PROBLEM 1: TIME DEPENDENT CONSOLIDATION OF FINE POWDERS

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Abstract

This document illustrates several important points related to the writing of your report. What follows is part of a report from a couple of years ago. The flow of granular materials has been a challenging mathematical endeavor since the advent of plasticity theory in the late 1950's. Jenike's original work in this field was built on foundations of continuum mechanics, plasticity theory, and soil mechanics. Advances in computer technology continually warrant a closer look at the mathematics that is the major limitation of rapid progress in this field. There are a lot of problems without adequate mathematical solutions. Some introductory material on stress and velocity fields in bins with flowing granular material will be presented as a precursor to the problem to be addressed.

The problem offered is time dependent consolidation of compressible fine powders. One application is predicting how air escapes from a powder when a bin or silo is filled and then allowed to deaerate with time. There is a parallel consolidation problem in soil mechanics that occurs in water saturated clays and a model is used to predict the settlement of foundations built on clay. For instance in Mexico City, settlement of up to 15 ft in some areas has occurred in the last 50 years due to building. Unfortunately powder/air behavior in a bin is much more complex than water/soil, so the mathematical models are very different. In large bins settlement time can be several days, in stockpiles it can be several weeks.

The original problem involves a Lagrangian frame of reference. The general three dimensional problem is too computationally intensive for a quick solution. Instead we focus on an axisymmetric problem to reduce the problem to two spatial and one time dimension. The problem will be considered first as a pseudo-2D spatial problem by introducing an approximation that effectively reduces the problem to one spatial dimension.

1 Introduction and Motivation

Jenike & Johanson is a specialized engineering firm which provides clients with solutions to bulk solids handling problems. One of the interests at Jenike & Johanson is computer modeling of the settlement of fine powders in bins. When aerated, fine powders behave like fluids, and so settlement properties become very important. The handling of fine powders presents some difficulties such as: flooding (uncontrolled flow), no-flow (occurs when deaerated), erratic flow, and so on. If a solid is not given enough time to settle, then flooding can occur. On the other hand, allowing solids to settle too long may result in no-flow. The amount of time for a powder to deaerate depends on the fill rate of a silo/bin. Understanding the settlement of fine powders over time facilitates more efficient handling of fine powders.

The original problem was to model the settlement of fine powders in a bin or silo of arbitrary, but simple geometric shape (i.e., cylinder or cone). If possible, we were to model the settlement process as the bin/silo is filled. This is a three dimensional problem and quite complex. We first simplified the problem by working with a cylinder that is filled instantaneously. Using the axisymmetry of the bin, we reduced the problem to one dimension.

From physical intuition, we expect the following behaviour over time. Air escapes from the top of the bin, and the pressure decreases over time, with the pressure at the top being the atmospheric pressure. Stress on the solid increases, with zero stress at the top of the bin. Density also increases over time.

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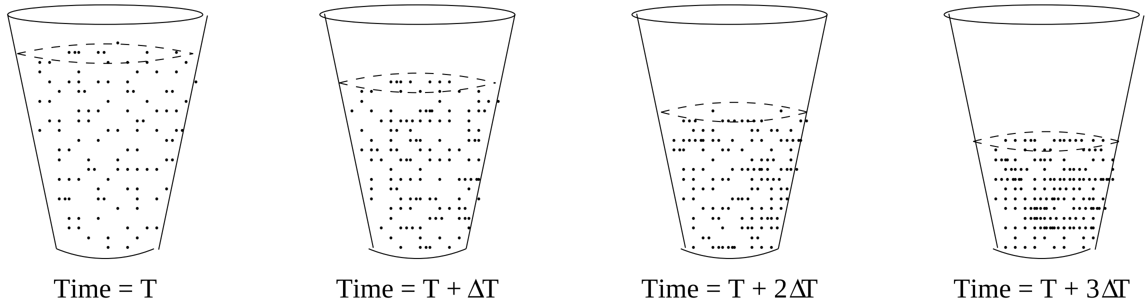

Figure 1: Compression of granular material over time.

