Statistical analysis of brain images using matrix decompositions

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Independent Component Analysis

\[ A - \text{temporal mixtures. Many methods assume } Q = T. \]
Group Independent Component Analysis

Subject 1  \[ X_1 \]  \[ A_1 \]  \[ S \]

Subject 2  \[ X_2 \]  =  \[ A_2 \]

\[ \ldots \quad \ldots \]

Subject I  \[ X_I \]  \[ A_I \]

- Reconstruct each row of \( S \) in 3D.
- Each 3D image is a brain network (Calhoun, 2001).
Children with Autism Spectrum Disorder (ASD) have difficulties performing motor tasks.

- Autism trait severity using total Raw SRS score.
- Imitation ability.
- Overall skilled gesture performance using praxis exam scores.

Goals:

- Is visual-motor synchrony different in ASD?
- Is visual-motor synchrony associated with imitation ability?
ICA based Connectivity Analysis - KKI

Motor system

- dorsomedial lower limb areas ("LL")
- more lateral upper limb areas ("UL")

Visual components

- visual processing areas ("VC1" and "VC2")
- lateral occipital cortex ("VC3")

Estimated by ICA for 50 children with ASD and 50 controls. Age 8-12 years.
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Data from UM was used for validation.
Independent Component Analysis

The ICA model and assumptions.

\[ X = AS + E \]

- The components \( S_1, \ldots, S_Q \) are statistically independent.
- The mixing matrix \( A \) is nonsingular.
- At most one of the components \( S_q \) is Gaussian.

When \( E = 0 \) the model is called noise-free.
A mapping $\phi(\cdot)$ from the set of densities $\{f_s, s \in \mathbb{R}^n\}$ to $\mathbb{R}$ is called a contrast function (Comon (1994)) if it satisfies the following requirements

- $\phi(f_s) = \phi(f_{Ps})$ where $P$ is a permutation matrix.
- $\phi(f_s) = \phi(f_{\Lambda s})$ where $\Lambda$ is a diagonal invertible matrix.
- $\phi(f_{As}) \leq \phi(f_s)$ if all elements of $s$ are independent and the matrix $A$ is invertible.

Commonly used contrast functions involve kurtosis, negentropy and mutual information.
Contrast Function

Negentropy as a measure of nongaussianity.

\[ J(f_x) = H(f_z) - H(f_x), \]

where \( E(z) = E(x) \) and \( \text{cov}(z) = \text{cov}(x) \), \( H(\cdot) \) differential entropy.

Hyvarinen (1997) negentropy of \( s \) can be approximated by

\[ J_G(f_s) = \left[ E_s(G(s)) - E_z(G(z)) \right]^2, \]

where \( z \) is a Gaussian random variable with mean zero and variance one, \( G(\cdot) \) is a nonquadratic function.
The R package fastICA is based on this method using

\[ G_1(u) = \log \cosh au, \quad G_2(u) = \exp\left(-\frac{u^2}{2}\right), \]

where \( a \) is a constant such that \( 1 \leq a \leq 2 \).

More information in Hyvarinen, Karhunen, and Oja (2003).

- Assume the mixing matrix $A$ is square,
- Define a structure for the mixing matrix,
- Model densities of underlying sources using Gaussian mixtures,
- Parameter estimation via EM-algorithm.

Shi and Guo (2016) incorporate covariates within group ICA.
ProDenICA, Distance Covariance, LCA

ProDen ICA proposed by Hastie and Tibshirani (2002)
- Model densities of underlying sources using exponentially tilted Gaussian densities,
- Estimate the mixing matrix using a fixed point algorithm.

ICA via Distance Covariance, Matteson and Tsay (2011)
- Estimate components by targeting independence,
- Define independence via Distance Covariance.

