Dynamic Multiscale Spatiotemporal Models for Poisson Data

Marco A. R. Ferreira (University of Missouri)

Thaís C. O. Fonseca (Federal University of Rio de Janeiro)
Outline

Motivation

Poisson multiscale factorization

Multiscale spatiotemporal model

Bayesian analysis

Applications

Conclusions
Outline

Motivation

Poisson multiscale factorization

Multiscale spatiotemporal model

Bayesian analysis

Applications

Conclusions
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1990
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1991
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1992
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1993
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1994
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1995
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1996
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1997
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1998
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1999
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2000
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2001
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2002
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2003
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2004
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2005
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2006
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2007
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2008
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2009
Introduction

- Here we are primarily concerned with Poisson spatiotemporal datasets available in the form of areal or regional data; that is, data available on a partition of the geographical domain of interest (Banerjee et al., 2004).
- Many methods have been recently proposed for the analysis of large point-referenced datasets (e.g., Banerjee et al., 2008; Paciorek and McLachlan, 2009; Lemos and Sansó, 2009).
- Here we propose a class of multiscale spatiotemporal models for Poisson areal data that lead to scalable, parallelizable, and computationally efficient inferential procedures.
Data Structure

The region of interest is divided in geographic subregions or blocks, and the data are number of occurrences of event of interest over these subregions.

Moreover, there is a nested multiscale structure that aggregates subregions at one resolution level onto coarser resolution levels.

Our framework can also handle spatiotemporal point process data.
Multiscale structure for the state of Missouri (Ferreira et al. 2011)
USA Tornado alley and multiscale structure

Motivation
Factorization
Model
Inference
Applications
Conclusions

USA Tornado alley and multiscale structure

<table>
<thead>
<tr>
<th>l=1</th>
<th>l=2</th>
<th>l=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:2</td>
<td>5:6</td>
<td>17:18:21:22</td>
</tr>
<tr>
<td>3:4</td>
<td>7:8</td>
<td>19:20:23:24</td>
</tr>
<tr>
<td>33:34</td>
<td>37:38</td>
<td>49:50:53:54</td>
</tr>
<tr>
<td>35:36</td>
<td>39:40</td>
<td>51:52:55:56</td>
</tr>
<tr>
<td>41:42</td>
<td>45:46</td>
<td>57:58:61:62</td>
</tr>
<tr>
<td>43:44</td>
<td>47:48</td>
<td>59:60:63:64</td>
</tr>
</tbody>
</table>

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Outline

Motivation

Poisson multiscale factorization

Multiscale spatiotemporal model

Bayesian analysis

Applications

Conclusions
Poisson multiscale factorization

At each time point we decompose the data into empirical spatiotemporal multiscale coefficients using the spatial multiscale modeling framework of Kolaczyk and Huang (2001). See also Chapter 9 of Ferreira and Lee (2007).

Interest lies in an inhomogeneous Poisson process with rate \( \{ \lambda(s) : s \in S \} \) on a domain \( S \subset \mathbb{R}^k \).

Because of measurement, resources or confidentiality restrictions, data are available only up to a given scale of resolution \( L \) on a partition of the domain \( S \). Denote this partition by \( \{ B_{L1}, \ldots, B_{L,n_L} \} \), with \( B_{Lj} \in S, j = 1, \ldots, n_L \), \( B_{Li} \cap B_{Lj} = \emptyset, i \neq j \), and \( \bigcup_{j=1}^{n_L} B_{Lj} = S \).
For each subregion $B_{Lj}$ there is a count $y_{Lj}$ of the number of occurrences of the event of interest.

Moreover, the expected number of counts on $B_{Lj}$ is

$$\mu_{Lj} = E(y_{Lj}) = \int_{B_{Lj}} \lambda(s) ds, \ j = 1, \ldots, n_L.$$ 

Further, similarly to Kolaczyk and Huang (2001) we assume that $y_{L1}, \ldots, y_{L,n_L}$ are conditionally independent given $\mu_{L1}, \ldots, \mu_{L,n_L}$.

