Online Scheduling for Resource Allocation of Differentiated Services: Optimal Settings and Sensitivity Analysis

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***Abstract*— In this paper we investigate in detail the properties of a dynamic resource allocation scheme that utilizes online measurements to optimally adjust scheduling weights and to achieve the required QoS, under a given pricing structure. Extending our previous work, we consider here an additional important QoS parameter, namely, the average queueing delay, in order to provision favorable QoS guarantees for *all* classes of traffic, especially the delay-sensitive one. The objective of this paper is to formally investigate optimal settings that guarantee an improved QoS performance, and develop fundamental insights based on a detailed, case-by-case mathematic model that takes into account all the relevant QoS parameter constraints.**

I. Introduction

Emerging bandwidth and delay-sensitive applications such as voice over IP (VoIP), video-conferencing, online gaming, and interactive television, have made imperative the development of scheduling algorithms that provide differentiated Quality of Service (QoS) guarantees to multiple classes of traffic.

The high variability of the traffic implies that *static* bandwidth reservation protocols accompanied by over-provisioning of network links leads to significant under-utilization of available resources. On the other hand, a *dynamic* allocation of bandwidth that closely tracks the prevailing traffic characteristics can achieve significant savings, while at the same time satisfying QoS guarantees (e.g., end-to-end delay, jitter or packet loss probability), as guaranteed by service-level agreements (SLAs). Implementation of such dynamic schemes requires *efficient traffic monitoring* and estimation policies coupled with adaptive bandwidth allocation mechanisms.

In the meanwhile, pricing schemes and charging methods are also assuming a central role, and have been attracting significant attention recently. From the perspective of the network provider, any pricing scheme should maximize the revenue or profit, while also maximizing the utilization and minimizing the cost from the viewpoint of applications or users. In this paper, we model the router or switch as a "profit center", in which the price for different classes of traffic is predefined and the QoS constraints are also included in the general optimization model.

Significant contributions have been made already in related areas such as traffic measurement and estimation [1], including effective bandwidths [2], measurement-based admission control (MBAC) [3], [4], [5], [6], self-sizing network frameworks [7], [8], [9] and QoS adaptive routing [10], [11], [12]. Shin *et al.* proposed the adaptive allocation of scheduler weights according to the average queue length of the premium service, in which only QoS constraints of premium service are considered [13]. More recently, Chandra *et al* [14] described a dynamic resource allocation technique that uses on-line measurements. In general, there have been limited advances in formally defined, control-theoretic closed-loop methodologies for adaptive scheduling.

In our previous paper [15], we extended these approaches by introducing an adaptive mechanism for generalized schedulers under periodic estimates of traffic and the system's state. Furthermore, we conducted experiments investigating the effect of various factors on performance and robustness, and we initiated a formal description of this scheme by sketching the analytical derivation of the optimal settings. However, only one of the QoS parameters, namely, the loss probability, was taken into account directly in our previous scheme because of the inherent characteristics of the incorporated measurement algorithm [6].

Continuing our previous work, we consider here another important QoS parameter, namely, the average queue delay, in order to provision favorable QoS guarantees for all classes of traffic, especially the delay-sensitive class. The objective of this paper is to formally investigate settings that guarantee an improved QoS performance, and develop fundamental insights based on a detailed mathematic model that takes into account all the relevant QoS parameter constraints.

The contribution of this paper is two-fold: First, we propose and describe the simultaneous optimization of the triplet (loss probability, average queue delay and profit) while taking into account the individual connections' cost and profit or "utility". Second, we specify a more detailed derivation of the fluid queueing model in [14] including a case-by-case analytical solution to the optimization problem.

The rest of the paper is organized as follows: In Section II the optimization problem corresponding to our adaptive scheduling under QoS and pricing is formulated. In Section III we discuss in detail the form of the solution in different cases depending on the constraints, while in Section IV investigate the calculation of the optimal values. In Section V we summarize the online measurement method used to obtain the effective bandwidth for the various traffic classes that is used in the solution of the optimal resource allocation problem. Section VI contains a brief sensitivity analysis of the optimization problem. Finally, in Section VII we summarize and conclude with open issues and future research directions.

II. Modeling Framework and Problem Formulation

It is assumed that the users of the system under study can be grouped into the following three service classes: delay-sensitive (class 1), loss-sensitive (class 2) and best effort (class 3).

