Elicitation of an Individual's Risk Tolerance: Theory and some Experimental Evidence

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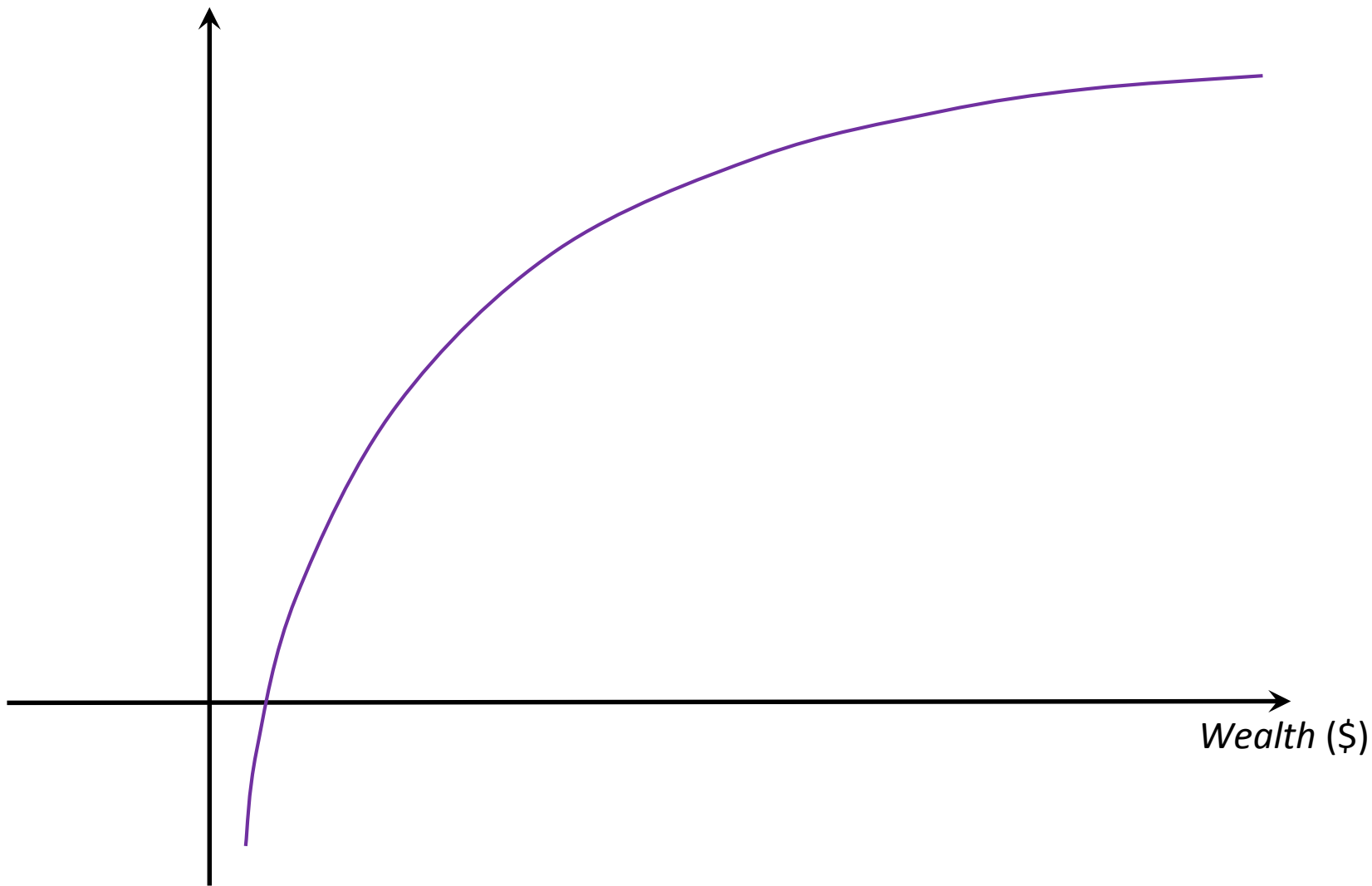
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" Mathematicians evaluate money in proportion to its quantity, while people with common sense in proportion to the usage that they can make of it."

