Statistical analysis of brain images using matrix decompositions

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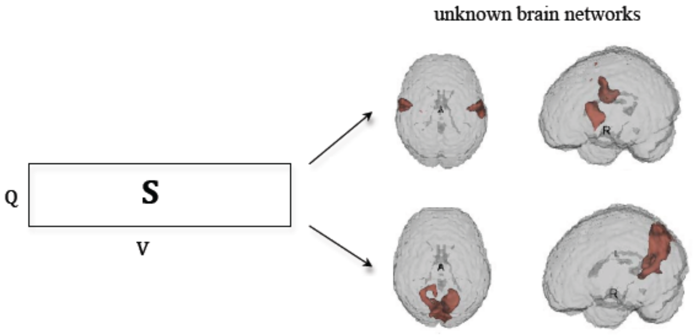
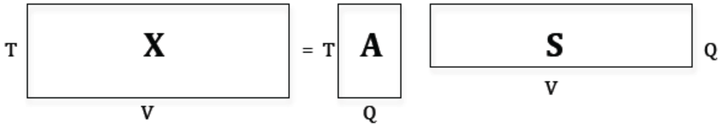
Independent Component Analysis





A - temporal mixtures. Many methods assume $Q = T$.

Group Independent Component Analysis

$$
\begin{array}{lcccc}
\text{Subject 1} & \boxed{\quad \boldsymbol{X}_1 \quad} & & \boxed{\boldsymbol{A}_1} & \boxed{\quad \boldsymbol{S} \quad} \\
\text{Subject 2} & \boxed{\quad \boldsymbol{X}_2 \quad} & = & \boxed{\boldsymbol{A}_2} & \\
 & \dots & & \dots & \\
\text{Subject I} & \boxed{\quad \boldsymbol{X}_{\mathrm{I}} \quad} & & \boxed{\boldsymbol{A}_{\mathrm{I}}} &
\end{array}
$$

- Reconstruct each row of $\boldsymbol{S}$ in 3D.
- Each 3D image is a brain network (Calhoun, 2001).

ICA in Brain Research

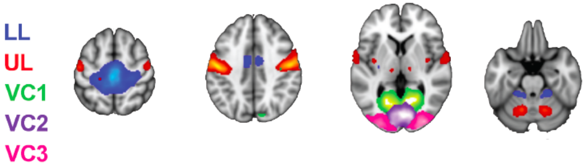
Children with Autism Spectrum Disorder (ASD) have difficulties performing motor tasks.

- Autism trait severity using total Raw SRS score.
- Imitation ability.
- Overall skilled gesture performance using praxis exam scores.

Goals:

- Is visual-motor synchrony different in ASD?
- Is visual-motor synchrony associated with imitation ability?

ICA based Connectivity Analysis - KKI



Motor system

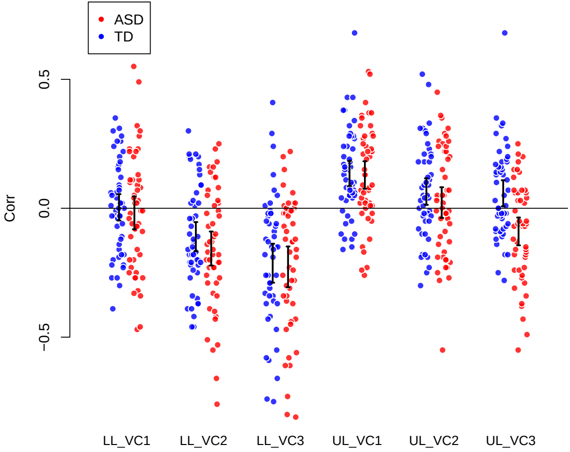
- dorsomedial lower limb areas ("LL")
- more lateral upper limb areas ("UL")

Visual components

- visual processing areas ("VC1" and "VC2")
- lateral occipital cortex ("VC3")

