Amanda L. Traud and Katie Newhall

Network Topology Effects on Cascade-Induced Synchrony in Stochastic Dynamical Systems

Outline

- Introduction
  - Motivations
  - Brief Introduction
  - Network Data
  - Models
- The Influence of Network Structure
- Simplifying Network Structure
- Theory for Average Cascade Size
- Simulations versus Theory

Motivations

1. How does the network topology interact with pulse coupled dynamics on each node?
   - Two Axis Study:
     - Different Point Neuron Models
     - Different Network Topologies
2. Given Real World Data, how simplified can the network or the dynamics be and still qualitatively capture the original behavior?
   - From Degree Correlation and Distribution down to Mean Degree

Brief Introduction

- Simplified Neuronal Networks
  - Nodes, Edges
  - Undirected, Unweighted
- Point Neuron Models
  - Stochastic Current-Based Integrate-and-Fire
  - Coarse Grain by Discretizing Voltage and Time
    - Time
      - Discrete Time
      - Continuous time
    - Voltage
      - Single Bin ,Two States (Firing and Not Firing)
      - Multiple Bins, Plus a Firing State

Network Data Under Consideration

- Many Data Types
  - Real World Network Data:
    - C. elegans Neuronal Network(297 Nodes, Avg. Deg. ~14)
    - Facebook Social Network(3068 Nodes, Avg. Deg. ~78)
  - Manufactured Data:
    - Preferential Attachment Networks (PA)
    - Watts and Strogatz Small World Networks (WS)
    - K-Regular Ring (RI)
    - Erdos Renyi Random Networks (ER)

Models

- Integrate-and-Fire
  - Continuous Voltage, Continuous Time
  - Voltage is tracked using an ODE for each neuron
  - Spiking Modeled with Reset Mechanism
- Multi-bin Continuous Time
  - Continuous Time Markov Chain Model (CTMC)
  - Q – Probability Transition Rate Matrix
  - Spiking Modeled with Probability Transition Matrix, $P^{(S)}$
- Multi-bin Discrete Time
  - Discrete Time Markov Chain Model (DTMC)
  - P – Probability Transition Matrix $P = e^{Q\Delta t}$
  - Spiking Modeled with Probability Transition Matrix

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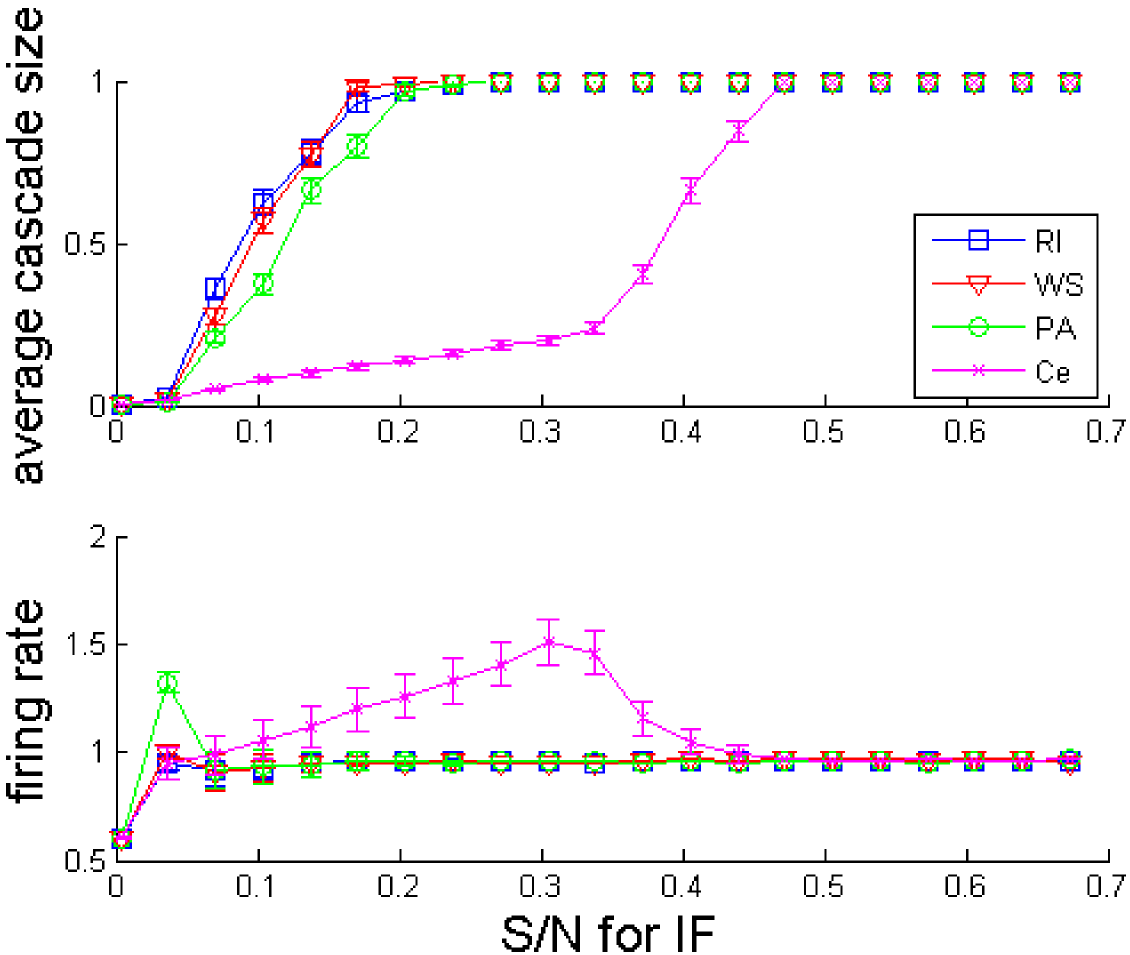
Multiple Networks, Same Model