In what follows, the latent spatial process $\lambda(s)$ is constructed in such a way that this latent process will be spatially correlated. Therefore, this spatial dependence will transfer to the counts $y_{L1}, \ldots, y_{L,n_L}$ and lead their marginal distribution to contain spatial dependence.
In addition to the mean process at the $L$th resolution level, we are also interested in the process at aggregated coarser scales.

At the $l$th scale of resolution, the domain $\mathcal{S}$ is partitioned in $n_l$ subregions $B_{l1}, \ldots, B_{ln_l}$, $l = 1, \ldots, L - 1$.

Moreover, the partition at level $l$ is assumed to be a refinement of the partition at level $l + 1$; that is, $B_{lj} = \bigcup (l+1,j') \in D_{lj} B_{l+1,j'}$, where $D_{lj}$ is the set of descendants of subregion $j$ at level $l$, and $D_{lj} \cap D_{li} = \emptyset$, $i \neq j$.

Additionally, let $A_l(L,j)$ be the ancestral at resolution level $l$ of subregion $(L,j)$.

Finally, denote by $d_{lj}$ the number of descendants of subregion $(l,j)$. 
The aggregated counts at the \( l \)th level of resolution are recursively defined as

\[
y_{l,j} = 1'_{d_{lj}} y_{D_{lj}}
\]

with corresponding aggregated mean process

\[
\mu_{l,j} = 1'_{d_{lj}} \mu_{D_{lj}}
\]

The mean \( \mu_{l,j} \) may be written as

\[
\mu_{l,j} = \lambda_{l,j} e_{l,j}
\]

where \( \lambda_{l,j} \) is the relative risk and \( e_{l,j} \) is either known or unknown up to a low-dimensional parameter vector. Similarly to the observed counts, \( e_{l,j} \) may be aggregated as

\[
e_{l,j} = 1'_{d_{lj}} e_{D_{lj}}
\]

and in that case the aggregation for the mean process implies that the relative risk process is aggregated as

\[
\lambda_{l,j} = e_{l,j}^{-1} \lambda'_{D_{lj}} e_{D_{lj}}
\]
Because of conditional independence, the likelihood function admits the multiscale factorization (Kolaczyk and Huang, 2001)

$$\prod_{j=1}^{n_L} p(y_{Lj}|\mu_{Lj}) = \prod_{j=1}^{n_1} p(y_{1j}|\mu_{1j}) \prod_{l=1}^{L-1} \prod_{j=1}^{n_l} p(y_{Dlj}|y_{lj}, \omega_{lj}), \quad (1)$$

where $y_{1j}|\mu_{1j} \sim \text{Poisson}(\mu_{1j})$.

Further, $y_{Dlj}|y_{lj}, \omega_{lj} \sim \text{Multinomial}(y_{lj}, \omega_{lj})$, where $y_{lj}$ plays the role of the sample size parameter of the multinomial distribution, and $\omega_{lj} = \mu_{Dlj}/\mu_{lj}$ is the vector of probabilities.

$\omega_{lj}$ describes how the counts at subregion $(l, j)$ are expected to be distributed among its descendants $D_{lj}$, and connects coarser to finer resolution levels.

In analogy to wavelet analysis we refer to $\omega_{lj}$ as a spatial multiscale coefficient.
Outline

Motivation

Poisson multiscale factorization

Multiscale spatiotemporal model

Bayesian analysis

Applications

Conclusions
Multiscale spatiotemporal model

The model for the number of counts at the finest resolution level $L$ at time $t$ is

$$y_{tLj} | \mu_{tLj} \sim \text{Poisson}(\mu_{tLj})$$

Further, we assume that $\mu_{tLj} = \lambda_{tLj} e_{tLj}$ where $\lambda_{tLj}$ is the risk on subregion $(L, j)$ at time $t$ and $e_{tLj}$ is either known or unknown up to a low-dimensional parameter vector.

Examples of known $e_{tLj}$ include the case when $e_{tLj} = 1$ and the case when $e_{tLj}$ is the known population size of subregion $(l, j)$ at time $t$.