The main components of our adaptive scheme are: a *traffic measurement module* that provides an accurate estimation of the future traffic load of the different classes under consideration, a *scheduling module* that deals with the packet forwarding mechanism and a *decision module* that determines how bandwidth is distributed among the various classes of traffic. A schematic representation of the system is shown in Figure 1.

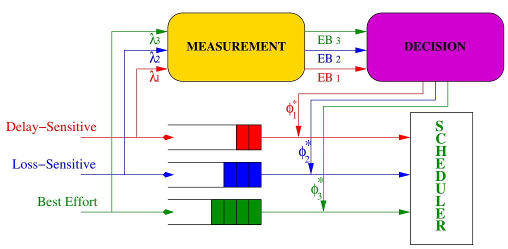

Fig. 1. Illustration of the adaptive framework under consideration and its components.

The coordination of these three components is described next: when a job/customer of class i arrives at the scheduler, it is assigned the queue of the corresponding queue, waiting to receive service from the scheduling module. At the same time, the measurement module updates the arrival rate statistics of the corresponding traffic class, and provides an estimate of the *effective bandwidth*. It should be noted that the measurement module performs the above operation over a pre-specified time interval (window). Finally, the decision module allocates the service rate (bandwidth) to the queues using information about the effective bandwidth and the queue length processes. We discuss next the problem that the decision module solves at every decision time instant.

We take a social point of view [16] and are interested in optimizing the network's provider profit, while satisfying the QoS requirements of the users' traffic classes. The provider's long-term profit consists of a revenue and a cost component. The revenue part is given by $R = \sum_i p_i \phi_i C$, where p_i is the price charged to the users per unit of time for the utilization of the system's resource, C is the capacity of the system and ϕ_i is the proportion of the resources (e.g. bandwidth) allocated to class $i = 1,2,3$. In our formulation, it is assumed that $p_i > p_j$ for $i > j$. The cost component is given by $\mathscr{C} = b_i \bar{q_i}/(C\phi_i)$, where b_i is the per unit of time cost incurred by class i users and $\bar{q_i}$ is the average queueing length over time of that class. The cost component basically captures the cost associated with queueing delays in the system. Hence, the provider's problem becomes

$$\max_{\phi} R - \mathscr{C} = \max_{\phi} \sum_{i=1}^{3} p_i \phi_i C - \frac{b_i \bar{q_i}}{\phi_i C}, \qquad (1)$$

subject to the following constraints

$$\begin{gathered} \textstyle\sum_{i=1}^{3} \phi_i \leq 1 \\ \phi_i \geq \max\{EB_i/C, \tfrac{\bar{q_i}}{Cd_i}\},\ \ i = 1,2 \\ \phi_3 \geq EB_3/C \end{gathered}$$

where d_i is the desired queueing delay for the $i-$the class and EB_i its effective bandwidth, and with $\phi = (\phi_1, \phi_2, \phi_3)$. The first constraint is a feasibility one, while the second constraint incorporates the QoS requirements of the users into the optimization problem.

The average queue length plays an important role in the above formulation and could be derived from the fluid model [17]. Notice that the instantaneous queue length process at time t for class i can be obtained through the formula

$$q_i(t) = \max[q_i^0 + (EB_i - \phi_i C)t, 0] \qquad (2)$$

where q_i^0 is initial queue length of the i-th class and t denotes the length of the time interval. The max operator prevents the process from taking negative values.

In our proposed scheme, the share of system resources (bandwidth) allocated to the various classes would be dynamically assigned over an adaptive window W. Thus, the average queue length of class i during an adaptive window W is given by

$$\begin{aligned} \bar{q_i} &= \frac{1}{W}\int_0^{\tau_i} q_i(t) \\ &= \frac{\tau_i}{W}[q_i^0 + \frac{\tau_i}{2}(EB_i - \phi_i C)] \qquad (3) \end{aligned}$$

where τ_i is the time instant when the queue length process becomes zero and determined by $\tau_i = \min\{t_i^0, W\}$ with t_i^0 being the time it takes to empty the queue. In turn, t_i^0 could be obtained by $t_i^0 = q_i^0/(\phi_i C - EB_i)$, given the initial queue length q_i^0. In the above derivation it is assumed that the length of the adaptive window W is such that the above result holds due to Little's law [16]. It can be seen that the average queue length depends on the τ_i and therefore we distinguish the following cases:

a) Condition 1: if $t_i^0 < W$, we obtain

$$\phi_i > \frac{q_i^0}{WC} + \frac{EB_i}{C} = \varphi_i^1 \qquad (4)$$

The average queue length $\bar{q_i}$ can be written as:

$$\bar{q_i} = \frac{[q_i^0]^2}{2W} \times \frac{1}{\phi_i C - EB_i}$$

Then, the delay component of the QoS can be rewritten as follows:

$$\phi_i \geq \frac{EB_i}{2C} + \sqrt{(\frac{EB_i}{2C})^2 + (\frac{q_i^0}{C})^2 \times \frac{1}{2d_i W}} = \varphi_i^2 \qquad (5)$$

Hence, equations 4 and equation 5, imply that $\phi_i = \max[\varphi_i^1, \varphi_i^2]$.

b) Condition 2: if $t_i^0 \geq W$, we obtain

$$\phi_i \leq \frac{q_i^0}{WC} + \frac{EB_i}{C} = \varphi_i^1 \qquad (6)$$

As before we also have that

$$\bar{q_i} = q_i^0 + \frac{W}{2}(EB_i - \phi_i C)$$

which in turn gives that

$$\phi_i \geq \frac{\frac{q_i^0}{WC} + \frac{EB_i}{2C}}{\frac{1}{2} + \frac{d_i}{W}} = \varphi_i^d \qquad (7)$$

However, in some cases φ_i^d may be greater than φ_i^1, which creates a conflict between the constraints. In such a case we must at least have $\phi_i \geq EB_i$, whereas if φ_i^d is no greater than than φ_i^1, then the lower bound of ϕ_i should be given by the maximum of EB_i and φ_i^d. In summary we have:

$$\varphi_i^3 = \begin{cases} EB_i & \text{if } \varphi_i^d > \varphi_i^1 \\ max[EB_i, \varphi_i^d] & \text{if } \varphi_i^d \leq \varphi_i^1 \end{cases}$$

Hence, under condition 2 we get the following constraint $\varphi_i^3 \leq \phi_i \leq \varphi_i^1$.

III. Optimal Allocation of Resources

Equation 1 shows that the problem under consideration is a nonlinear optimization one with inequality constraints. In the ensuing discussion it is assumed that the system is *stable*, in the sense that the sum of the input rates does not exceed the capacity of the router; i.e. $\sum_{i=1}^{3} EB_i < C$.

In this subsection we outline how the optimal solution is obtained. It is important to note that the constraints change the nature of the objective function over different regions of the parameter space. We start by considering the nature of the constraints.

Notice that at the optimum we must have $\sum_{i=1}^{3} \phi_i = 1$; otherwise, resources would be wasted. Furthermore, since the best effort class (3rd class) pays the lowest price and has no constraints on its delay, we get that at the optimum $\phi_3 = EB_3/C$. These two facts show that the optimal solution satisfies

$$\phi_1 + \phi_2 = 1 - EB_3/C.$$

Furthermore, the problem has been reduced to one involving only two decision variables, namely ϕ_1 and ϕ_2.

Since the average queue length from equation 3 is non-differentiable, our optimization problem can be divided into the following four cases:

1) Case 1: Suppose that both decision variables (i.e. ϕ_1 and ϕ_2) satisfy condition 1. The optimization problem can be written as:

$$f(\phi_1, \phi_2) = p_1\phi_1 C + p_2\phi_2 C - \frac{b_1 C[q_1^0]^2}{2W(\phi_1 C - EB_1)\phi_1 C} - \frac{b_2 C[q_2^0]^2}{2W(\phi_2 C - EB_2)\phi_2 C}$$

subject to the constraints

$$\sum_{i=1}^{2} \phi_i = 1 - EB_3/C$$
$$\phi_1 \geq \max[\varphi_1^1, \varphi_1^2]$$
$$\phi_2 \geq \max[\varphi_2^1, \varphi_2^2]$$

2) Case 2: Suppose that ϕ_1 satisfies condition 1, whereas ϕ_2 satisfies condition 2. In this case the optimization problem can be written as:

$$\max_{\phi_1,\phi_2} f(\phi_1, \phi_2) = p_1\phi_1 C + p_2\phi_2 C - \frac{b_1 \times [q_1^0]^2}{2W(\phi_1 C - EB_1)\phi_1 C} - b_2 \times (\frac{q_2^0 + \frac{W}{2}EB_2}{\phi_2 C} - \frac{W}{2})$$

subject to the constraints

$$\sum_{i=1}^{2} \phi_i = 1 - EB_3/C$$
$$\phi_1 \geq \max[\varphi_1^1, \varphi_1^2]$$
$$\varphi_2^3 \leq \phi_2 \leq \varphi_2^1$$