- Ring, Watts Strogatz and Preferential Attachment Networks Match Average Degree and Number of Nodes
- Real World C. elegans Data Behaves Differently



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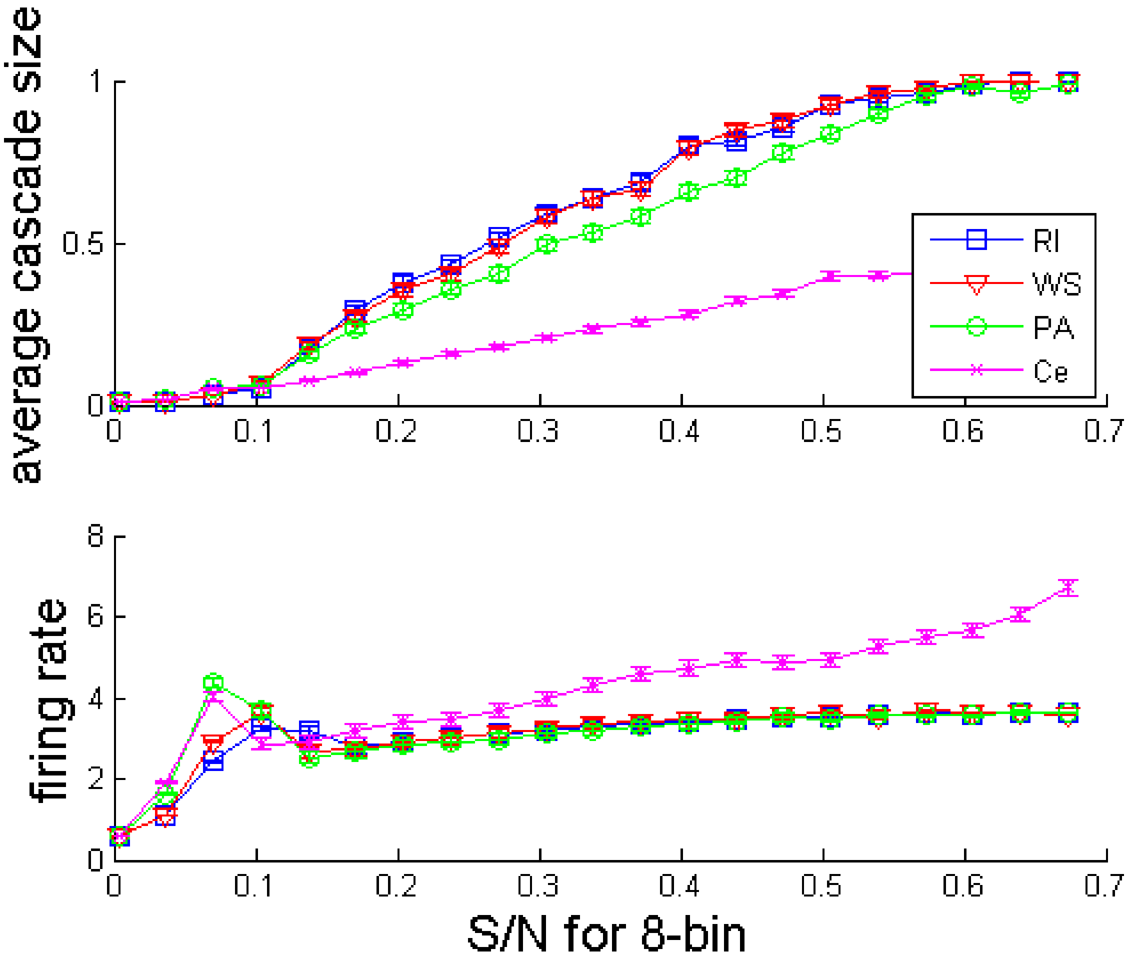

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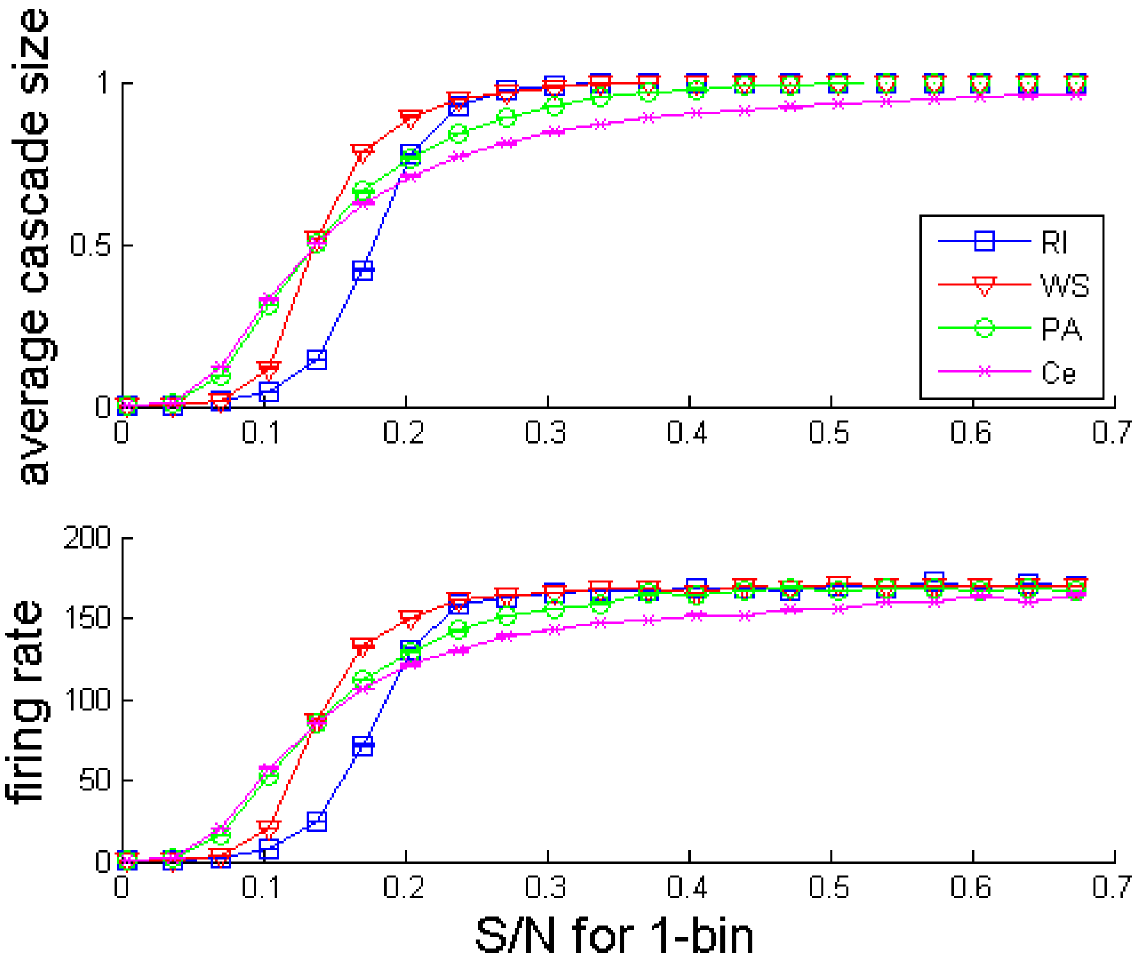