An example of $e_{tLj}$ unknown up to a low-dimensional parameter vector is $e_{tLj} = \exp(x_t' \beta_{Lj})$, where $x_t$ is a known vector of regressors common to all regions at time $t$ and $\beta_{Lj} = \beta_{A1(L,j)}$. 
It follows from Equation (1) that the multiscale factorization at time $t$ for the Poisson model is

$$\prod_{j=1}^{n_L} p(y_{tLj}|\mu_{tLj}) = \prod_{j=1}^{n_1} p(y_{t1j}|\mu_{t1j}) \prod_{l=1}^{L-1} \prod_{j=1}^{n_l} p(y_{t,Dlj}|y_{tlj},\omega_{tlj}),$$

where $y_{t1j}|\mu_{t1j} \sim \text{Poisson}(\mu_{t1j})$ and

$$y_{t,Dlj}|y_{tlj},\omega_{tlj} \sim \text{Multinomial}(y_{tlj},\omega_{tlj}), \text{ with } \omega_{tlj} = \mu_{t,Dlj}/\mu_{tlj}.$$ 

The parameter $\omega_{tlj}$ is the spatiotemporal multiscale coefficient which represents the vector of probabilities associated with how the counts in $y_{tlj}$ are distributed for each descendant in $D_{lj}$.

Let $\omega_{tlj}^e = y_{tDlj}/y_{tlj}$ be an estimator of $\omega_{tlj}$. We refer to $\omega_{tlj}^e$ as an empirical spatiotemporal multiscale coefficient.
Beta evolution for coarse level risk (Smith and Miller, 1986)

The beta temporal evolution for $\lambda_{t1j}$ is defined as

$$\lambda_{t1j} = \lambda_{t-1,1j} \gamma_j^{-1} \eta_{tj}, \quad (4)$$

where

$$\eta_{tj} | \mathcal{D}_{t-1}, \gamma_j \sim \text{Beta}(\gamma_j a_{t-1,j}, (1 - \gamma_j) a_{t-1,j}),$$

$0 < \gamma_j \leq 1$ is a discount factor parameter, and $a_{t-1,j} > 0$. 
Dirichlet evolution for multiscale coefficients

The stochastic temporal evolution for $\omega_{tlj}$ is defined as

$$\omega_{tlj} = \frac{1}{S_{t-1,lj}} \phi_{t-1,lj} \odot \omega_{t-1,lj}. \quad (5)$$

with $\phi_{t-1,lj} = (\phi_{t-1,lj1}, \ldots, \phi_{t-1,lj,dlj})'$, where

$\phi_{t-1,lj1}, \ldots, \phi_{t-1,lj,dlj}$ are i.i.d. $Beta(\delta_{lj} c_{t-1,lji}, (1 - \delta_{lj}) c_{t-1,lji})$,

$S_{t-1,lj} = \phi_{t-1,lj}' \omega_{t-1,lj}$, $0 < \delta_{lj} \leq 1$ is a discount factor parameter, and $c_{t-1,lji} > 0$, $i = 1, \ldots, d_{lj}$. 
Initial conditions and priors for the discount factors

\[ \lambda_{01j} | D_0 \sim \text{Gamma}(a_{0j}, b_{0j}) \]

\[ \omega_{0lj} | D_0 \sim \text{Dirichlet}(c_{0lj}) \]

\[ \gamma_j \sim \text{Beta}(a_\gamma, b_\gamma) \]

\[ \delta_{lj} \sim \text{Beta}(a_\delta, b_\delta) \]
Outline

Motivation

Poisson multiscale factorization

Multiscale spatiotemporal model

Bayesian analysis

Applications

Conclusions
Theorem

Consider the multiscale spatiotemporal model for Poisson data defined by Equations (2), (3), (4), and (5). Given the discount factor parameters $\gamma_j, j = 1, \ldots, n_1$, and $\delta_{lj}, l = 1, \ldots, L - 1, j = 1, \ldots, n_l$, the vectors $\lambda_{1:T,11}, \ldots, \lambda_{1:T,1n_1}$, $\omega_{1:T,11}, \ldots, \omega_{1:T,1n_1}$, $\omega_{1:T,L-1,1}, \ldots, \omega_{1:T,L-1,n_{L-1}}$ are conditionally independent a posteriori.
Filtering for $\lambda_1: T,1j$