3) Case 3: Suppose that ϕ_1 satisfies condition 2, whereas ϕ_2 satisfies condition 1. Thus, the optimization problem can be written as:

$$\max_{\phi_1,\phi_2} f(\phi_1, \phi_2) = p_1\phi_1 C + p_2\phi_2 C - b_1 \times (\frac{q_1^0 + \frac{W}{2}EB_1}{\phi_1 C} - \frac{W}{2}) - \frac{b_2 \times [q_2^0]^2}{2W(\phi_2 C - EB_2)\phi_2 C}$$

subject to the constraints

$$\sum_{i=1}^{2} \phi_i = 1 - EB_3/C$$
$$\varphi_1^3 \leq \phi_1 \leq \varphi_1^1$$
$$\phi_2 \geq \max[\varphi_2^1, \varphi_2^2]$$

4) Case 4: Suppose that both decision variables satisfy condition 2. The optimization problem becomes:

$$\max_{\phi_1,\phi_2} f(\phi_1,\phi_2) = p_1\phi_1 C + p_2\phi_2 C - b_1 \times \left(\frac{q_1^0 + \frac{W}{2}EB_1}{\phi_1 C} - \frac{W}{2}\right) - b_2 \times \left(\frac{q_2^0 + \frac{W}{2}EB_2}{\phi_2 C} - \frac{W}{2}\right)$$

subject to the following constraints

$$\sum_{i=1}^{2} \phi_i = 1 - EB_3/C$$
$$\varphi_1^3 \le \phi_1 \le \varphi_1^1$$
$$\varphi_2^3 \le \phi_2 \le \varphi_2^1$$

Insight about the nature of the problem under consideration is obtained by examining the following plots. Suppose that the intersection of the constraints from Case 1 occurs inside the region determined by the inequality $\phi_1 + \phi_2 \le 1 - EB_3/C$ (see Figure 2). It is then easy to see that we do not have to consider the optimization problem given in Case 4.

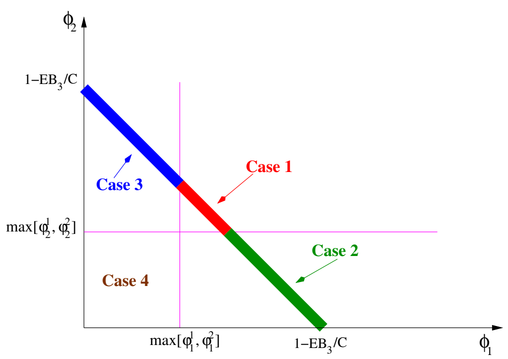

Fig. 2. Structure of the overall optimization problem when case 1 is feasible.

If, on the other hand, the intersection of the constraints from Case 1 occurs outside the region determined by $\phi_1 + \phi_2 \le 1 - EB_3/C$ (see Figure 3), then Case 1 becomes infeasible and we have only to consider the optimal solutions for the remaining 3 cases. In the latter case, we need to further examine the constraints used in cases 2 and 3. Given the magnitude of the lower bounds φ_1^3 and φ_2^3 we end up either with restricted feasibility regions for the optimization problem defined in cases 2 and 3, as Figure 4 indicates, or with only case 4 being feasible, as Figure 5 shows.

IV. Calculating the Optimal Solution

In this section we continue our investigation into the solution of the optimization problems given in cases 1-4. In principle, the problem can be solved by nonlinear optimization methods. For the objective function derived in cases 1-4 it can be shown that the Hessian matrix of second

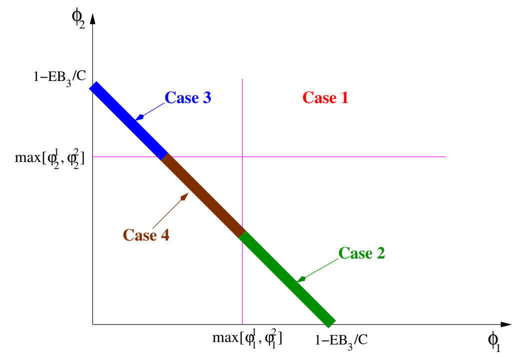

Fig. 3. Structure of the overall optimization problem when case 1 is not feasible.