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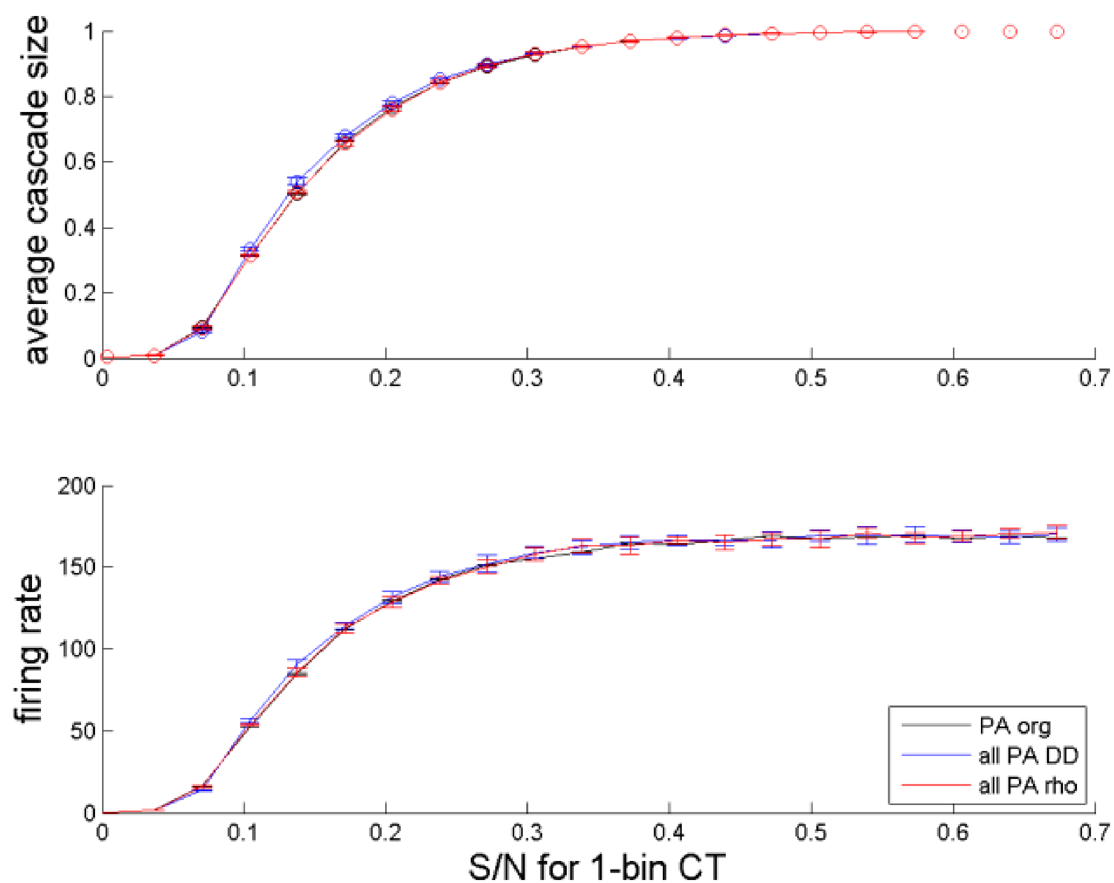
Three Models on Preferential Attachment

- Keeping Joint Degree Distributions (rho) constant matches model behavior*
- Keeping Degree Distribution (DD) constant matches model behavior