**Theorem**

Assume the initial distribution $\lambda_{01j}|D_0 \sim \text{Gamma}(a_{0j}, b_{0j})$ and consider the Observation Equation (3) and the beta evolution for $\lambda_{t1j}$ given by Equation (4). Then, for $t = 1, \ldots, T$:

(i) **Posterior for $\lambda_{t-1,1j}$**:

$$\lambda_{t-1,1j}|D_{t-1}, \gamma_j, \beta_j \sim \text{Gamma}(a_{t-1,j}, b_{t-1,j}).$$

(ii) **Prior for $\lambda_{t1j}$**:

$$\lambda_{t1j}|D_{t-1}, \gamma_j, \beta_j \sim \text{Gamma}(a_{t|t-1,j}, b_{t|t-1,j}),$$

where $a_{t|t-1,j} = \gamma_j a_{t-1,j}$ and $b_{t|t-1,j} = \gamma_j b_{t-1,j}$.

(iii) **Posterior for $\lambda_{t1j}$**:

$$\lambda_{t1j}|D_t, \gamma_j, \beta_j \sim \text{Gamma}(a_{tj}, b_{tj}),$$

where $a_{tj} = \gamma_j a_{t-1,j} + y_{t1j}$ and $b_{tj} = \gamma_j b_{t-1,j} + e_{t1j}$. 
Filtering for $\omega_{1:T,lj}$

**Theorem**

Assume the initial distribution $\omega_{0lj}|\mathcal{D}_0 \sim \text{Dirichlet}(c_{0lj})$, and consider the Observation Equation (3) and the Dirichlet temporal evolution for the spatiotemporal multiscale coefficient $\omega_{tlj}$ given by Equation (5). Then, for $t = 1, \ldots, T$:

(i) **Posterior for $\omega_{t-1,lj}$**: $\omega_{t-1,lj}|\mathcal{D}_{t-1}, \delta_{lj} \sim \text{Dirichlet}(c_{t-1,lj})$.

(ii) **Prior for $\omega_{tlj}$**: $\omega_{tlj}|\mathcal{D}_{t-1}, \delta_{lj} \sim \text{Dirichlet}(c_{t|t-1,lj})$, where $c_{t|t-1,lj} = \delta_{lj}c_{t-1,lj}$.

(iii) **Posterior for $\omega_{tlj}$**: $\omega_{tlj}|\mathcal{D}_t, \delta_{lj} \sim \text{Dirichlet}(c_{tlj})$, where $c_{tlj} = \delta_{lj}c_{t-1,lj} + y_{tlj}\omega_{tlj}^e$. 
Smoothing for $\lambda_{1:T,1j}$

Proposition

Assume the Observation Equation (3) and the beta evolution for $\lambda_{t1j}$ given by Equation (4). Then, the conditional smoothing distribution of $\lambda_{t-1,1j}$ given $\lambda_{t1j}$ is equal to

$$p(\lambda_{t-1,1j}|D_T, \lambda_{t1j}, \gamma_j)$$

$$= \frac{b_{t-1,j}^{(1-\gamma_j)a_{t-1,j}}}{\Gamma((1-\gamma_j)a_{t-1,j})} (\lambda_{t-1,1j} - \gamma_j \lambda_{t1j})^{(1-\gamma_j)a_{t-1,j}-1}$$

$$\times \exp\{-b_{t-1,j}(\lambda_{t-1,1j} - \gamma_j \lambda_{t1j})\},$$

where $\lambda_{t-1,1j} - \gamma_j \lambda_{t1j} > 0$. 