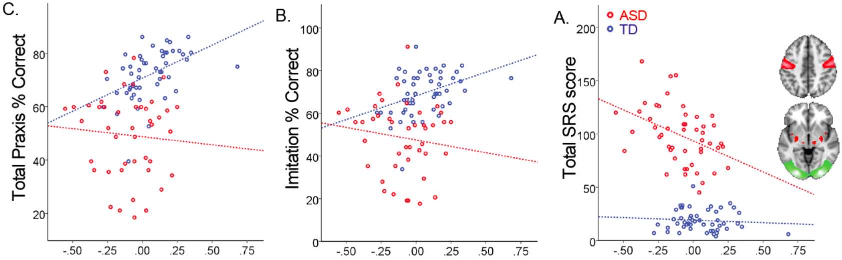
Estimated by ICA for 50 children with ASD and 50 controls.
Age 8-12 years.

ICA based Connectivity Analysis - KKI



Nebel, M.B., Eloyan, A., Nettles, C., Ament, K., Sweeney, K., Ward, R., Barber, A.D., Choe. A., Pekar, J.J., and Mostofsky, S.H. (2016) Reduced intrinsic visual-motor synchrony relates to autism severity. Biological Psychiatry.

ICA based Connectivity Analysis - KKI



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Validation

ABIDE
Autism Brain Imaging
Data Exchange

indi
International Neuroimaging
Data-Sharing Initiative

Data from UM was used for validation.

Independent Component Analysis

The ICA model and assumptions.

$$\boldsymbol{X} = \boldsymbol{AS} + \boldsymbol{E}$$

- The components $S_1, \ldots, S_Q$ are statistically independent.
- The mixing matrix $\boldsymbol{A}$ is nonsingular.
- At most one of the components S_q is Gaussian.

When $\boldsymbol{E} = 0$ the model is called noise-free.

Contrast Function

A mapping $\phi(\cdot)$ from the set of densities $\{f_s, \boldsymbol{s} \in \mathbb{R}^n\}$ to $\mathbb{R}$ is called a contrast function (Comon (1994)) if it satisfies the following requirements

- $\phi(f_s) = \phi(f_{\boldsymbol{P}s})$ where $\boldsymbol{P}$ is a permutation matrix.
- $\phi(f_s) = \phi(f_{\Lambda s})$ where Λ is a diagonal invertible matrix.
- $\phi(f_{As}) \leq \phi(f_s)$ if all elements of s are independent and the matrix $\boldsymbol{A}$ is invertible.

Commonly used contrast functions involve kurtosis, negentropy and mutual information.

Contrast Function

Negentropy as a measure of nongaussianity.

$$J(f_x) = H(f_z) - H(f_x),$$

where $E(\boldsymbol{z}) = E(\boldsymbol{x})$ and $\mathrm{cov}(\boldsymbol{z}) = \mathrm{cov}(\boldsymbol{x})$, $H(\cdot)$ differential entropy.

Hyvarinen (1997) negentropy of $\boldsymbol{s}$ can be approximated by

$$J_G(f_s) = [E_s(G(s)) - E_z(G(z))]^2,$$

where z is a Gaussian random variable with mean zero and variance one, $G(\cdot)$ is a nonquadratic function.

FastICA

The R package `fastICA` is based on this method using

$$G_1(u) = \log \ \cosh \ au, \qquad\qquad G_2(u) = \exp(-\frac{u^2}{2}),$$

where a is a constant such that $1 \leq a \leq 2$.

More information in Hyvarinen, Karhunen, and Oja (2003).

Unified Framework for Group ICA

Guo and Pagnoni (2008) and Guo (2011) - EM based algorithm.

- Assume the mixing matrix $\boldsymbol{A}$ is square,
- Define a structure for the mixing matrix,
- Model densities of underlying sources using Gaussian mixtures,
- Parameter estimation via EM-algorithm.

Shi and Guo (2016) incorporate covariates within group ICA.

ProDenICA, Distance Covariance, LCA

ProDen ICA proposed by Hastie and Tibshirani (2002)

- Model densities of underlying sources using exponentially tilted Gaussian densities,
- Estimate the mixing matrix using a fixed point algorithm.