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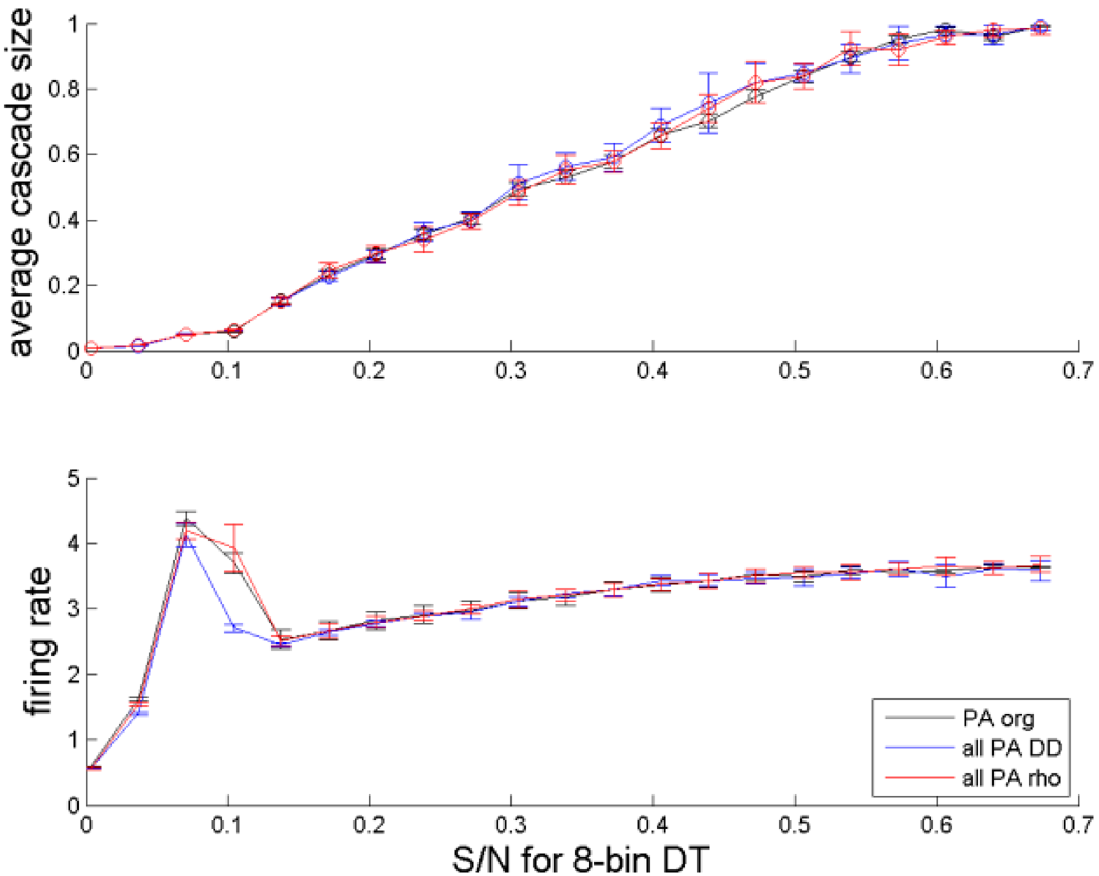

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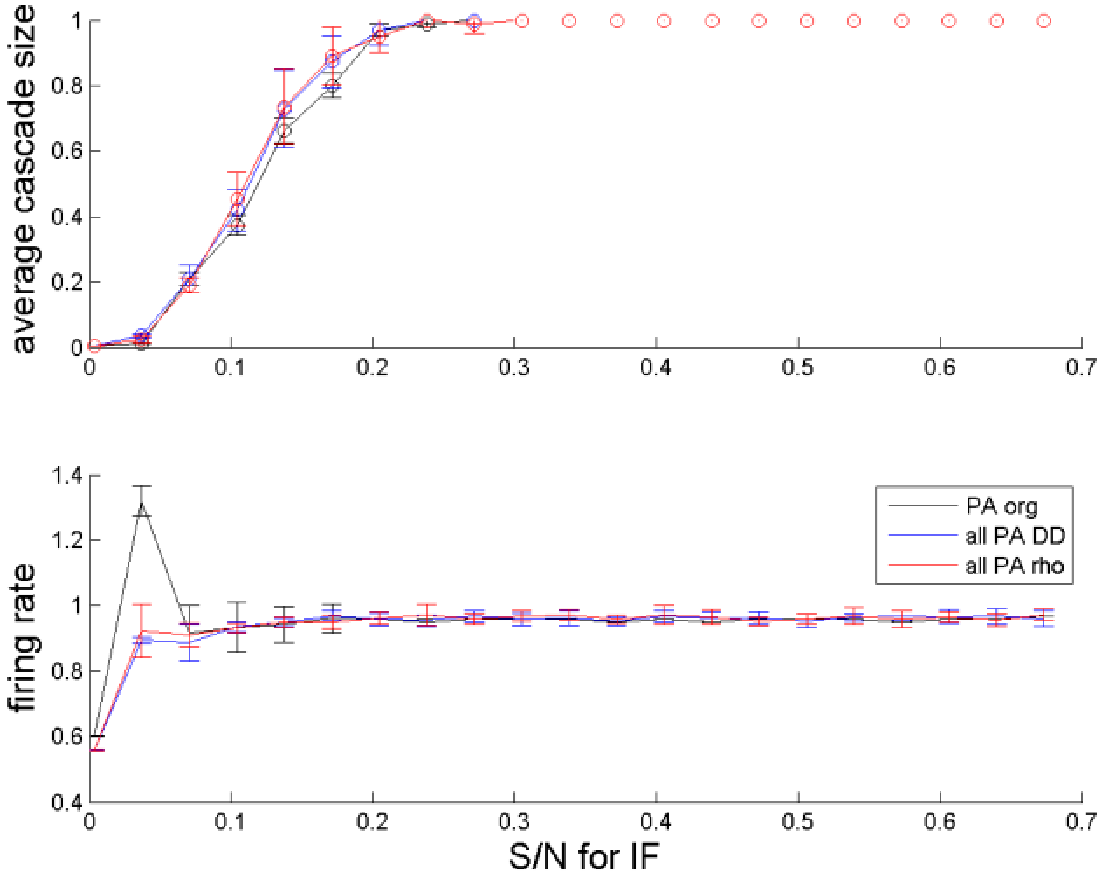