Poisson Multiscale Spatiotemporal Models
Marco A. R. Ferreira
Smoothing for $\omega_{1:T,lj}$

Proposition

Consider the Observation Equation (3) and the Dirichlet evolution for the spatiotemporal multiscale coefficient $\omega_{tlj}$ given by Equation (5). Then,

(i) $\omega_{t-1,lj} | D_T, S_{t-1,lj}, \omega_{tlj}, \delta_{lj} \sim \text{Mod-Dirichlet}((1 - \delta_{lj})c_{t-1,lj}, 0, S_{t-1,lj} \omega_{tlj})$

(ii) $S_{t-1,lj} | D_T, \omega_{tlj}, \delta_{lj} \sim \text{Beta}(\delta_{lj} \tilde{c}_{t-1,lj}, (1 - \delta_{lj}) \tilde{c}_{t-1,lj})$

where $\tilde{c}_{t-1,lj} = \sum_{i=1}^{d_{lj}} c_{t-1,lji}$ and Mod-Dirichlet denotes a modified Dirichlet distribution.
Likelihood function for the discount factor $\gamma_j$ and the regression coefficients $\beta_j$

\[
p(y_1:T,1_j|\gamma_j, \beta_j, D_0)
= \prod_{t=\tau_1}^{T} p(y_{t1j}|D_{t-1}, \gamma_j, \beta_j)
= \prod_{t=\tau_1}^{T} \int_0^\infty p(y_{t1j}|\gamma_j, \beta_j, \lambda_{t1j}) \ p(\lambda_{t1j}|D_{t-1}, \gamma_j, \beta_j) \ d\lambda_{t1j}
= \prod_{t=\tau_1}^{T} \left\{ \frac{\Gamma(\gamma_j a_{t-1,j}+y_{t1j})}{\Gamma(\gamma_j a_{t-1,j})\Gamma(y_{t1j}+1)} \left( \frac{\gamma_j b_{t-1,j}}{e^{x'_tj} \beta_j} \right)^{\gamma_j a_{t-1,j}} \left(1 + \frac{\gamma_j b_{t-1,j}}{e^{x'_tj} \beta_j} \right)^{-(\gamma_j a_{t-1,j}+y_{t1j})} \right\}.
\]
Likelihood function for the discount factor $\delta_{lj}$

\[ p(y_{1:T}, D_{lj} | D_0, y_{1:T}, l_j, \delta_{lj}) \]

\[ = \prod_{t=\tau_2}^T p(y_t, D_{lj} | D_{t-1}, y_{tlj}, \delta_{lj}) \]

\[ = \prod_{t=\tau_2}^T \int p(y_t, D_{lj} | y_{tlj}, \omega_{tlj}) p(\omega_{tlj} | D_{t-1}, \delta_{lj}) d\omega_{tlj} \]

\[ = \prod_{t=\tau_2}^T \left\{ \frac{\Gamma(\sum_i \delta_{lj} c_{t-1,lji}) \Gamma(y_{tlj} + 1)}{\Gamma(\sum_i \delta_{lj} c_{t-1,lji} + y_{tlj})} \prod_{i=1}^{d_{lj}} \frac{\Gamma(\delta_{lj} c_{t-1,lji} + y_{tlj} \omega_{tlji}^e)}{\Gamma(y_{tlj} \omega_{tlji}^e + 1) \Gamma(\delta_{lj} c_{t-1,lji})} \right\} . \]
Outline

Motivation

Poisson multiscale factorization

Multiscale spatiotemporal model

Bayesian analysis

Applications

Conclusions
Priors

In all applications, we have used the same default prior distributions:

- For $\lambda_{0j} | D_0$, we assume $a_{0j} = b_{0j} = 0.01$,
- For $\omega_{0lj} | D_0$, we assume $c_{0lj} = 0.01 \mathbf{1}_{d_{lj}}$,
- For $\gamma_{j}$, we assume $a_{\gamma} = b_{\gamma} = 1$,
- For $\delta_{lj}$, we assume $a_{\delta} = b_{\delta} = 1$. 
Simulated dataset

Coarsest level: 9 subregions

Intermediate level: 36 subregions

Finest level: 72 subregions

\( T = 200 \)

At time 0, \( \mu_{0Lj} = 30 + 5 \times \text{longitude}_j + 8 \times \text{latitude}_j \).