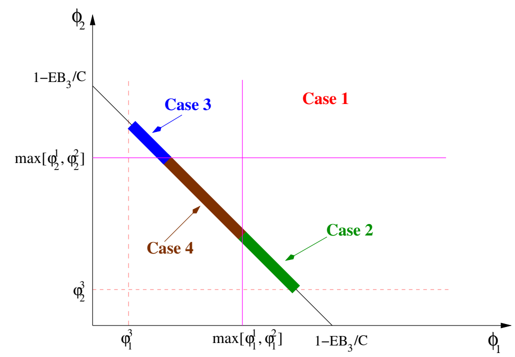

Fig. 4. A more refined view of the feasible cases, when case 1 is not feasible.

partial derivatives is negative definite. For example, for the objective function in case 4 the Hessian is given by

$$\mathbf{H} = \begin{pmatrix} -\frac{2b_1(q_1^0 + \frac{W}{2}EB_1)}{\phi_1^3 C} & 0 \\ 0 & -\frac{2b_2(q_2^0 + \frac{W}{2}EB_2)}{\phi_2^3 C} \end{pmatrix}$$

which, under the feasibility constraints, is negative definite (since all its eigenvalues are negative). Therefore, it can be concluded that the objective function is jointly concave and hence possesses a unique maximum (maybe at boundary point), which can be obtained by solving for the classical

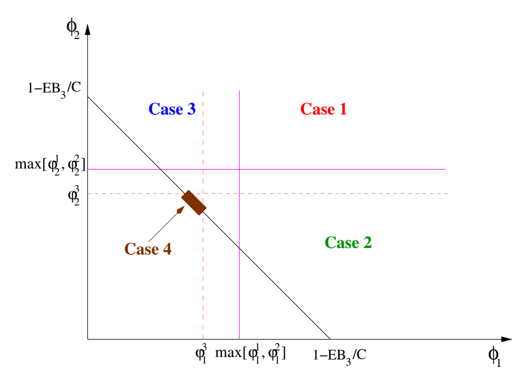

Fig. 5. Another more refined view of the feasible cases, when case 1 is not feasible.

Kuhn-Tucker conditions.

However, by further exploring the structure of the problem at hand we can obtain the optimal solution in a more inexpensive and easy to implement manner. We illustrate the main steps of the proposed approach on the problem defined in case 4. The other optimization problems (cases 1-3) can be solved in an analogous manner (the details can be found in [18]). By solving the feasibility constraint $\phi_1 + \phi_2 = 1 - EB_3/C$ for ϕ_2 and substituting that value in the objective function we find a new objective function of a single variable given by

$$g(\phi_1) = p_1\phi_1 C + p_2(1 - \frac{EB_3}{C} - \phi_1)C - b_1(\frac{q_1^0 + \frac{W}{2}EB_1}{\phi_1 C} - \frac{W}{2}) - b_2(\frac{q_2^0 + \frac{W}{2}EB_2}{(1 - \frac{EB_3}{C} - \phi_1)C} - \frac{W}{2})$$

Its first and second derivatives are given next:

$$g'(\phi_1) = (p_1 - p_2)C + \frac{b_1(q_1^0 + \frac{W}{2}EB_1)}{\phi_1^2 C} - \frac{b_2(q_2^0 + \frac{W}{2}EB_2)}{(1 - \frac{EB_3}{C} - \phi_1)^2 C}$$

$$g''(\phi_1) = -2b_1 \times \frac{q_1^0 + \frac{W}{2}EB_1}{\phi_1^3 C} - 2b_2 \times \frac{q_2^0 + \frac{W}{2}EB_2}{(1 - \frac{EB_3}{C} - \phi_1)^3 C}$$

It can easily be seen that $g''(\phi_1) < 0$, which implies that $g(\phi_1)$ is a concave function. Plots of the objective function $g(\phi_1)$ and its first derivative $g'(\phi_1)$ are shown in Figure 6. The derivation of the $g'(\phi_1)$ helps us determine the

