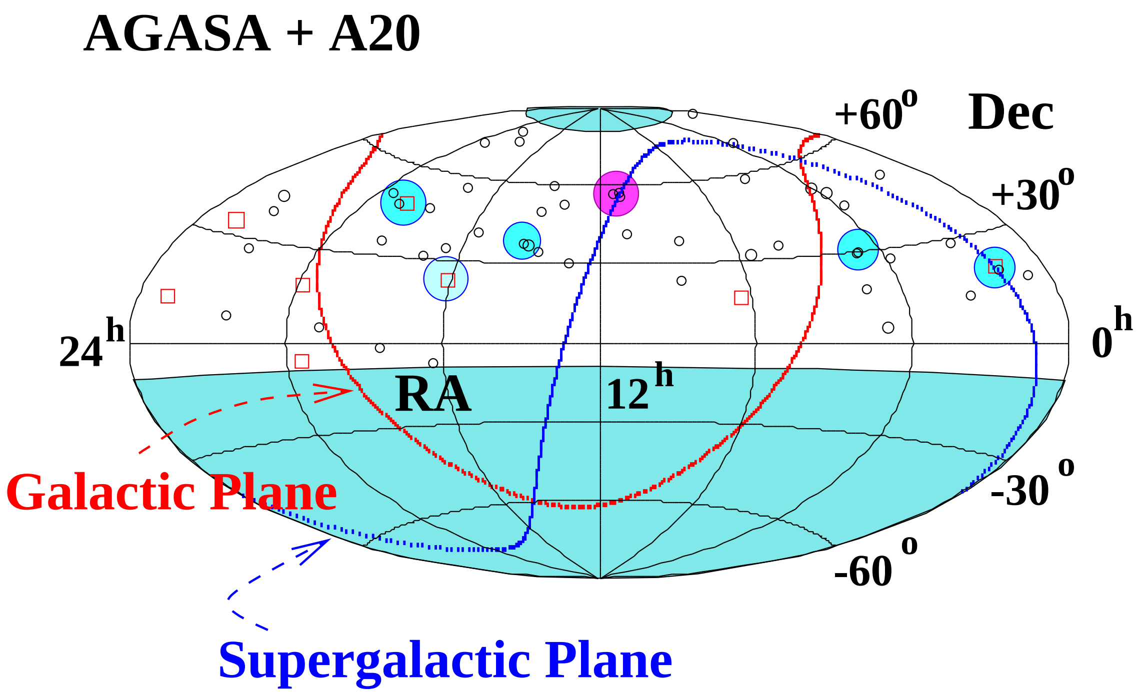


BATSE GRB directions have 5–25° uncertainties→ direction likelihood functions:

$$\mathcal{L}_i(\mathbf{n}) = p(D_i|\mathbf{n}, M)$$

Coincidences Among UHE Cosmic Rays?

AGASA data above GZK cutoff (Hayashida et al. 2000)



- 58 events with $E > 4 \times 10^{19}$ eV
- Energy-dependent direction uncertainty $\sim 2^\circ$
- Significance test — Search for coincidences $< 2.5^\circ$:
  - 6 pairs; $\lesssim 1$% significance
  - 1 triplet; $\lesssim 1$% significance

Frequentist nearest neighbor analysis—two objects:

Null hypothesis H_0: no repetition, isotropic source dist'n

Statistic: Angle to nearest neighbor, θ_{12}

Sampling Dist'n:

$$\begin{aligned} p(\cos\theta_{12}, \phi_{12}) &= \frac{1}{4\pi}, \\ \rightarrow p(\theta_{12}) &= \frac{\sin\theta_{12}}{2} \\ p(<\theta_{12}) &= \frac{1-\cos\theta_{12}}{2} \end{aligned}$$

independent of uncertainty

Reject H_0 if this probability is small; e.g.:

- $\theta_{12} = 26^\circ \quad \rightarrow \quad p(< 26^\circ) = 0.05$
- $\theta_{12} = 0^\circ \quad \rightarrow \quad p(< 0^\circ) = 0$

Bayesian coincidence assessment—two objects:

Direction uncertainties accounted for via likelihoods for object directions:

$$\mathcal{L}_i(\mathbf{n}) = p(d_i|\mathbf{n}), \quad \text{normalized w.r.t. } \mathbf{n}$$

