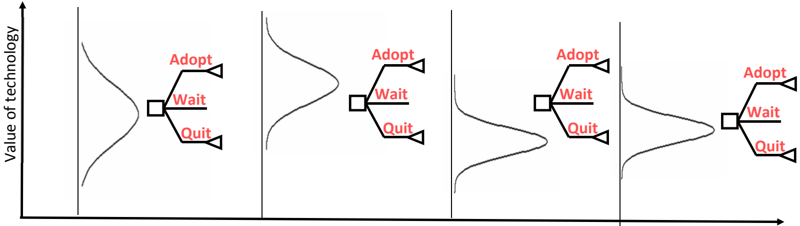
Risk Aversion, Information Acquisition, and Technology Adoption

Canan Ulu
McDonough School of Business
Georgetown University

Jim Smith
Fuqua School of Business
Duke University

SAMSI Games and Decisions in Reliability and Risk Workshop
May 2016

Problem: Should Jim buy a Tesla? Or should he wait and learn more?



Hear about it

See 99 rating in consumer reports

Read about cars catching fire after a crash

103 rating in consumer reports


Tesla's New Car Is So Good, It Literally Broke the Consumer Reports Scale

Other examples: farmer planting a new variety of soybean, utility building a power plant based on a new technology, or doctors changing treatments

We study a DP model of information acquisition in technology adoption decisions.

- We build on McCardle (1985) and Ulu and Smith (2009), adding *risk aversion*.
- In each period, the consumer can adopt the technology, gather information about the technology, or quit.

  State variables: probability distribution on technology benefits
  wealth

  Beliefs are updated over time using Bayes' Rule.

  Arbitrary distributions are allowed.

  Information gathering is costly.

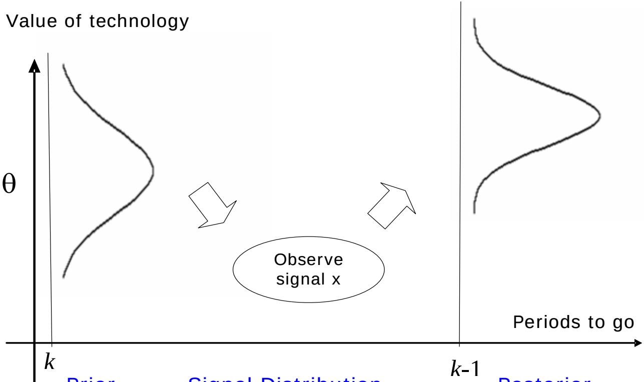
- We focus on structural properties of the model:

  Properties of the value function (increasing, convex, ...)

