The Current State of Image Processing and Reconstruction with Future Directions

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Image Processing

On a single slice full $k$-space array.
Image Processing

We can stack freq. rows of reals over rows of imaginaries, make one IFT reconstruction matrix from the two, to get the rows of reals over rows of imaginaries.

\[ v = O_I O_R O_k f \]

\[ y_R = O_I \times \]

image processing

\[ y_I \]

\[ f_R \]

\[ f_I \]

k-space processing
Image Processing

\[ v = O_I O_R O_k f \]

**\( O_k \) - k-space Processing**
- Nyquist Ghost Correction
- Static B0 Field Correction
- Zero Fill Interpolation
- Non-Cartesian Interpolation
- Ramp Sampling Interpolation
- Homodyne Interpolation
- Apodization
- And many more…

**\( O_R \) - Image Reconstruction**
- 2D inverse Fourier transform
- In-Plane SENSE/GRAPPA
- Through-Plane SENSE
- And many more…

**\( O_I \) - Image Processing**
- Motion Correction
- Global Normalization
- Image Smoothing
- And many more…

**\( O_T \) - Time Series Processing**
- Dynamic B0 Correction
- Slice Timing
- Filtering/Smoothing
- Physiologic Regressors
- And many more…

Show dark blue.
Done dark teal.
Empirically dark red.
Not red.
Implications

In statistics, we know the rule that says:

If a vector \( f \) has a mean \( \delta \), and a covariance \( \Gamma \),

Then \( y=Of \) has a mean \( \mu=O\delta \), and a covariance \( \Sigma=O\Gamma O^T \).

Then \( \Sigma \) can converted into a correlation matrix \( R=D^{-1/2}\Sigma D^{-1/2} \).

Where \( D^{-1/2}=1/\sqrt{\text{diag}(\Sigma)} \).

Assume \( k \)-space measurements independent so \( \Gamma=I \).
Implications
Correlation, $R = D^{-1/2} \Sigma D^{-1/2}$.

\[
R = \begin{bmatrix}
R_{RR} & R_{RI} \\
R_{IR} & R_{II}
\end{bmatrix}
\]

\[ TH = .001 \]

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$H_0$ Implications

a) $O = \Omega$
b) $O = \Omega Z$
c) $O = \Omega A$
d) $O = S \Omega$
e) $O = \Omega AZ$
f) $O = S \Omega A$
g) $O = S \Omega Z$
h) $O = S \Omega AZ$
Image Processing

On a single slice full $k$-space array.

Local correlations induced!
Image Processing

On a single slice sub-sampled $k$-space array.
Reconstruction
Multi-Coil Acquisition

Each coil measures $k$-space.

\[ N_c = 4, \ A = 3 \]

\[ a_4 = S_{41}v_1 + S_{42}v_2 + S_{43}v_3 \]
\[ a_1 = S_{11}v_1 + S_{12}v_2 + S_{13}v_3 \]
\[ a_3 = S_{31}v_1 + S_{43}v_2 + S_{33}v_3 \]
\[ a_2 = S_{21}v_1 + S_{22}v_2 + S_{23}v_3 \]
Reconstruction/Processing

SENSE

Image vector

\[ O_k = U_I P_U \]

permute to by folded voxel

reconstruct \( N_c = 4 \) images

\( k \)-space vector of \( N_c \) images
Implications
SENSE induces long-range in-plane correlation.

Theoretical Results

<table>
<thead>
<tr>
<th></th>
<th>R-R</th>
<th>I-I</th>
<th>R-I</th>
<th>M^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center voxel</td>
<td></td>
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<td>fold</td>
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</tbody>
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SENSE $A=3$ smoothed

Basically multiplying voxel values $a_t$ by same 3 numbers over time $t$ to lead to correlated voxels.

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Implications
SENSE Reconstruction induces long-range correlation.

Experimental Results  SENSE $A=3$ smoothed
Care is needed!

Anderson, JS et al. PNAS 2010;107:20110-20114
Greicus, MD et al. PNAS 2003;100:4637-4642

Number of citations:
PubMed – 754
Google Scholar – 2423
Image Processing

One single slice sub-sampled $k$-space array.

Long-range correlations induced!

Of no biological origin!
Image Processing

One slice packet full sampled $k$-space array.
Image Processing
Image Processing
Image Processing
Image Processing

One slice packet full sampled $k$-space array.

Long-range correlations induced!

Of no biological origin!
Image Processing

One slice packet full sampled $k$-space array.
Example. $N_S = 4$

Slice Aliasing Pattern

Acquired Images

Hadamard pattern repeats every $N_S$ TRs

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Image Processing

True Magnitude

Mean Magnitude Separated

Activation Maps
Image Processing

One slice packet full sampled $k$-space array.

Proper separation (no leakage) and activation!

No visible induced correlations!

Knew about potentially induced correlations and avoided them.
Image Processing
Expand processing to include Time Series

\[
\begin{align*}
\mathbf{O}_{I1} & \quad \Omega_1 & \quad \mathbf{O}_{k1} & \quad 0 \\
\vdots & \quad \vdots & \quad \vdots & \quad \vdots \\
\mathbf{O}_{In} & \quad \Omega_n & \quad \mathbf{O}_{kn} & \quad 0 \\
\end{align*}
\]
0/1 permutation matrix reorder from by image to by voxel

ordered by voxel
Image Processing

One slice packet full sampled $k$-space array.
Image Processing
Image Processing
Image Processing

On a slice full sampled $k$-space array.

Homology is preserved.
Discussion

Processing changes the images we analyze.

The mean, variance, and correlation is changed.

Long range artificial correlations can be induced.

Null hypothesis implications.

How do we incorporate the processing into analysis?