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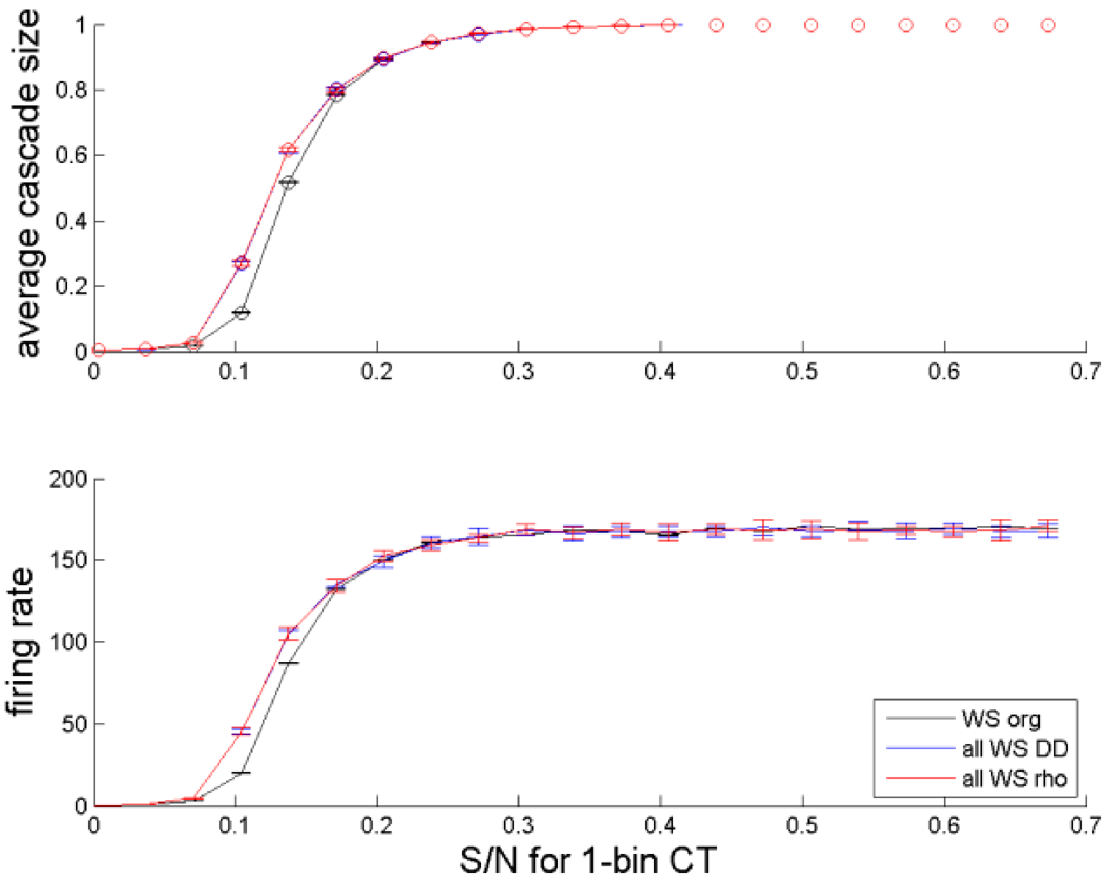

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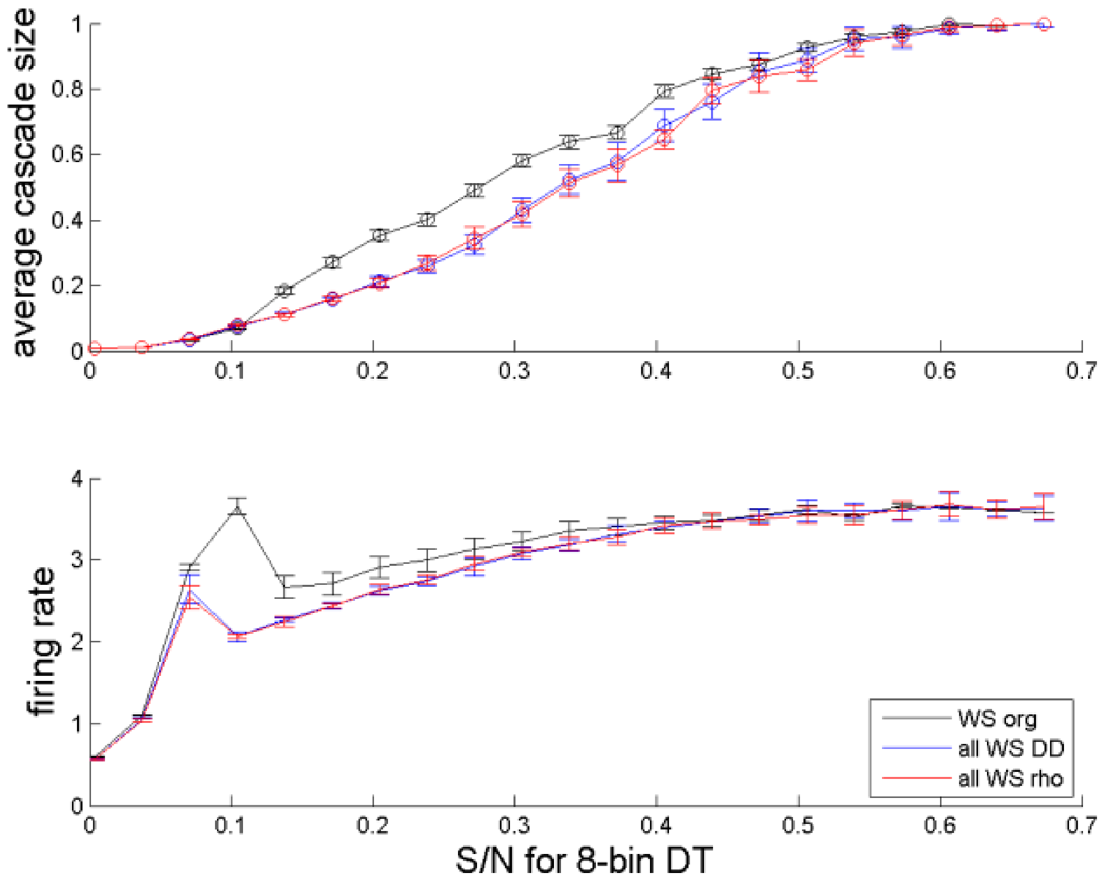