  Monotonicity properties of the optimal policies

  Effects of risk aversion

Modeling learning: Notation

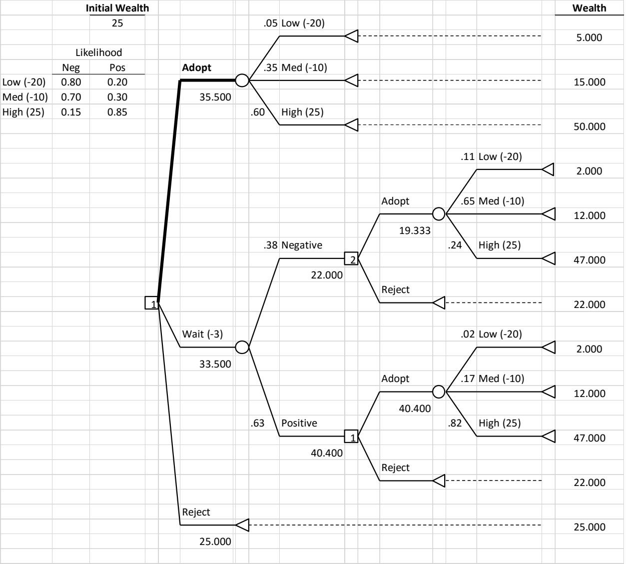


Prior | Signal Distribution | Posterior

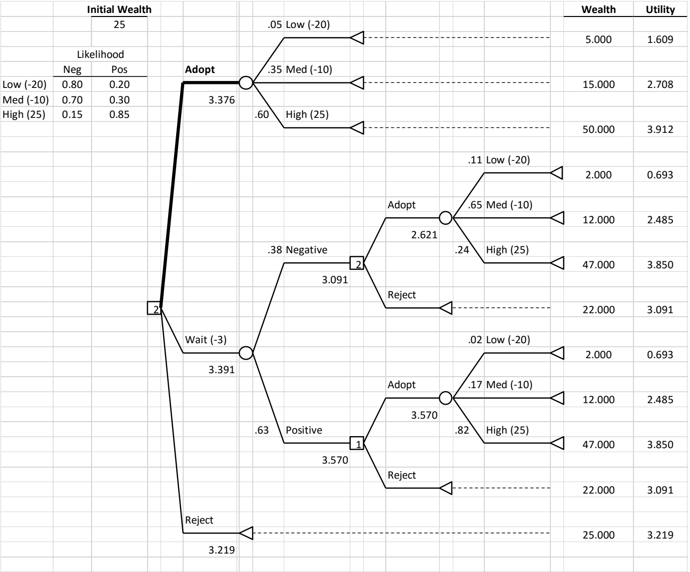
$(\ )$ $\qquad f(x;\ ) = \int_\theta L(x|\ )\ (\ )d$ $\qquad (\ ;\ ;x) = \frac{L(x|\theta)\pi(\theta)}{f(x;\pi)}$

$f(\ )$ $\qquad (\ ;x)$

Decision Tree Example: Risk Neutral



Decision Tree Example: Risk Averse



The value function:

- c = cost of waiting ($c > 0$)
- $u(w)$ = DM's utility for wealth w
- Value (or derived utility) function with k periods remaining:

$$U_0(w;\quad) = u(w)$$

$$U_k(w;\quad) = \max \begin{cases} E[u(w + \tilde{\ })\,|\quad] & \text{(adopt)} \\ E[U_{k-1}(w - c;\ (\quad;\ \tilde{x}))\,|\,f(\quad)] & \text{(wait)} \\ u(w) & \text{(quit)} \end{cases}$$

where

$$E[u(w + \tilde{\ })\,|\quad] = \int u(w + \quad)\ (\quad)\, d$$

$$E[U_{k-1}(w;\ (\quad;\ \tilde{x}))\,|\,f(\quad)] = \int_x U_{k-1}(w;\ (\quad; x))\, f(x;\quad)\, dx$$

Illustrative example: beta-Bernoulli model

- = p 0:5 where (p) / p(1)(1 p)(1) with p 2 [0; 1]
- Expected bene t (E[ ]) = = (+) 0:5; \precision" = (+)
- Signals are + or with probability p or (1 p). Precision increases by one each period. Given prior with (;),

  + signal =) (+ 1 ;)

  signal =) (; + 1)
- Example: Start with (;) = (2 :25; 1:75), observe (; + ; ; + ; ;):

Illustrative example: risk-neutral results

- $u(w) = w$; initial wealth = 1.06; $c = 0.01$; long time horizon

Policy regions

Value function with + = 10

Illustrative example: risk-averse results

- $u(w) = 1{:}2 \quad 0{:}2w^{(1 \quad)}$ where $= 6$; initial wealth = 1.06; $c = 0{:}01$

Policy regions

Value function with + = 10

General results: De ning \better" priors

- De nition: $_2$ likelihood-ratio (LR) dominates $_1$ ($_2$ $_{LR}$ $_1$) if $_2$( )= $_1$( ) is increasing in .
- Examples of LR improvements:

  Beta: Increasing while holding the precision (+) constant
  Normal: Increasing the mean while holding the variance constant

- LR-dominance implies FOSD, but the reverse is not true.

Not a LR-improvement

A LR-improvement

- The LR-order survives Bayesian updating: given a signal x

$$_2 \quad _{LR} \quad _1 , \quad (\quad _2 ; x) \quad _{LR} \quad (\quad _1 ; x); \text{ for all } x \; 2 \; X:$$

General results: Ordered signal processes

- De nition: The signal process $L(xj\ )$ satis es the monotone-likelihood-ratio (MLR) property if the signal space X is ordered and

$$L(xj\ _2)\quad _{LR}\ L(xj\ _1) \text{ for all } \ _2 \quad _1 :$$

- Examples: Bernoulli signals; normal signals

- If the signal process satis es the MLR property, then:

$$_2 \quad _{LR} \quad _1 \ ) \quad f(\ _2) \quad _{LR}\ f(\ _1)$$

For any prior , $x_2 \quad x_1$, $(\ ; x_2) \quad _{LR} \quad (\ ; x_1)$:

General results: Increasing value functions

- De nition: $V(\ )$ is LR-increasing if $V(\ _2)$ $V(\ _1)$ whenever $_2$ $_{LR}$ $_1$.