ICA via Distance Covariance, Matteson and Tsay (2011)

- Estimate components by targeting independence,
- Define independence via Distance Covariance.

Likelihood Component Analysis, Risk, et. al (2016)

- Allows for non-square mixing matrices,
- Options for modeling densities of underlying sources.

Group ICA

$$\boldsymbol{S}(q, v) = \boldsymbol{W}(q, .)\boldsymbol{X}(., v),$$

$\boldsymbol{W} = \boldsymbol{A}^{-1}$, $\boldsymbol{W}(q, .)$ - *qth* row of $\boldsymbol{W}$, $\boldsymbol{X}(., v)$ - *vth* column of $\boldsymbol{X}$.

$$\boldsymbol{S}(q, 1), \ldots, \boldsymbol{S}(q, V) \sim f_q(\cdot).$$

The likelihood function for ICA model

$$L(\boldsymbol{S}) = \prod_{v=1}^{V} \prod_{q=1}^{Q} f_q[\boldsymbol{S}(q, v)],$$

$$L(\boldsymbol{W}, f) = \prod_{v=1}^{V} \prod_{q=1}^{Q} f_q[\sum_{l=1}^{Q} \boldsymbol{W}(q, l)\boldsymbol{X}(l, v)].$$

Estimate the matrix $\boldsymbol{W}$ and densities f_q given the observed $\boldsymbol{X}$.

Mixtures for Estimating Densities

We parameterize the density of $\boldsymbol{S}_q$ as a mixture density:

$$f_q(s) = \sum_{j=1}^{N} \theta_{qj} \phi\left(\frac{s - \mu_{qj}}{\sigma_q}\right) \frac{1}{\sigma_q},$$

where $\phi(\cdot)$ is the standard normal density function.

- The means μ_{qj} and the standard deviations σ_j - fixed.
- The estimation of $\theta_{q1}, \ldots, \theta_{qN}$ is performed via a modified EM algorithm.

Eloyan, A. and Ghosh, S.K. (2011) *Smooth Density Estimation with Moment Constraints Using Mixture Distributions.* J. of Nonparametric Statistics. 23, 2, 513-531.

Independent Component Analysis

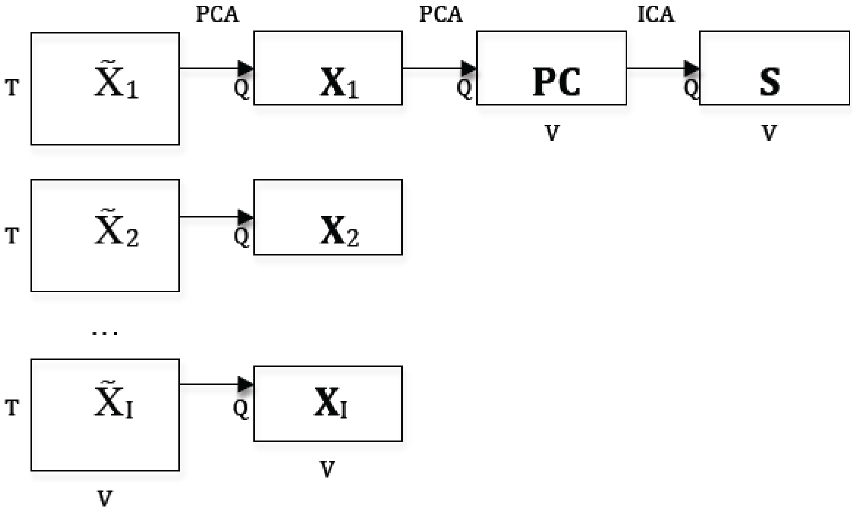
The log-likelihood of ICA is obtained as

$$l(\boldsymbol{W}, \widehat{f}) = \sum_{v=1}^{V} \sum_{q=1}^{Q} \log\{\hat{f}_q(\sum_{l=1}^{Q} x_{vl} w_{lq})\} + V \log|\det \boldsymbol{W}|.$$

Estimate the mixing matrix $\boldsymbol{W} = \boldsymbol{A}^{-1}$ and the densities $\hat{f}_q$ via an iterative optimization algorithm.