Gabriel Cramer (1704-1752)

Swiss mathematician

(Correspondence with Nicolas Bernoulli, 1728)

u(*Wealth*)

Wealth ($)

4

Risk Tolerance of the Exponential Utility function

Exponential utility $u(x) = 1 - \mathrm{e}^{-x/\rho}$

$\rho > 0$ the **Risk Tolerance** coefficient, a constant

ρ in same units as x (i.e., often monetary units)

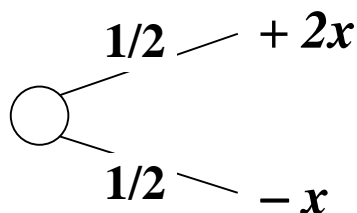Lower ρ → higher risk aversion (lower tolerance to risk)

The most used utility function in the world!

(cf. Corner & Corner 1995, Walls & Clyman 1999, Kirkwood 2004)

Simple questions to assess the Risk Tolerance coefficient, ρ

❑ **The 50-50 gamble**

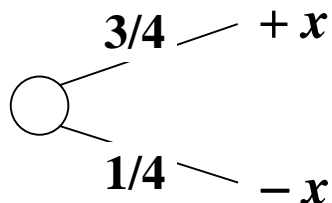What is the largest stake, x_{max}, for which you will accept:

$$\overset{1/2}{\diagup} + 2x$$
$$\underset{1/2}{\diagdown} - x$$

yields: $\boxed{\rho \approx 2x_{max}}$

❑ **The 75-25 gamble**

What is the largest stake, x_{max}, for which you will accept:

$$\overset{3/4}{\diagup} + x$$
$$\underset{1/4}{\diagdown} - x$$

yields: $\boxed{\rho \approx x_{max}}$

Utility for financial payoffs: <u>total</u> wealth vs. <u>change</u> in wealth

Expected Utility theory

Utility

$w_1 \longleftrightarrow w_2$ *Total Wealth*

0 := no wealth, extreme poverty
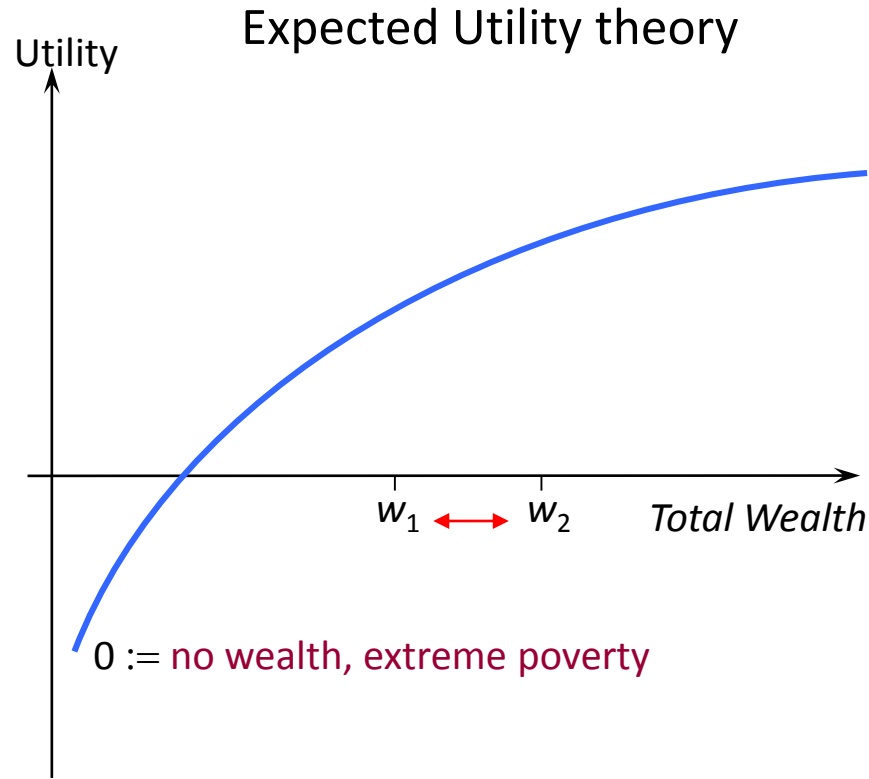
Carrier of value: **total** wealth

$u(w)$

Prospect theory (K&T '79)