- Proposition [Increasing]: Suppose the DM's utility function $u(w)$ is increasing in w and the signal process satis es the MLR property. Then, for all k and w, the value function $U_k(w;\ )$ is LR-increasing in .

General results: Increasing value functions

- Value (or derived utility) function with k periods remaining:

$$U_0(w;\ ) = u(w)$$

$$U_k(w;\ ) = \max \begin{cases} E[u(w + \tilde{\ }) \mid\ ] & \text{(adopt)} \\ E[U_{k\ 1}(w\ \ c; (\ ; \tilde{x})) \mid f(\ )] & \text{(wait)} \\ u(w) & \text{(quit)} \end{cases}$$

Value function with + = 10

General results: Increasing policies

- Proposition: Suppose the DM's utility function is increasing and the signal process satis es the MLR property.

  Rejection: If it is optimal to reject with prior $_2$, it is also optimal to reject with any prior $_1$ such that $_2$ $_{LR}$ $_1$.

  Proof: Follows from LR-increasing value functions.

  Adoption : Suppose the DM is risk neutral (or risk seeking). If it is optimal to adopt with prior $_1$, then it is also optimal to adopt with any prior $_2$ such that $_2$ $_{LR}$ $_1$.

  Proof: Utility di erence between adoption and waiting is LR-increasing.

- With risk neutrality, policies \increase" from quit to wait to adopt as LR-improves.

Illustrative example revisited

- Policies are LR-increasing.

Policy regions

Value function with + = 10

Illustrative example revisited

- Policies are LR-increasing.

Policy regions

Value function with + = 10

Illustrative example revisited

Risk neutral

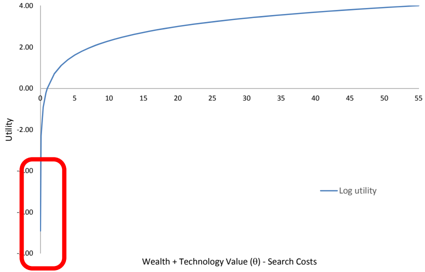
Risk averse

- With risk aversion, can adopt and wait cross twice as we LR-improve ?

If the DM is risk averse, adoption policies may not be monotonic in .

- Example: log utility; three technology values; signals satisfy MLR property

Comparing waiting and adopting in this example:



- With log utility, a bad technology outcome + search costs can be catastrophic if the resulting wealth level is near zero.

Search costs push the DM "over the edge"

Can we ensure monotonicity by limiting the degree of risk aversion?

Increasing Adoption Policies

Proposition Suppose the DM is risk averse and her utility function u exhibits decreasing absolute risk aversion (DARA), i.e., her risk tolerance $\ _u(w)$ is increasing. Then, if

$$_u(w_0 + \_ \quad c) \qquad \_$$

where $w_0 = w \quad kc$ and $\_ = \min$ (“Not too risk averse”), then adoption policies are monotonic.
If u is CARA, no risk tolerance bound is required.

- We define a new property: “sLR-increasing”:

  LR-increasing functions are sLR-increasing

  sLR-increasing functions are single-crossing

  Bayesian updating preserves sLR-increasing property

- We show utility difference between adoption and waiting is sLR-increasing

  Then, utility difference between adoption and waiting is single crossing.

Summary:

- With a DARA utility function that is "not too risk averse," we have:

- The same structural properties as in the risk-neutral model (*existence of thresholds, etc.*), but risk aversion leads to quitting sooner and adopting later (*if CARA*).

Generalizations:

- Model with discounting, $u(w + \text{NPV of costs/benefits})$

  Delay is costly; also risk reducing

  Results and proofs follow the same pattern

- Other applications of s-increasing: Monotonic policies in DPs

  Submodularity (increasing differences) conditions (Topkis (1979), Lovejoy (1987a,b)) are sometimes hard to establish

  Single-crossing conditions (e.g., Milgrom and Shannon (1994), Quah and Strulovici (2012),. . .) are hard to use in DPs