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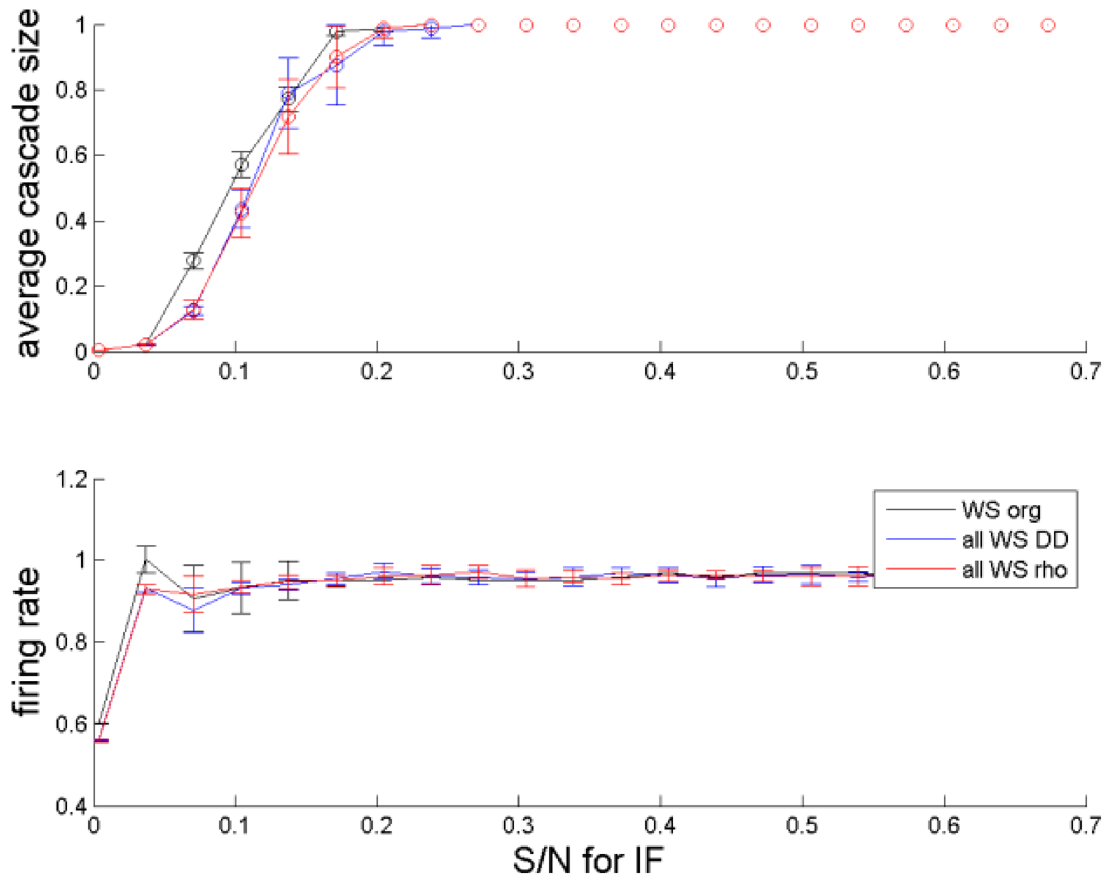

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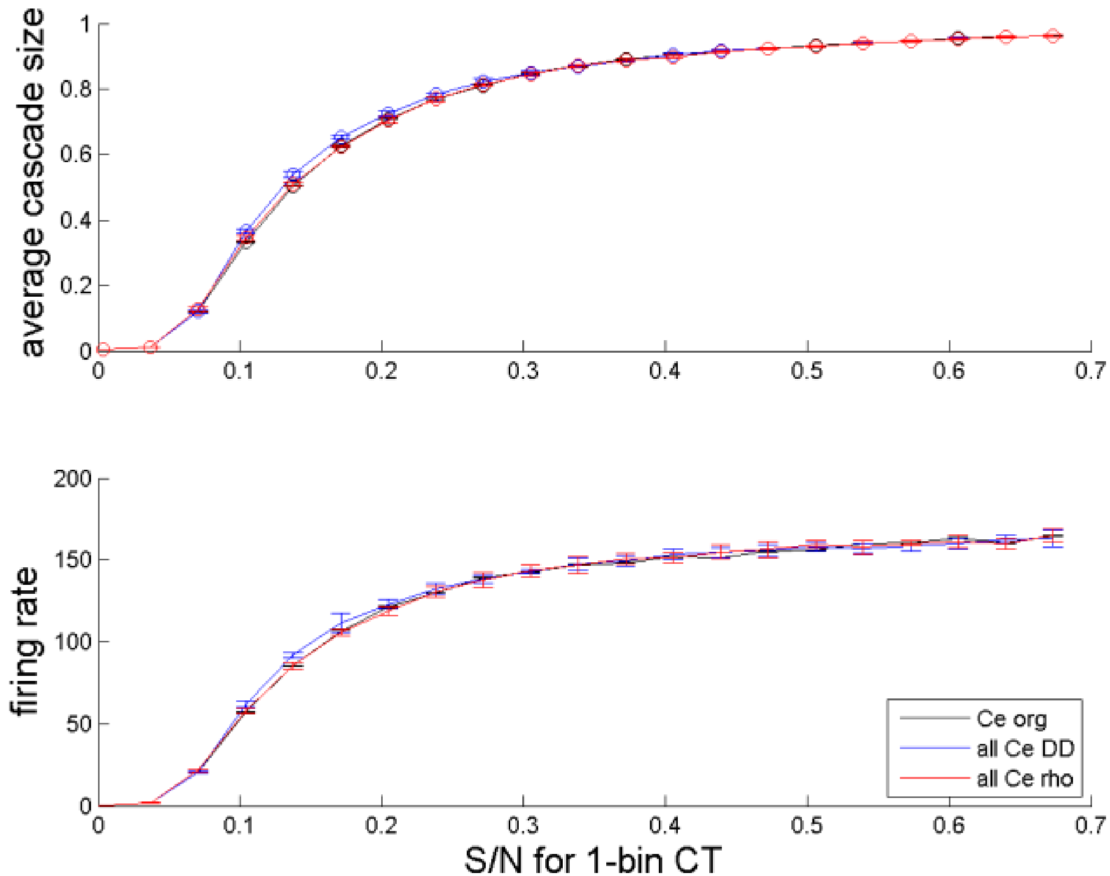