Value

Gains

Losses   *Change in Wealth*

0 := no change
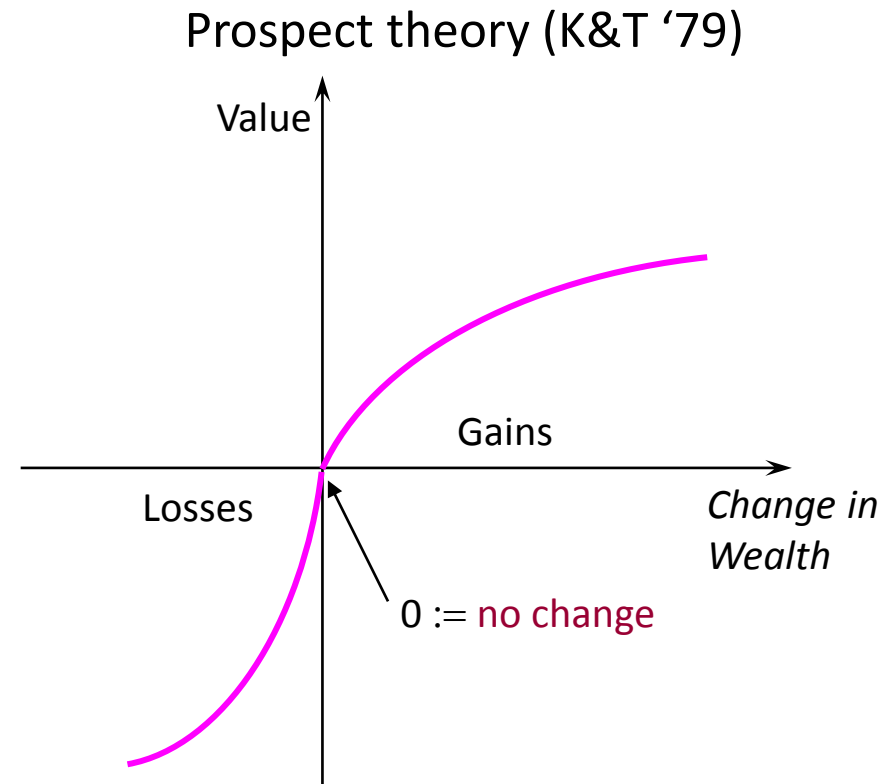
Carrier of value: **change** in wealth

$v(x), \; x = \delta w$

What are we measuring? $u(w), v(x),$ a mix of the two?...

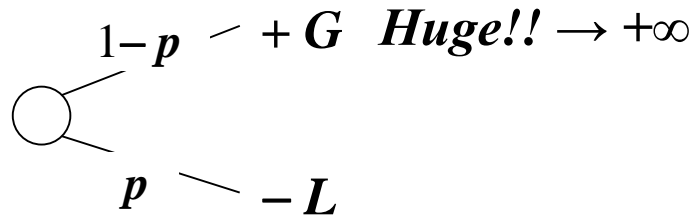# Risk Tolerance as Maximum Acceptable Loss

❑ **The "big" gamble:**

$$1-p \quad \diagup \quad +G \quad \textit{Huge!!} \rightarrow +\infty$$

$$p \quad \diagdown \quad -L$$

For what loss, L, does this gamble become unacceptable?

$$L : u(\text{Accept}) = u(\text{Reject}) \qquad \textbf{yields:} \qquad \boxed{\rho = -L/\ln(p)}$$

Risk Tolerance as Maximum Acceptable Loss

Maximum tolerable loss as a function of probability of loss:

Max acceptable loss (in multiples of RT)