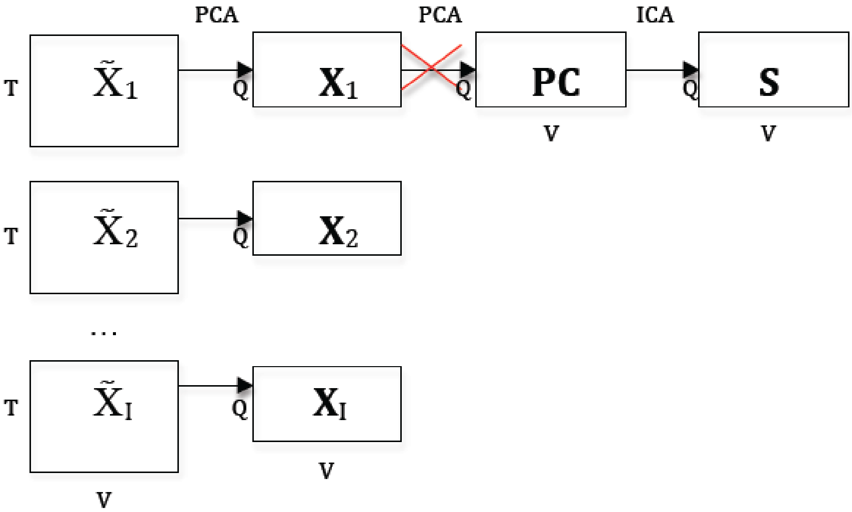
Eloyan, A. and Ghosh, S.K. (2013) *A Semiparametric Approach to Source Separation using Independent Component Analysis.* Comp. Stat. and Data Analysis. 58, 383-396.

Group Independent Component Analysis



Two-stage singular value decomposition.

Group Independent Component Analysis



Two-stage singular value decomposition.

Iterative Algorithm, HDICA

1. $\boldsymbol{S}_i = \widehat{\boldsymbol{W}_i}\boldsymbol{X}_i$
2. Estimate the weights of the density for each independent component. $\boldsymbol{\theta}_q$ is estimated by an EM algorithm.
3. Compute the derivative and Hessian for the log-likelihood.

$$L(\widehat{\boldsymbol{W}}) = \sum_{i=1}^{\mathrm{I}} \sum_{v=1}^{V} \sum_{q=1}^{Q} \log[f_q(\widehat{\boldsymbol{W}}_{iq}\boldsymbol{X}_{iv})] + V \log|\det \widehat{\boldsymbol{W}_i}|.$$

4. $\widehat{\boldsymbol{W}_i}^{new} = \widehat{\boldsymbol{W}_i} - L''(\widehat{\boldsymbol{W}_i})^{-1} L'(\widehat{\boldsymbol{W}_i})$
5. Stopping rule

$$\max \|\widehat{\boldsymbol{W}_i} - \widehat{\boldsymbol{W}_i}^{new}\| < \delta$$

Eloyan, A., Crainiceanu, C.M., and Caffo, B.S. (2013) Likelihood based population independent component analysis. *Biostatistics*. 14, 3, 514-527.
Chen, S. Huang, L., Qui, H., Nebel, M.B., Mostofsky, S.H., Pekar, J.J., Lindquist, M.A., Eloyan, A., and Caffo, B.S. (submitted) Parallel Group Independent Component Analysis for Massive fMRI Data Sets.