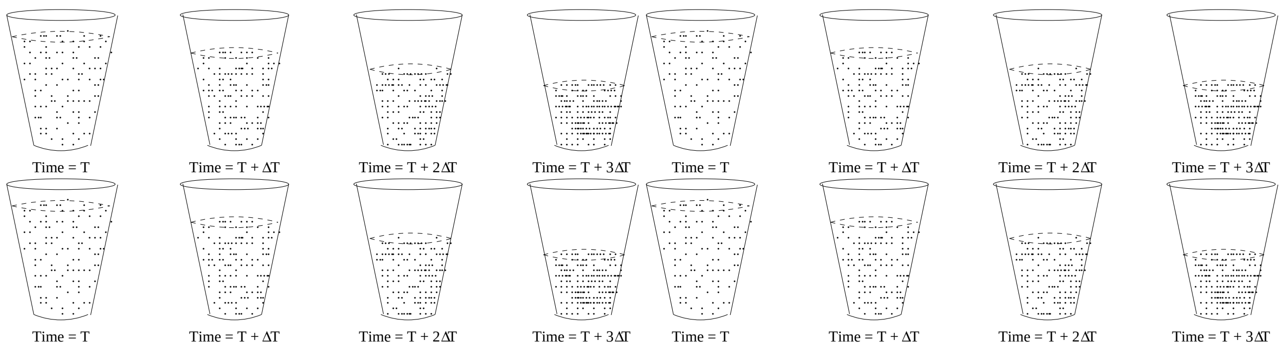

Figure 2: Silly example of how to use minipage.

Figures should be included using the *includegraphics* command. Several figures can be grouped using the *minipage* environment.

2 Modeling Equations

In order to have both equations in the same coordinate framework, we derived a new model that is very close to the model proposed originally. The coordinate system for this model is not material specific, but rather is an external coordinate system in which the material moves, i.e., an Eulerian framework.

For materials that are nearly incompressible, this extra term is small and may perhaps be neglected. For compressible material, this term is important. The resulting Eulerian system is as follows:

$$\begin{aligned} &\left(1-\frac{\gamma}{\Gamma}\right)\partial_t p - \frac{p}{\Gamma}\partial_t\gamma - \partial_z\left(\frac{p}{\gamma}\int_0^z \partial_t\gamma\, d\tilde{z}\right) - \partial_z\left(\frac{pK}{\gamma}\partial_z p\right) = 0 \\ &\partial_z\sigma + \partial_z p + \sigma\left(A_2 - A_3\right) + \gamma = 0 \end{aligned} \tag{1}$$

with the additional condition

$$\gamma = \gamma_m\left(1+\frac{\sigma}{\sigma_m}\right)^{\beta_m}.$$

We see that equation (1) can then be referenced using the automatic cross referencing capabilities of LaTeX. So can the above Figure 1.

Notes:

(1) Avoid the use of macros since this leads to confusion when the files from individual teams are combined. Please, use the conventions presented here.

Region:	**Equation:**	**Domain:**
Annulus	$w(r) = \frac{qr^4}{64D} + \frac{C_1 r^2}{4} + C_2 \ln\left(\frac{r}{R_0}\right) + C_3$	$R_i < r \leq R_0$
Composite	$\widetilde{w}(r) = \frac{qr^4}{64D_c} + \frac{C_4 r^2}{4} + C_5 \ln\left(\frac{r}{R_0}\right) + C_6$	$0 \leq r \leq R_i$

Table 1: Components of a circular plate composite.

(2) Employ the cross referencing and reference citation capabilities of LaTeX. When numbering your equations or figures, use the convention that your team number appears before the equation number (e.g., the label for equation (1), obtained with the command `\label{1_eq1}`, indicates equation 1 for team 1). This will eliminate the double labeling of certain equations when reports are combined into a final document.

(3) A good reference for Latex is the book the book by Lamport [1]. This item also illustrates the automatic cross referencing capability which LaTeXprovides.

(4) Some of the publications related to the Workshop can be found here `http://www.ncsu.edu/crsc/immw/publication.html`

(5) For many groups, it will be necessary to create tables. This can be easily accomplished using the tabular command.

Example:

Table may be referenced the same way equations and figures are.

References

[1] L. Lamport, *LaTeX: A Document Preparation System,* Addison-Wesley Publishing Company, New York, 1994.