Region of **unacceptable** gambles

Probability of Loss
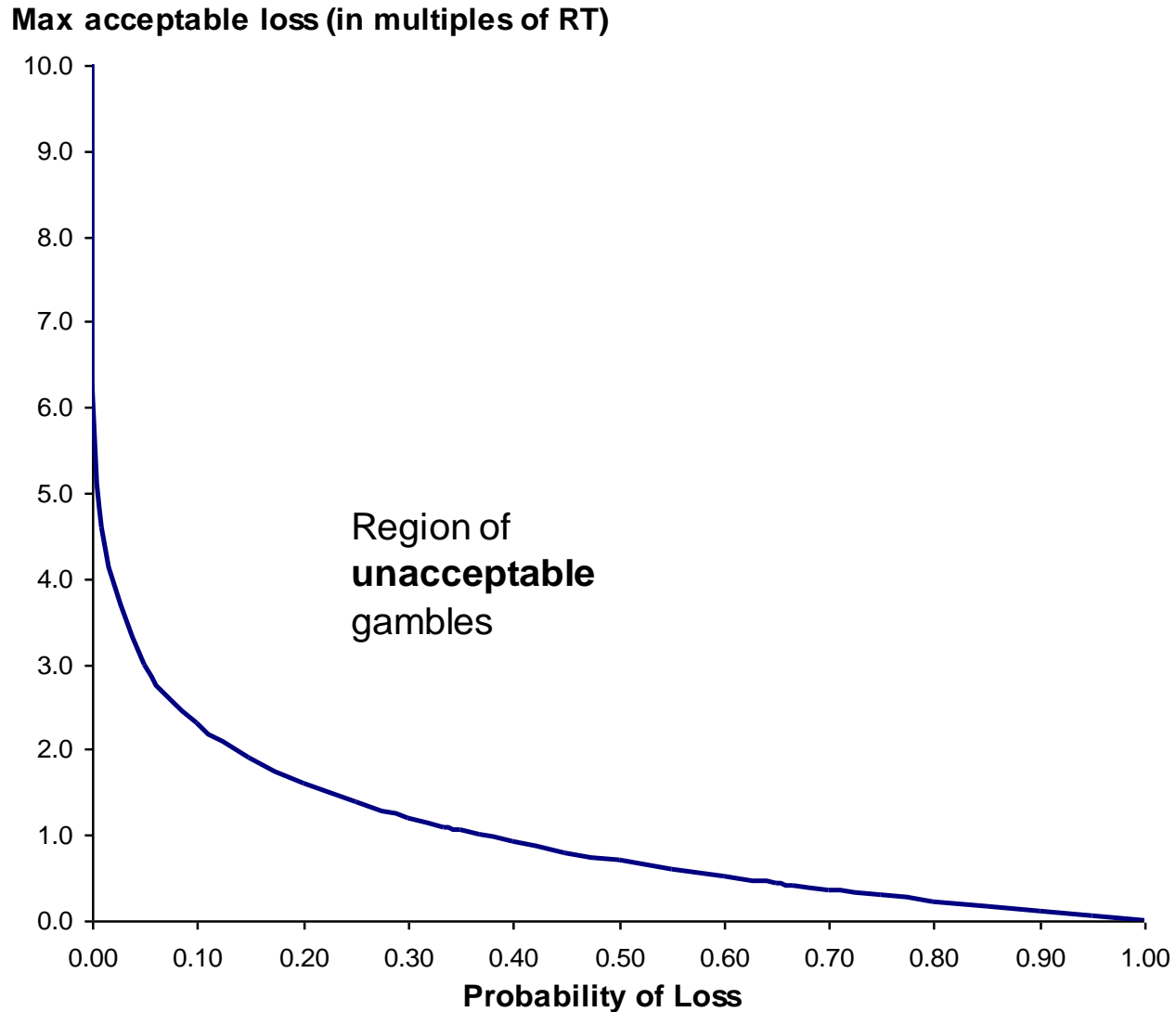
Risk Tolerance as Maximum Acceptable Loss

Examples

You have the opportunity to invest in a venture with "sky-is-the-limit" upside and odds 1-to-2 of losing your investment. You indicate that the maximum you would be willing to invest is X. Then your Risk Tolerance is approximately X.

You are managing a portfolio of projects which has a 1 in 20 chance of realizing a loss. You indicate that L is the maximum loss you could stand, no matter what the upside is, then your Risk Tolerance is L/3.

A company sets a Value-at-Risk at the 0.05-level equal to $VaR_{0.05}$.
Then its Risk Tolerance might be: $\rho = 1/3 \times VaR_{0.05}$

Assessing RT: Indifference vs. Most Preferred

- ***Typically preference assessment seeks to elicit <u>Indifference</u>, i.e., <u>"null" preference</u>***

    - What value makes you indifferent? What is your limit?
      What is your break-even value?...

- ***Alternatively, could ask about <u>Optimal preference</u>***

    - What is your ideal risk-return profile?

    - What is your most preferred stake in this gamble?