Bayesian Independent Component Analysis

The noisy ICA model:

$$\boldsymbol{X} = \boldsymbol{A}\boldsymbol{S} + \boldsymbol{E},$$

$$X_{tv}|\boldsymbol{A}, \boldsymbol{S}, \sigma_e \sim N(\boldsymbol{A}_{t.}\boldsymbol{S}_{.v}, \sigma_e^2)$$

Proposed priors:

$$\sigma_e^2|\alpha_e, \beta_e \sim \mathit{InverseGamma}(\alpha_e, \beta_e).$$

$$S_{qv}|Z_{qv} = k \sim N(\mu_k, \sigma_N^2),$$

$$P[Z_{qv} = k] = \theta_{qk},$$

$$\boldsymbol{\theta}_q = (\theta_{q1}, \theta_{q2}, \ldots, \theta_{qN})|\alpha \sim \mathit{Dirichlet}(\alpha, \alpha, \ldots, \alpha),$$

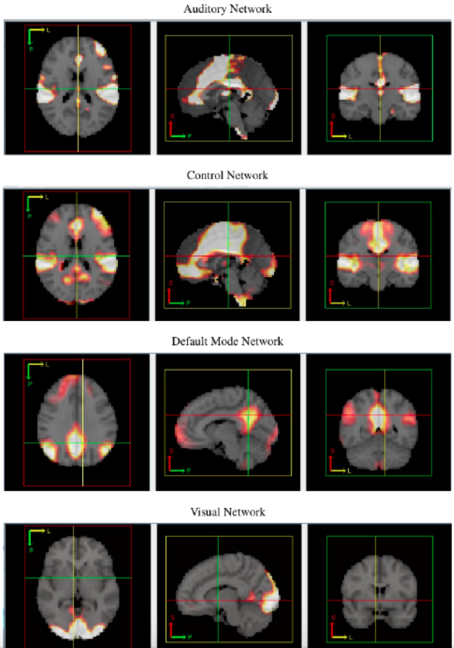
where $v = 1, \ldots, V$, $t = 1, \ldots, Q$ and $k = 1, 2, \ldots, N$.

The 1000 Functional Connectomes Project Dataset

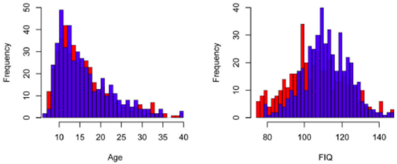
- More than 1400 scans available online.
- The scans are collected using a 3T scanner.
- For the subset used in this analysis the number of time points was $T = 119$.
- Standard image processing was performed to register the data to the MNI standard brain space.

$\boldsymbol{W}_i$ and $\boldsymbol{S}$ are estimated via the parallel HDICA algorithm.

Results for 301 Subjects

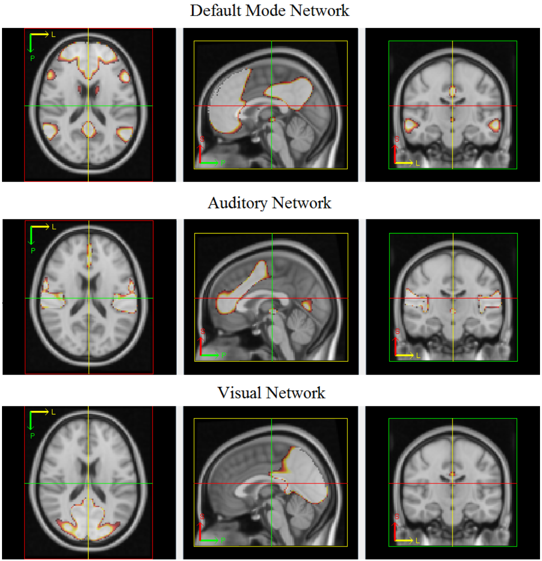


ICA based Connectivity Analysis - ABIDE



- 379 ASD.
- 400 typically developing
- For the subset used in this analysis the number of time points was $T = 220$.
- Standard image processing was performed to register the data to the MNI standard brain space.

ICA based Connectivity Analysis - ABIDE



Summary and Extensions

- Connectivity using ICA in Autism looking at motor function.
- New methods for finding brain networks for large groups of fMRI data.
- Extensions of existing methods for novel types of data.

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SMART group
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