Likelihood Component Analysis, Risk, et. al (2016)
- Allows for non-square mixing matrices,
- Options for modeling densities of underlying sources.
Group ICA

\[ S(q, \nu) = \mathbf{W}(q,.) \mathbf{X}(.,\nu), \]

\[ \mathbf{W} = \mathbf{A}^{-1}, \mathbf{W}(q,.) - qth\ row\ of\ \mathbf{W}, \mathbf{X}(.,\nu) - \nu th\ column\ of\ \mathbf{X}. \]

\[ S(q, 1), \ldots, S(q, V) \sim f_q(\cdot). \]

The likelihood function for ICA model

\[ L(S) = \prod_{v=1}^{V} \prod_{q=1}^{Q} f_q[S(q, \nu)], \]

\[ L(\mathbf{W}, f) = \prod_{v=1}^{V} \prod_{q=1}^{Q} \left[ \sum_{l=1}^{Q} \mathbf{W}(q, l) \mathbf{X}(l, \nu) \right]. \]

Estimate the matrix \( \mathbf{W} \) and densities \( f_q \) given the observed \( \mathbf{X} \).
We parameterize the density of $S_q$ as a mixture density:

$$f_q(s) = \sum_{j=1}^{N} \theta_{qj} \phi\left(\frac{s - \mu_{qj}}{\sigma_q}\right) \frac{1}{\sigma_q},$$

where $\phi(\cdot)$ is the standard normal density function.

- The means $\mu_{qj}$ and the standard deviations $\sigma_j$ - fixed.
- The estimation of $\theta_{q1}, \ldots, \theta_{qN}$ is performed via a modified EM algorithm.

The log-likelihood of ICA is obtained as

$$l(W, \hat{f}) = \sum_{v=1}^{V} \sum_{q=1}^{Q} \log \{ \hat{f}_q(\sum_{l=1}^{Q} x_{vl} w_{lq}) \} + V \log | \det W |.$$ 

Estimate the mixing matrix $W = A^{-1}$ and the densities $\hat{f}_q$ via an iterative optimization algorithm.

Group Independent Component Analysis

Two-stage singular value decomposition.
Group Independent Component Analysis

Two-stage singular value decomposition.
Iterative Algorithm, HDICA

1. $S_i = \hat{W}_i X_i$

2. Estimate the weights of the density for each independent component. $\theta_q$ is estimated by an EM algorithm.

3. Compute the derivative and Hessian for the log-likelihood.

$$L(\hat{W}) = \sum_{i=1}^{I} \sum_{v=1}^{V} \sum_{q=1}^{Q} \log[f_q(\hat{W}_{iq} X_{iv})] + V \log |\text{det} \hat{W}_i|.$$  

4. $\hat{W}_i^{\text{new}} = \hat{W}_i - L''(\hat{W}_i)^{-1}L'(\hat{W}_i)$

5. Stopping rule

$$\max ||\hat{W}_i - \hat{W}_i^{\text{new}}|| < \delta$$


Bayesian Independent Component Analysis

The noisy ICA model:

\[ X = AS + E, \]

\[ X_{tv} | A, S, \sigma_e \sim N(A_t.S.v, \sigma^2_e) \]

Proposed priors:

\[ \sigma^2_e | \alpha_e, \beta_e \sim \text{InverseGamma}(\alpha_e, \beta_e). \]

\[ S_{qv} | Z_{qv} = k \sim N(\mu_k, \sigma^2_N), \]

\[ P[Z_{qv} = k] = \theta_{qk}, \]

\[ \theta_q = (\theta_{q1}, \theta_{q2}, \ldots, \theta_{qN}) | \alpha \sim \text{Dirichlet}(\alpha, \alpha, \ldots, \alpha), \]

where \( v = 1, \ldots, V, t = 1, \ldots, Q \) and \( k = 1, 2, \ldots, N \).
The 1000 Functional Connectomes Project Dataset

- More than 1400 scans available online.
- The scans are collected using a 3T scanner.
- For the subset used in this analysis the number of time points was $T = 119$.
- Standard image processing was performed to register the data to the MNI standard brain space.

$W_i$ and $S$ are estimated via the parallel HDICA algorithm.
Results for 301 Subjects

Auditory Network

Control Network

Default Mode Network

Visual Network
ICA based Connectivity Analysis - ABIDE

- 379 ASD.
- 400 typically developing
- For the subset used in this analysis the number of time points was $T = 220$.
- Standard image processing was performed to register the data to the MNI standard brain space.
ICA based Connectivity Analysis - ABIDE
Summary and Extensions

- Connectivity using ICA in Autism looking at motor function.
- New methods for finding brain networks for large groups of fMRI data.
- Extensions of existing methods for novel types of data.
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