Most preferred vs. Maximum investment for the 50-50 Gamble (under exponential utility)
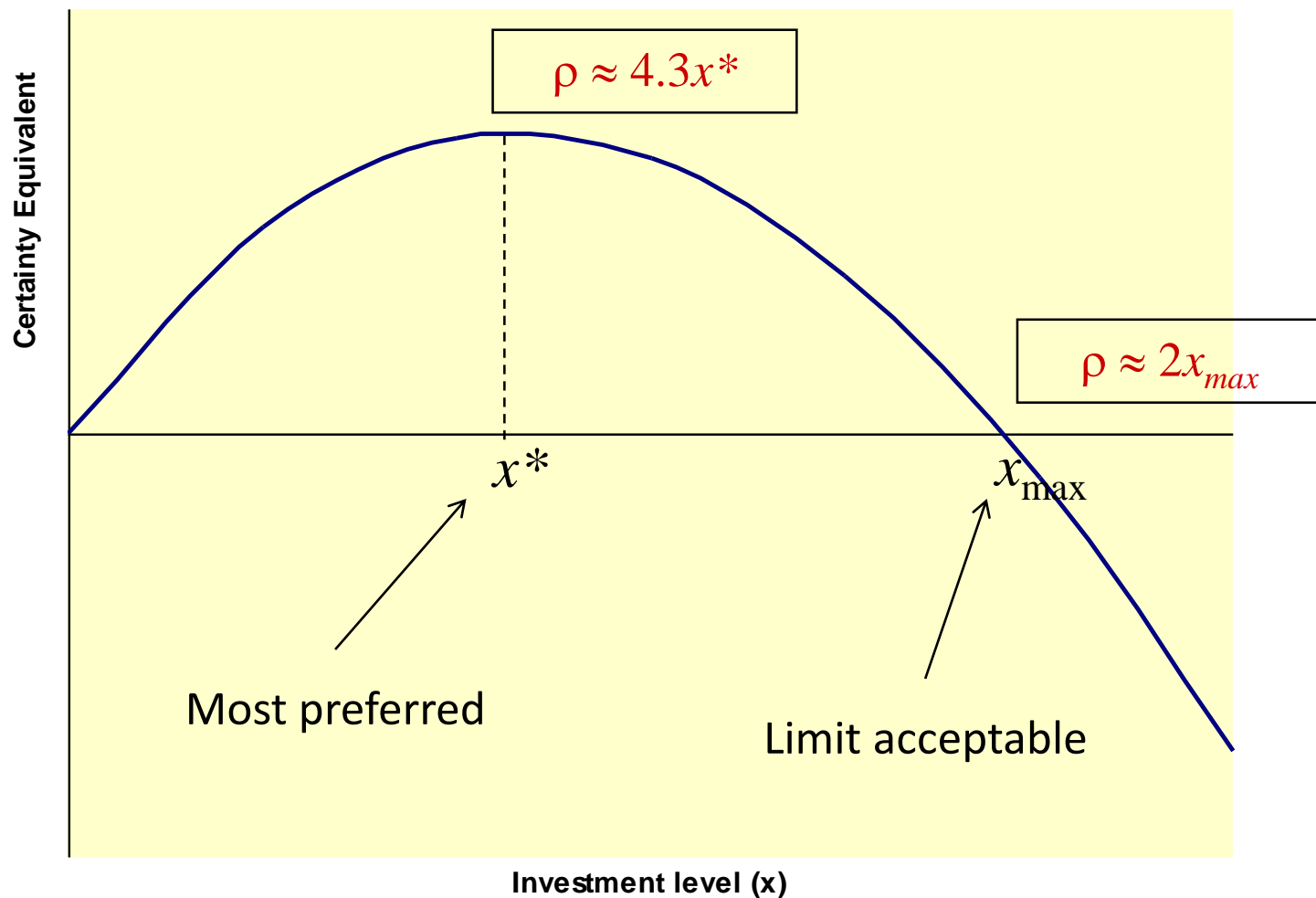
Surveys Set-up

- Short questionnaire administered in 15-minute session

- Random assignment to conditions

- Instructions provided

- Part of a general questionnaire on judgment and decision making

- Results debriefed during session

- Participants:

       Business Executives

       MBA and EMBA students

Scenario for **Big Gamble question**

You have the opportunity to invest in a start-up venture. You are well acquainted with the project, and know that it will be executed competently. The venture has a tremendous upside, offering the possibility of creating immense wealth, but it also involves a sizeable probability of failure. If the venture is successful, your net financial payoff (after subtracting your investment) will be more than one hundred times your initial investment within about four years from now. If it fails, all of your initial investment will be lost.

Based on the best expert assessment, the start-up has:

- a 2/3 chance of success, in which case your net discounted gain will be in excess of 100 times the amount you invested;

- a 1/3 chance of failure, in which case your net loss will be equal to the amount you invested.

You regard the above summary assessment as accurate and realistic, that is, neither optimistic, nor pessimistic.

Similar scenario for 50-50 Gamble and 75-25 Gamble

Assessment Questions

Maximum amount you would invest?

What is the _largest_ amount of money that you would invest in this venture? This is the amount such that any lower investment level would be acceptable to you, and any higher level would be unacceptable.

Think through a wide range of values, high and low, in order to pinpoint your _highest_ investment level, and fill in your answer below:

_I would be willing to invest up to, but no more than, $ _____ in this venture._

Favorite amount you would invest?

What is the amount of money that you would _most prefer_ to invest in this venture? This is the amount that produces the best balance for you between a large enough upside and risk exposure. Any other investment level, higher or lower, would be less attractive to you.

Think through a wide range of values, high and low, in order to pinpoint your _most desirable_ investment level, and fill in your answer below:

_I would most prefer to invest $ _____ in this venture._

Results: Big Gamble vs. 50-50 Gamble

Question: Maximum amount you would invest?

Summary measures of RT (in $)

	Big 2/3-1/3 Gamble N = 16	Standard 50-50 gamble N = 14
Mean RT	619,063	52,769
Median RT	150,000	20,000
Stdev	1,274,543	59,327

→ RT much larger with Big Gamble approach

Results: Big Gamble vs. 50-50 Gamble

Question: Maximum amount you would invest?

Summary measures of RT (in $)

	Big 2/3-1/3 Gamble N = 16	Standard 50-50 gamble N = 15
Mean RT	226,563	58,997
Median	100,000	52,000

→ RT much larger with Big Gamble approach

Results: 75-25 Gamble vs. 50-50 Gamble vs. Big Gamble

Elicitation Question: **Maximum amount you would invest?**

N=106 Full-time MBA students

Summary measures of RT (in $)

	75-25 Gamble $\{+X, 3/4; -X, 1/4\}$	50-50 Gamble $\{+2X, 1/2; -X, 1/2\}$	Big Gamble $\{+\text{Infty}, 2/3; -L, 1/3\}$
Mean RT	22,013	44,879	151,712
Median RT	9,102	13,520	45,099
Std Dev	27,985	96,055	242,809
N	37	36	33

Results: Most desirable vs. Highest stake

Big Gamble $\{+\text{Infty}, 2/3; -L, 1/3\}$

Summary measures in $

N = 25	Most preferred investment	Highest investment
Mean	**42,438**	**15,992**
Median	**15,000**	**7,500**
Std Dev	**54,934**	**25,020**
N	12	13

→ Most Preferred stake > Highest Acceptable stake !!

Results: Most desirable vs. Highest stake

50-50 Gamble $\{+2X, 1/2; -X, 1/2\}$

Summary measures in $

	Most preferred	Highest
Mean	**17,705**	**8,301**
Median	**3,000**	**7,500**
N	15	15

→ Most Preferred stake > Highest Acceptable stake !

Remember:

- multiply "Highest" by 2.08 to get RT coefficient

- multiply "Most preferred" by 4.3 to get RT coefficient

Results: Most desirable vs. Highest stake

N=21 Executive MBAs

50-50 Gamble {+2X, 1/2; -X, 1/2}

Summary measures in USD

	Most preferred	**Highest**
Mean stake	**19,182**	**23,000**
Median stake	**10,000**	**15,000**
Std Dev	**21,222**	**19,889**
N	11	10

Remember:

- multiply "Highest" by 2.08 to get RT coefficient

- multiply "Most preferred" by 4.3 to get RT coefficient

Results: Most desirable and Highest (Within Subject)

50-50 Gamble {+2X, 1/2; -X, 1/2}

Elicit both favorite and highest amounts (order counterbalanced)

Summary measures in $

N = 39	Most preferred	Highest	Difference
Mean	30,041	49,051	18,535
Median	10,000	20,000	3,000
Std Dev	44001	77819	50679

Difference = Highest – Most preferred > 0 (*p-value* < 0.03)

Summary of Empirical Results

■ ***Big gamble vs. "small" gamble questions***

+ Big gamble question elicits higher risk tolerance

+ May naturally induce decision maker to think in terms of <u>total wealth</u>

+ Domain of assessment <u>encompasses domain of use</u>

 (better to extrapolate from large to small risks, than the reverse)

− Thinking about extreme outcomes is a stretch, of course…

■ ***Maximal preference vs. Null preference questions***

− Optimal investment > Maximum acceptable investment !!

+ "Most desired level" question elicits higher risk tolerance

+ May provide more realistic estimates

 (combating exaggerated risk aversion (Rabin 2000, Smith 2004)