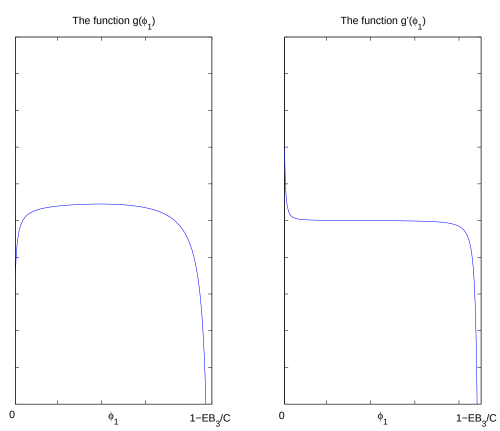

Fig. 6. The graph of $g(\phi_1)$ and $g'(\phi_1)$

optimal solution, as follows. First denote the lower bound of the feasible region by B_L and the upper bound by B_U. If $g'(B_L) > 0$ and $g'(B_U) > 0$, then the optimal solution is given at the boundary by B_L, whereas if $g'(B_L) < 0$ and $g'(B_U) < 0$, then the optimal solution is given at the other boundary point B_U. Finally, if $g'(B_L) > 0$ and $g'(B_U) < 0$, then the optimal solution lies in the interior of the interval (B_L, B_U) and must be found by numerical root finding methods, such as the bisection method, or Newton's method [19].

The globally optimal solution is then obtained by calculating first the optimal solution $\phi_1^*(k),\ k = 1,2,3,4$ for the 4 cases and then keeping the maximum amongst the four. As discussed in the previous section, some of the cases may not be feasible, a fact that leads to a speed-up of the algorithm, which is given in pseudo-code form next.

Algorithm 1 Obtaining the optimal solution

```
Identify the N feasible cases by checking the underlying
feasibility constraints
while N > 0 do
  Obtain B_L^i, B_U^i for case i
  if g'(B_L^i) > 0 and g'(B_U^i) > 0 then
    φ_1^{i*} = B_L^i
  else if g'(B_L^i) < 0 and g'(B_U^i) < 0 then
    φ_1^{i*} = B_U^i
  else
    Use root finding method for solving g'(φ_1) = 0 to
    obtain φ_1^{i*}.
  end if
  Obtain the maximum for case i, max f^{i*}(φ_1^{i*}, φ_2^{i*})
  N = N − 1
end while
max f^*(φ_1^{i*}, φ_2^{i*}) = max_{i∈N} f^{i*}(φ_1^{i*}, φ_2^{i*})
```

Remark: In case the system becomes unstable over a window W (i.e. $\sum_{i=1}^{3} EB_i \geq 1$), then there is not adequate capacity to satisfy the QoS requirements of all three traffic classes. What would happen is that the constraint of the best effort class should be relaxed and replaced by $\phi_3 \leq EB_3$, which implies that the QoS requirements of that class would be sacrificed in order for the more profitable classes to be accommodated. Thus, this scenario can also be accommodated with our modeling framework and the details are given in [18].

V. Measurement Algorithm

The traffic envelope [6] has proved useful in obtaining online measurements. Furthermore, it turns out to be robust to the time dependence structure of traffic (e.g. Long-Range Dependence vs Short-Range Dependence). A brief description of the traffic envelope approach is given next.

Its basic measurement unit is the measurement slot, τ. A measurement window is adaptive, and comprised of varying number of measurement slots, $W_k = k\tau (k = 1, 2, \cdots, T)$. In a certain measurement window W_k, let $A[t, t+W_k]$ denote the counting process of arrivals in the interval $[t, t+W_k]$; thus, $A[t, t+W_k]/W_k$ is the arrival rate over that interval. The maximal rate for W_k over this time interval could be defined as $R_k = \max_t A[t, t+W_k]/W_k$.

Suppose $A_t = A[t\tau, (t+1)\tau]$ are the arrivals in the time slot starting from t. In this way, the maximal rate over the certain measurement window with the size of $k\tau$, for the past $T\tau$ from the current time t could be obtained by

$$R_k^1 = \frac{1}{k\tau} \max_{t-T+k \le s \le t} \sum_{u=s-k+1}^{s} A_u \quad \text{for} \quad k = 1, 2, \cdots, T. \quad (8)$$

This equation is introduced for considering burstiness over small time scales.