H_0: No repetition

$$\begin{aligned}
p(d_1, d_2|H_0) &= \int d\mathbf{n}_1\, p(\mathbf{n}_1|H_0)\, \mathcal{L}_1(\mathbf{n}_1) \quad \times \int d\mathbf{n}_2 \cdots \\
&= \frac{1}{4\pi} \int d\mathbf{n}_1\, \mathcal{L}_1(\mathbf{n}_1) \quad \times \frac{1}{4\pi} \int d\mathbf{n}_2 \cdots \\
&= \frac{1}{(4\pi)^2}
\end{aligned}$$

H_1: Repeating (same direction!)

$$p(d_1, d_2|H_0) \;=\; \int d\mathbf{n}\; p(\mathbf{n}|H_0)\, \mathcal{L}_1(\mathbf{n})\, \mathcal{L}_2(\mathbf{n})$$

Odds favoring repetition:

$$\begin{aligned} O &= 4\pi \int d\mathbf{n}\; \mathcal{L}_1(\mathbf{n})\, \mathcal{L}_2(\mathbf{n}) \\ &\approx \frac{2C}{\sigma_{12}^2} \exp\left[-\frac{C\theta_{12}^2}{2\sigma_{12}^2}\right]; \quad \sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 \end{aligned}$$

E.g.: $\sigma_1 = \sigma_2 = 10^\circ$ $O \approx 1.5$ for $\theta_{12} = 26^\circ$

$O \approx 75$ for $\theta_{12} = 0^\circ$

$\sigma_1 = \sigma_2 = 25^\circ$ $O \approx 7$ for $\theta_{12} = 26^\circ$

$O \approx 12$ for $\theta_{12} = 0^\circ$

The Fisher Distribution

"Gaussian on the sky"

Let $\hat{\mathbf{n}}$ be the best-fit direction. For azimuthally-symmetric uncertainties, use:

$$\mathcal{L}(\mathbf{n}) = \frac{\kappa}{4\pi \sinh \kappa} e^{\kappa \mathbf{n} \cdot \hat{\mathbf{n}}}$$

$\kappa =$ concentration parameter. For small uncertainties,

$$\kappa \approx \frac{C}{\sigma^2}, \qquad\qquad C \approx 2.3$$

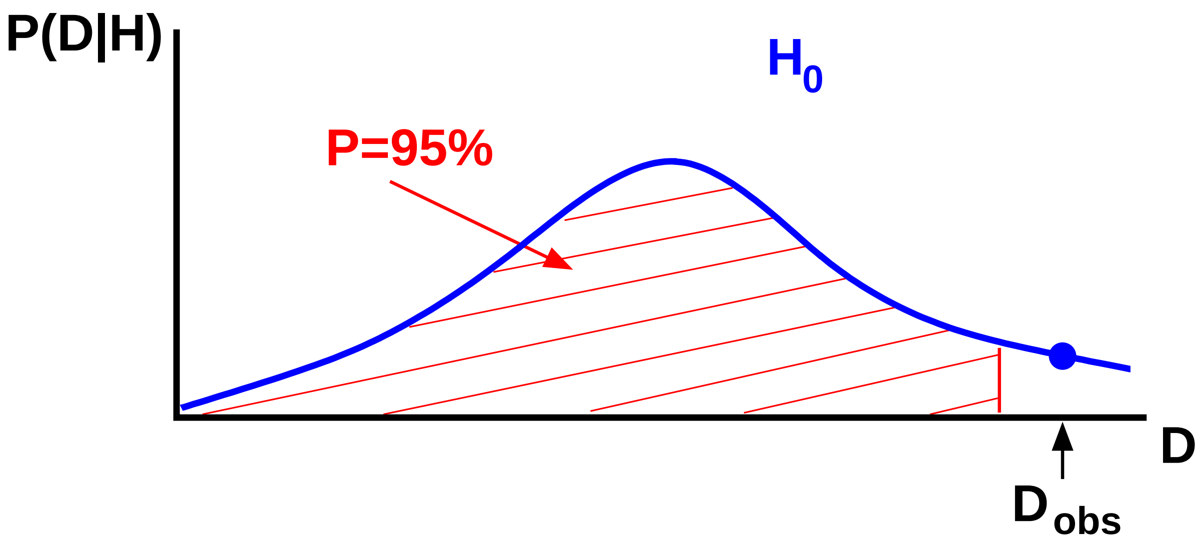
If $\mathbf{n}$ is near $\hat{\mathbf{n}}$ (angle θ)

$$\mathcal{L}(\mathbf{n}) \sim \exp\left[-\frac{C\theta^2}{2\sigma^2}\right]$$

Compare or Reject Hypotheses?