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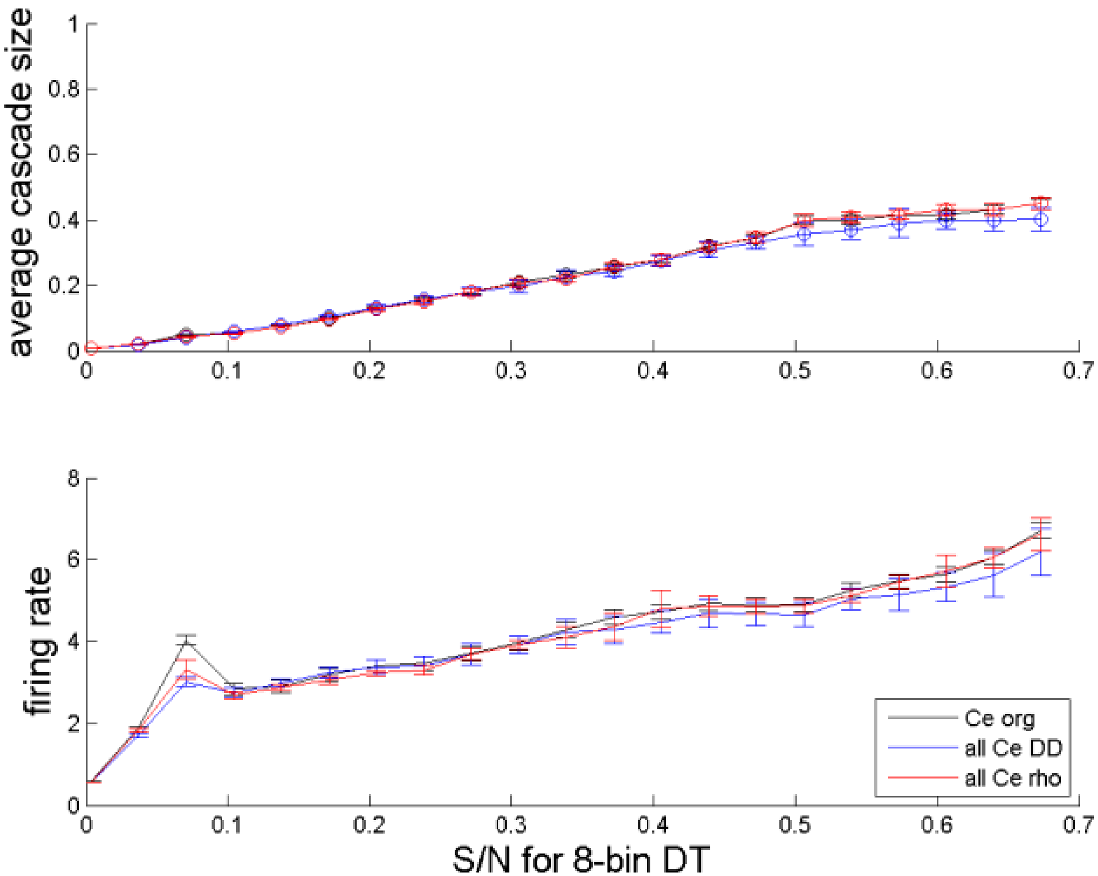