The current envelope R_k^1 is measured and updated every $T \cdot \tau$ measurement window, $R_k^n \leftarrow R_k^{(n-1)}$ for $k = 1, 2, \cdots, T$ and $n = 2, 3, \cdots, N$. The variance between envelopes over the past N windows could be computed by the following equation:

$$\sigma_k^2 = \frac{1}{N-1} \sum_{n=1}^{N} (R_k^n - \bar{R}_k)^2 \quad (9)$$

where $\bar{R}_k = \frac{1}{N} \sum_{n=1}^{N} R_k^n$ is the mean of past N envelopes.

The effective bandwidth in traffic envelope can also be calculated in both the small and the large time scales [20]. For the large time scale, the effective bandwidth is obtained by

$$EB_{large} = \bar{R}_T + \alpha_{large} \sigma_T \quad (10)$$

where $\bar{R}_T$ and σ_T are the mean and deviation for past N envelopes with the measurement window size of $T \cdot \tau$. And α_{large} is used to specify the confidence interval. It can be computed by the inverse of complementary CDF of an N(0,1) Gaussian distribution, $\alpha_{large} = Q^{-1}(\frac{\varepsilon \bar{R}_T}{\sigma_T})$.

For the small time scale, the EB is computed by

$$EB_{small} = \max_{k=1,2,\cdots,T} \frac{(\bar{R}_k + \alpha_{small} \sigma_k) k\tau}{k\tau - B/C} \quad (11)$$

where B and C are buffer size and capacity respectively. The mean $\bar{R}_k$ and deviation σ_k is for measurement window $k \cdot \tau$. And $\alpha_{small} = Q^{-1}(\frac{\varepsilon \bar{R}_k}{\sigma_k})$ is computed by using the same approach as α_{large}.

The algorithm gives the worst case effective bandwidth by choosing the maximum between the small-scale effective bandwidth and the large-scale one:

$$EB = \max\{EB_{large}, EB_{small}\} \quad (12)$$

VI. A Brief Discussion on Sensitivity Analysis

In this section we briefly explore the effect of the prices charged to the users (p_1, p_2) and the costs associated with queueing delays (b_1, b_2) on the optimal solution.

By regarding the objective function $g(\phi_1)$ as a function of two arguments , i.e. $\tilde{g}(\phi_1, p_1)$, and taking its partial derivative we get (for case 4 and analogously for all the other cases as well)

$$\frac{\partial^2 \tilde{g}(\phi_1, p_1)}{\partial p_1 \partial \phi_1} = C > 0.$$

Analogously we get that $\frac{\partial^2 \tilde{g}(\phi_1, p_2)}{\partial p_2 \partial \phi_1} = -C < 0$. The effect of the prices on the shape of the function $g(\phi_1)$ and its first derivative $g'(\phi_1)$ is illustrated in Figures 7-8. The above simple derivations (as well as the plots) indicate that the higher the price charged to the delay sensitive class, the higher (ceteris paribus) the bandwidth allocated to that class would be, if the optimal solution is located in the interior of the feasibility region. Analogously, the higher the price of the loss sensitive class, the lower the bandwidth allocated to the delay sensitive class and consequently the higher the bandwidth allocated to the loss sensitive case, in the presence of an optimal solution in the interior of the feasible region.

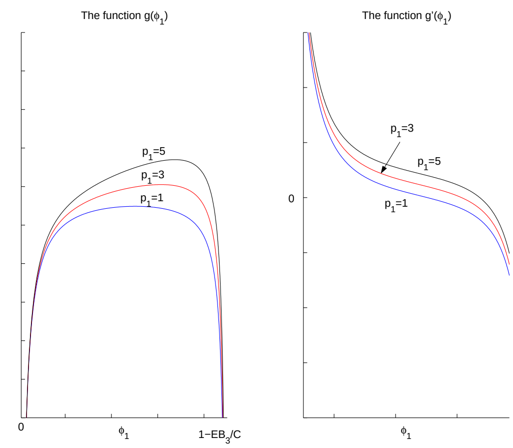

Fig. 7. The graph of $g(\phi_1)$ and $g'(\phi_1)$ for different prices of the delay-sensitive class.

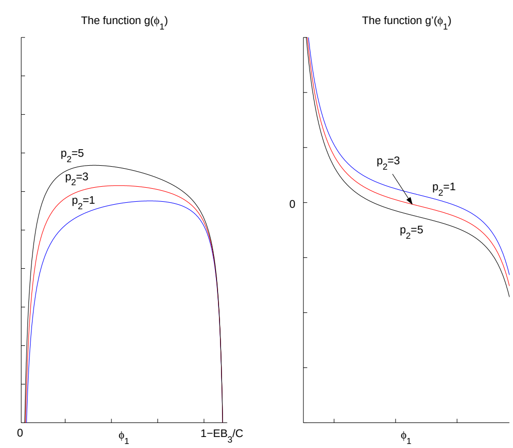

Fig. 8. The graph of $g(\phi_1)$ and $g'(\phi_1)$ for different prices of the loss sensitive class.