Frequentist Significance Testing (G.O.F. tests):

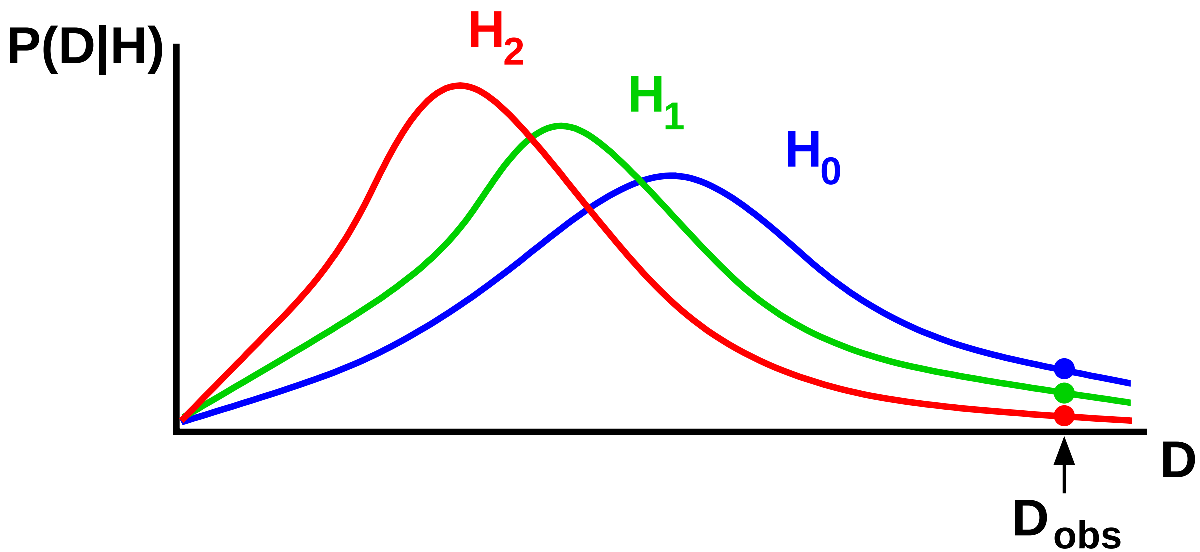
- Specify simple null hypothesis H_0 such that rejecting it implies an interesting effect is present
- Divide sample space into probable and improbable parts (for H_0)
- If $D_{\rm obs}$ lies in improbable region, reject H_0; otherwise accept it



Compare or Reject Hypotheses?

Bayesian Model Comparison:

- Favor the hypothesis that makes the observed data most probable (up to a prior factor)



If the data are improbable under H_0, the hypothesis *may* be wrong, *or* a rare event may have occured. GOF tests reject the latter possibility at the outset.

Challenge: Large hypothesis spaces

For $N = 2$ events, there was a single coincidence hypothesis, M_1 above.

For $N = 3$ events:

- Three doublets: $1 + 2$, $1 + 3$, or $2 + 3$
- One triplet

For N events, # of hypotheses with n_k k-tuplets (n_2 doublets, n_3 triplets...)

$$\mathcal{N} = \frac{N!}{\prod_{k=1}^{K} (k!)^{n_k} n_k!}$$

E.g. for $n_2 = 2$, $\mathcal{N} \approx N^4/8$.

Bayesian Analysis of AGASA Cosmic Rays

M_0: $N = 58$ different directions

M_1: Unknown number of pairs (n_2) and triplets (n_3)

$\rightarrow O_{10} = 1.4$ favoring clusters (i.e., no significant evidence)

If indeed clusters are present, we can constrain the number by calculating $p(n_2, n_3|D, M_1)$: