Elicitation of an Individual’s Risk Tolerance: Theory and some Experimental Evidence

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“Mathematicians evaluate money in proportion to its quantity, while people with common sense in proportion to the usage that they can make of it.”

Gabriel Cramer (1704-1752)
Swiss mathematician
(Correspondence with Nicolas Bernoulli, 1728)
$u(\text{Wealth})$
Risk Tolerance of the Exponential Utility function

**Exponential utility**  \[ u(x) = 1 - e^{-x/\rho} \]

- \( \rho > 0 \) the **Risk Tolerance** coefficient, a constant
- \( \rho \) in same units as \( x \) (i.e., often monetary units)
- Lower \( \rho \) \( \rightarrow \) higher risk aversion (lower tolerance to risk)

**The most used utility function in the world!**

Simple questions to assess the Risk Tolerance coefficient, $\rho$

- **The 50-50 gamble**
  What is the largest stake, $x_{\text{max}}$, for which you will accept:
  \[
  \frac{1}{2} + 2x - \frac{1}{2} - x
  \]
  yields: $\rho \approx 2x_{\text{max}}$

- **The 75-25 gamble**
  What is the largest stake, $x_{\text{max}}$, for which you will accept:
  \[
  \frac{3}{4} + x - \frac{1}{4} - x
  \]
  yields: $\rho \approx x_{\text{max}}$
Utility for financial payoffs: total wealth vs. change in wealth

Expected Utility theory

Prospect theory (K&T ’79)

Carrier of value: total wealth

\[ u(w) \]

Carrier of value: change in wealth

\[ v(x), \ x = \delta w \]

What are we measuring? \( u(w) \), \( v(x) \), a mix of the two?...
The “big” gamble:

\[ \begin{align*} 1 - p & \sim + G \\
\rho & \sim -L \end{align*} \]

For what loss, \( L \), does this gamble become unacceptable?

\[ L : u(\text{Accept}) = u(\text{Reject}) \quad \text{yields:} \quad \rho = -L/\ln(p) \]
Risk Tolerance as Maximum Acceptable Loss

Maximum tolerable loss as a function of probability of loss:
Examples

You have the opportunity to invest in a venture with “sky-is-the-limit” upside and odds 1-to-2 of losing your investment. You indicate that the maximum you would be willing to invest is X. Then your Risk Tolerance is approximately X.

You are managing a portfolio of projects which has a 1 in 20 chance of realizing a loss. You indicate that L is the maximum loss you could stand, no matter what the upside is, then your Risk Tolerance is L/3.

A company sets a Value-at-Risk at the 0.05-level equal to $\text{VaR}_{0.05}$. Then its Risk Tolerance might be: $\rho = 1/3 \times \text{VaR}_{0.05}$
Assessing RT: Indifference vs. Most Preferred

- **Typically preference assessment seeks to elicit Indifference, i.e., “null” preference**
  
  - What value makes you indifferent? What is your limit?
    What is your break-even value?...

- **Alternatively, could ask about Optimal preference**
  
  - What is your ideal risk-return profile?
  
  - What is your most preferred stake in this gamble?
Most preferred vs. Maximum investment for the 50-50 Gamble (under exponential utility)

\[ \rho \approx 4.3x^* \]

\[ \rho \approx 2x_{\text{max}} \]

Certainty Equivalent vs. Investment level (x)

Most preferred

Limit acceptable

\( x^* \)

\( x_{\text{max}} \)
Surveys Set-up

- Short questionnaire administered in 15-minute session
- Random assignment to conditions
- Instructions provided
- Part of a general questionnaire on judgment and decision making
- Results debriefed during session
- Participants:
  - Business Executives
  - MBA and EMBA students
Scenario for Big Gamble question

You have the opportunity to invest in a start-up venture. You are well acquainted with the project, and know that it will be executed competently. The venture has a tremendous upside, offering the possibility of creating immense wealth, but it also involves a sizeable probability of failure. If the venture is successful, your net financial payoff (after subtracting your investment) will be more than one hundred times your initial investment within about four years from now. If it fails, all of your initial investment will be lost.

Based on the best expert assessment, the start-up has:

- a 2/3 chance of success, in which case your net discounted gain will be in excess of 100 times the amount you invested;

- a 1/3 chance of failure, in which case your net loss will be equal to the amount you invested.

You regard the above summary assessment as accurate and realistic, that is, neither optimistic, nor pessimistic.

Similar scenario for 50-50 Gamble and 75-25 Gamble
Assessment Questions

Maximum amount you would invest?

What is the largest amount of money that you would invest in this venture? This is the amount such that any lower investment level would be acceptable to you, and any higher level would be unacceptable.

Think through a wide range of values, high and low, in order to pinpoint your highest investment level, and fill in your answer below:

I would be willing to invest up to, but no more than, $______________ in this venture.

Favorite amount you would invest?

What is the amount of money that you would most prefer to invest in this venture? This is the amount that produces the best balance for you between a large enough upside and risk exposure. Any other investment level, higher or lower, would be less attractive to you.

Think through a wide range of values, high and low, in order to pinpoint your most desirable investment level, and fill in your answer below:

I would most prefer to invest $______________ in this venture.
## Results: Big Gamble vs. 50-50 Gamble

**Question:** Maximum amount you would invest?

*Summary measures of RT (in $)*

<table>
<thead>
<tr>
<th></th>
<th>Big 2/3-1/3 Gamble</th>
<th>Standard 50-50 Gamble</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N = 16</td>
<td>N = 14</td>
</tr>
<tr>
<td>Mean RT</td>
<td>619,063</td>
<td>52,769</td>
</tr>
<tr>
<td>Median RT</td>
<td>150,000</td>
<td>20,000</td>
</tr>
<tr>
<td>Stdev</td>
<td>1,274,543</td>
<td>59,327</td>
</tr>
</tbody>
</table>

→ RT much larger with Big Gamble approach
## Results: Big Gamble vs. 50-50 Gamble

**Question:** Maximum amount you would invest?

### Summary measures of RT (in $)

<table>
<thead>
<tr>
<th>Gamble</th>
<th>Mean RT</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big 2/3-1/3</td>
<td>226,563</td>
<td>100,000</td>
</tr>
<tr>
<td>N = 16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard 50-50</td>
<td>58,997</td>
<td>52,000</td>
</tr>
<tr>
<td>Gamble</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N = 15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

→ RT much larger with Big Gamble approach
Results: 75-25 Gamble vs. 50-50 Gamble vs. Big Gamble

Elicitation Question: Maximum amount you would invest?

N=106 Full-time MBA students

<table>
<thead>
<tr>
<th>Summary measures of RT (in $)</th>
<th>75-25 Gamble (+X, 3/4; -X, 1/4)</th>
<th>50-50 Gamble (+2X, 1/2; -X, 1/2)</th>
<th>Big Gamble (+\text{Infty}, 2/3; -L, 1/3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean RT</td>
<td>22,013</td>
<td>44,879</td>
<td>151,712</td>
</tr>
<tr>
<td>Median RT</td>
<td>9,102</td>
<td>13,520</td>
<td>45,099</td>
</tr>
<tr>
<td>Std Dev</td>
<td>27,985</td>
<td>96,055</td>
<td>242,809</td>
</tr>
<tr>
<td>N</td>
<td>37</td>
<td>36</td>
<td>33</td>
</tr>
</tbody>
</table>
Results: Most desirable vs. Highest stake

Big Gamble \{+\text{Infty}, \frac{2}{3}; -L, \frac{1}{3}\}

\textit{Summary measures in $}

<table>
<thead>
<tr>
<th></th>
<th>Most preferred investment</th>
<th>Highest investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>42,438</td>
<td>15,992</td>
</tr>
<tr>
<td>Median</td>
<td>15,000</td>
<td>7,500</td>
</tr>
<tr>
<td>Std Dev</td>
<td>54,934</td>
<td>25,020</td>
</tr>
<tr>
<td>N</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

$\rightarrow$ Most Preferred stake > Highest Acceptable stake !!
50-50 Gamble \{+2X, \frac{1}{2}; -X, \frac{1}{2}\}

**Summary measures in $**

<table>
<thead>
<tr>
<th></th>
<th>Most preferred</th>
<th>Highest</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>17,705</td>
<td>8,301</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>3,000</td>
<td>7,500</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

→ Most Preferred stake > Highest Acceptable stake!

**Remember:**
- multiply “Highest” by 2.08 to get RT coefficient
- multiply “Most preferred” by 4.3 to get RT coefficient
Results: Most desirable vs. Highest stake

N=21 Executive MBAs

50-50 Gamble \{+2X, 1/2; -X, 1/2\}

**Summary measures in USD**

<table>
<thead>
<tr>
<th></th>
<th>Most preferred</th>
<th>Highest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean stake</td>
<td>19,182</td>
<td>23,000</td>
</tr>
<tr>
<td>Median stake</td>
<td>10,000</td>
<td>15,000</td>
</tr>
<tr>
<td>Std Dev</td>
<td>21,222</td>
<td>19,889</td>
</tr>
<tr>
<td>N</td>
<td>11</td>
<td>10</td>
</tr>
</tbody>
</table>

Remember:

- multiply “Highest” by 2.08 to get RT coefficient
- multiply “Most preferred” by 4.3 to get RT coefficient
Results: Most desirable and Highest (Within Subject)

50-50 Gamble \{+2X, 1/2; -X, 1/2\}

Elicit both favorite and highest amounts (order counterbalanced)

Summary measures in $

<table>
<thead>
<tr>
<th></th>
<th>Most preferred</th>
<th>Highest</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>30,041</td>
<td>49,051</td>
<td>18,535</td>
</tr>
<tr>
<td>Median</td>
<td>10,000</td>
<td>20,000</td>
<td>3,000</td>
</tr>
<tr>
<td>Std Dev</td>
<td>44001</td>
<td>77819</td>
<td>50679</td>
</tr>
</tbody>
</table>

Difference = Highest – Most preferred > 0 (p-value < 0.03)
Summary of Empirical Results

- **Big gamble vs. “small” gamble questions**
  - Big gamble question elicits higher risk tolerance
  - May naturally induce decision maker to think in terms of **total wealth**
  - Domain of assessment **encompasses domain of use**
    - (better to extrapolate from large to small risks, than the reverse)
  - Thinking about extreme outcomes is a stretch, of course...

- **Maximal preference vs. Null preference questions**
  - Optimal investment > Maximum acceptable investment !!
  - “Most desired level” question elicits higher risk tolerance
  - May provide more realistic estimates
    - (combating exaggerated risk aversion (Rabin 2000, Smith 2004))