At time 0, \( \omega_{0lj} = d_{lj}^{-1} \mathbf{1}_{d_{lj}} \).

\( \gamma_j = 0.95, j = 1, \ldots, n_1. \)

\( \delta_{lj} = 0.90, l = 1, 2, j = 1, \ldots, n_l. \)
Posterior mean (dot) and 95% credible interval (vertical dashed line) for the discount factors

(a) $\gamma_j$  

(b) $\delta_{1j}$  

(c) $\delta_{2j}$
Time series plots for $\mu_{1:T,3j}$

(a) $j = 6$

(b) $j = 25$

(c) $j = 63$
Mortality in Missouri for 45 to 64 years old age group
Posterior densities for discount factors
Estimated risk level for counties within the Saint Louis metropolitan area
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1990

Observed

Fitted
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1991

Observed

Fitted

Legend:
- under 530
- 530 – 624
- 624 – 718
- 718 – 812
- 812 – 906
- 906 – 1000
- 1000 – 1094
- over 1094

Poisson Multiscale Spatiotemporal Models
Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1992

Observed  Fitted

Poisson Multiscale Spatiotemporal Models
Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1993

Observed

Fitted

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1994

Observed

Fitted

Poisson Multiscale Spatiotemporal Models
Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1995

Observed

Fitted

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1996

Observed

Fitted

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1997

Observed

Fitted

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1998
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

1999

Observed

Fitted

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2000

Observed

Fitted

Legend:
- under 530
- 530 – 624
- 624 – 718
- 718 – 812
- 812 – 906
- 906 – 1000
- 1000 – 1094
- over 1094
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2002

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2003

Observed

Fitted

Poisson Multiscale Spatiotemporal Models
Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2004

Observed

Fitted

Poisson Multiscale Spatiotemporal Models
Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2006

Observed

Fitted

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2007

Observed

Fitted

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2008

Observed

Fitted

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group

2009

Observed

Fitted

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Tornados in the American Midwest

Poisson Multiscale Spatiotemporal Models

Marco A. R. Ferreira
Posterior densities for $\beta_j$, $\gamma_j$, and $\delta_{1j}$
Posterior densities for the discount factors $\delta_{2j}$ corresponding to the descendants of subregion (1,3) and subregion (1,4)
Posterior median (solid line) and 95% credible interval (dashed line) for each element of $\omega_{t,10}$
Model comparison

Competing models:

- Model I: Multiscale spatiotemporal model for Poisson data.

- Model II: Model that assumes that each finest level subregion has its own temporal evolution according to the Poisson state-space model.

- Model III: Spatiotemporal model: \( y_{tj} | \mu_{tj} \sim \text{Poisson}(\mu_{tj}) \), with 
  \[
  \log(\mu_{tj}) = \log(e_{tj}) + \alpha_0 + a_j + b_t, \quad b_t = \phi b_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, W),
  \]
  and the \( a_j \)'s are spatial random effects that follow a CAR specification.
Model comparison

Log conditional Bayes factor of Model I against Models II and III

<table>
<thead>
<tr>
<th></th>
<th>Model II</th>
<th>Model III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Missouri data</td>
<td>283.9</td>
<td>215.1</td>
</tr>
<tr>
<td>Tornado data</td>
<td>254.8</td>
<td>139.8</td>
</tr>
</tbody>
</table>
Outline

Motivation

Poisson multiscale factorization

Multiscale spatiotemporal model

Bayesian analysis

Applications

Conclusions
Conclusions

- Novel multiscale spatiotemporal model for Poisson data.

- Modeling strategy naturally respects nonsmooth transitions between geographic subregions.

- Analysis of the discount factors provides a powerful way to identify regions with spatiotemporal dynamics that warrant further investigation.

- Divide-and-conquer modeling strategy leads to computational procedures that are parallelizable, scalable, and fast.