We now turn our attention to the cost component. Defining a function of two variables $\tilde{g}(\phi_1, b_1)$ (for case 4) and taking its derivative with respect to both arguments we get

$$\frac{\partial^2 \tilde{g}(\phi_1, b_1)}{\partial b_1 \partial \phi_1} = \frac{q_1^0 + \frac{W}{2} EB_1}{\phi_1^2 C} > 0.$$

An analogous derivation shows that $\frac{\partial^2 \tilde{g}(\phi_1, b_2)}{\partial b_2 \partial \phi_1} < 0$. It is easy then to conclude that the higher the cost of the delay for the delay sensitive case, the higher the bandwidth allocated to it, as Figure 9 also indicates. The intuitive explanation behind this result goes as follows: the higher the delay cost for the

1st class, the bigger the incentive of the provider to *decrease* the delay of that class' customers; hence, the higher the bandwidth allocated to the delay sensitive class. A similar reasoning applies to the loss sensitive class, which shows that the higher its delay cost, the higher the bandwidth allocated to it should be, which in turn implies (*ceteris paribus*) the lower the bandwidth allocated to the delay sensitive class (as can also be seen from Figure 10).

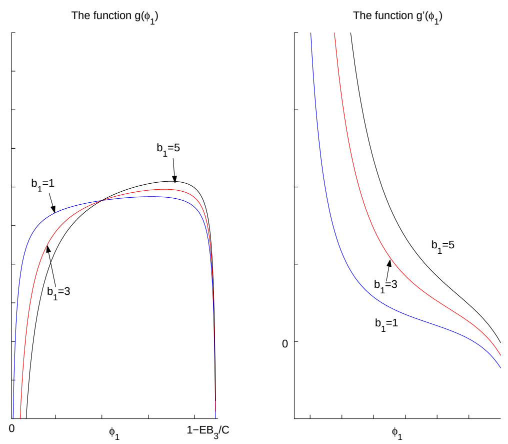

Fig. 9. The graph of $g(\phi_1)$ and $g'(\phi_1)$ for different values of the cost of the delay sensitive class.

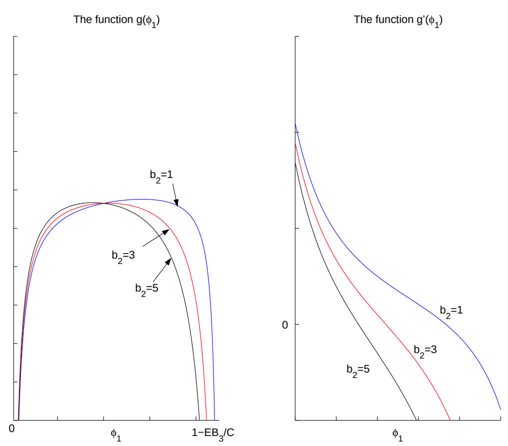

Fig. 10. The graph of $g(\phi_1)$ and $g'(\phi_1)$ for different values of the cost of the loss sensitive class.

VII. Conclusions and Future Work

Adaptive scheduling based on measurements of traffic and queueing state have the potential of greatly improving the efficiency of resource allocation techniques. Previously we have introduced a measurement-based adaptive scheduler and validated its performance with extensive simulation results. In this paper, we have formulated the online setting of adaptive schedulers as a formal optimization problem taking into account QoS constraints and the underlying pricing scenario. We then proceeded to study its solutions on a case-by-case basis, establishing the fundamental understanding required to be able to implement and utilize such schemes.

Continuing and extending our efforts in this area, we are working to analyze the behavior of the adaptive scheduler over time. In this paper the optimization problem given in section III is solved at every decision instant – which corresponds to the beginning of a new window W. Notice that the window size affects which constraints become binding in our optimization problem. In our previous work [15] we have empirically investigated the problem of dynamically adapting the size of the window W to changing traffic conditions. A topic of current research is to study the dynamics over time of the window size, as well as the long-term performance of the system under changing traffic patterns.

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