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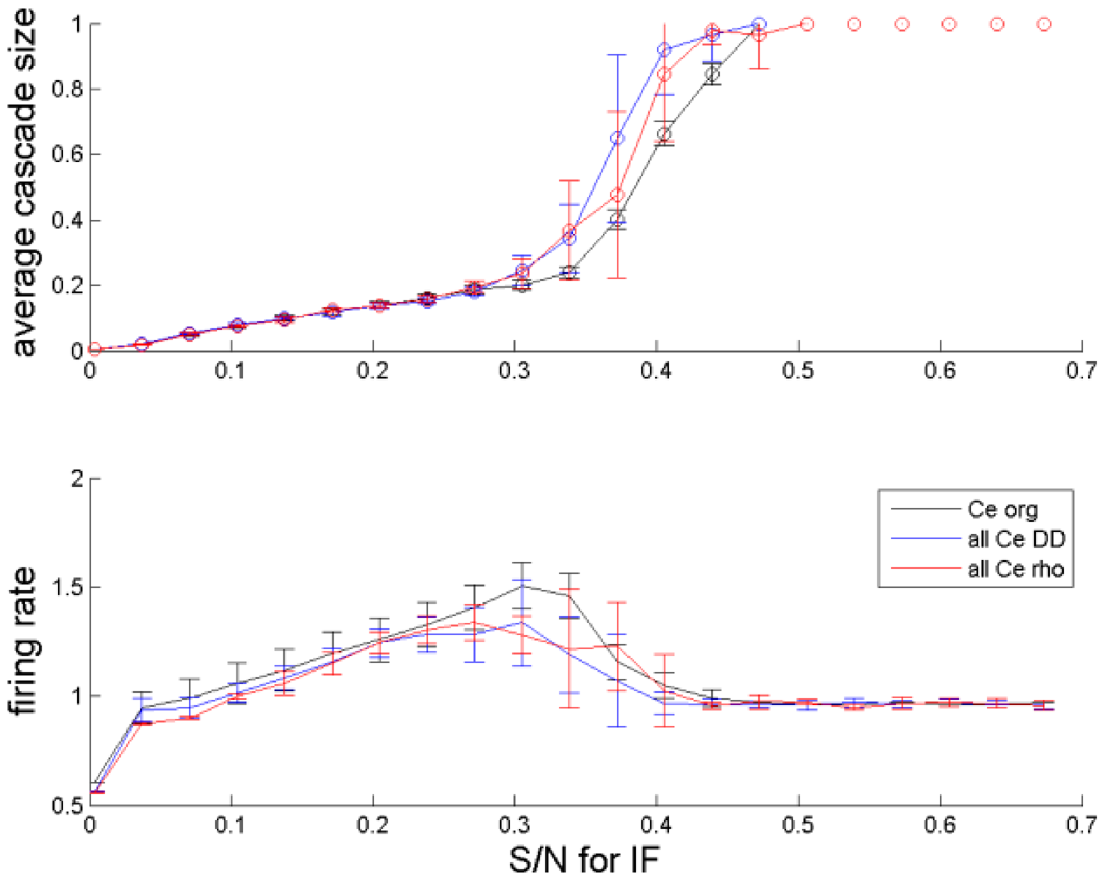

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Average Cascade Size Theory

- Assumption: Either one neuron fires or all neurons fire
- Calculate the probability that exactly one neuron fires
  - We include the degree distribution by conditioning on the edges of the first neuron to fire
  - We include the model dynamics by calculating the voltage distribution of the remaining neurons at the time the first neuron fires

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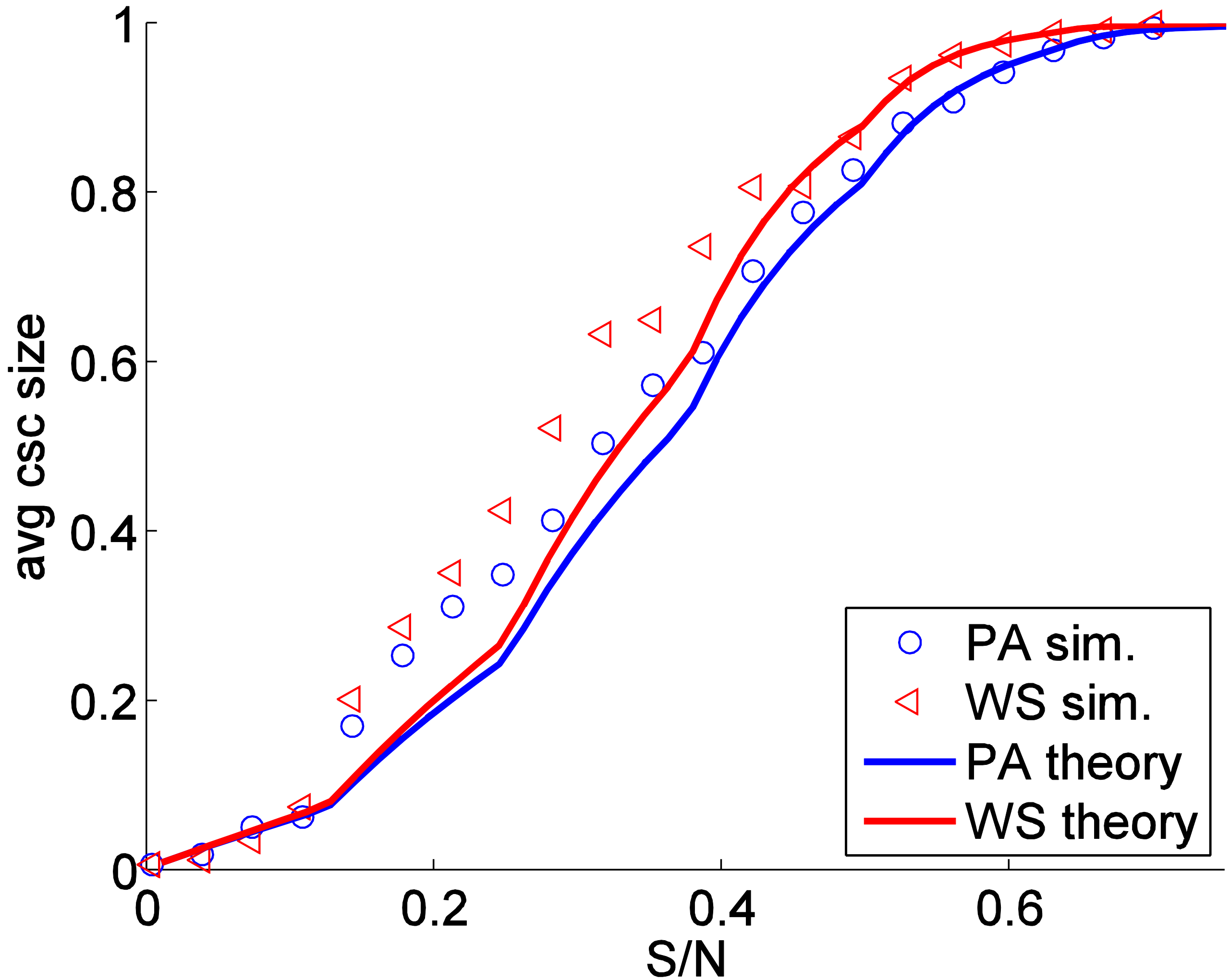
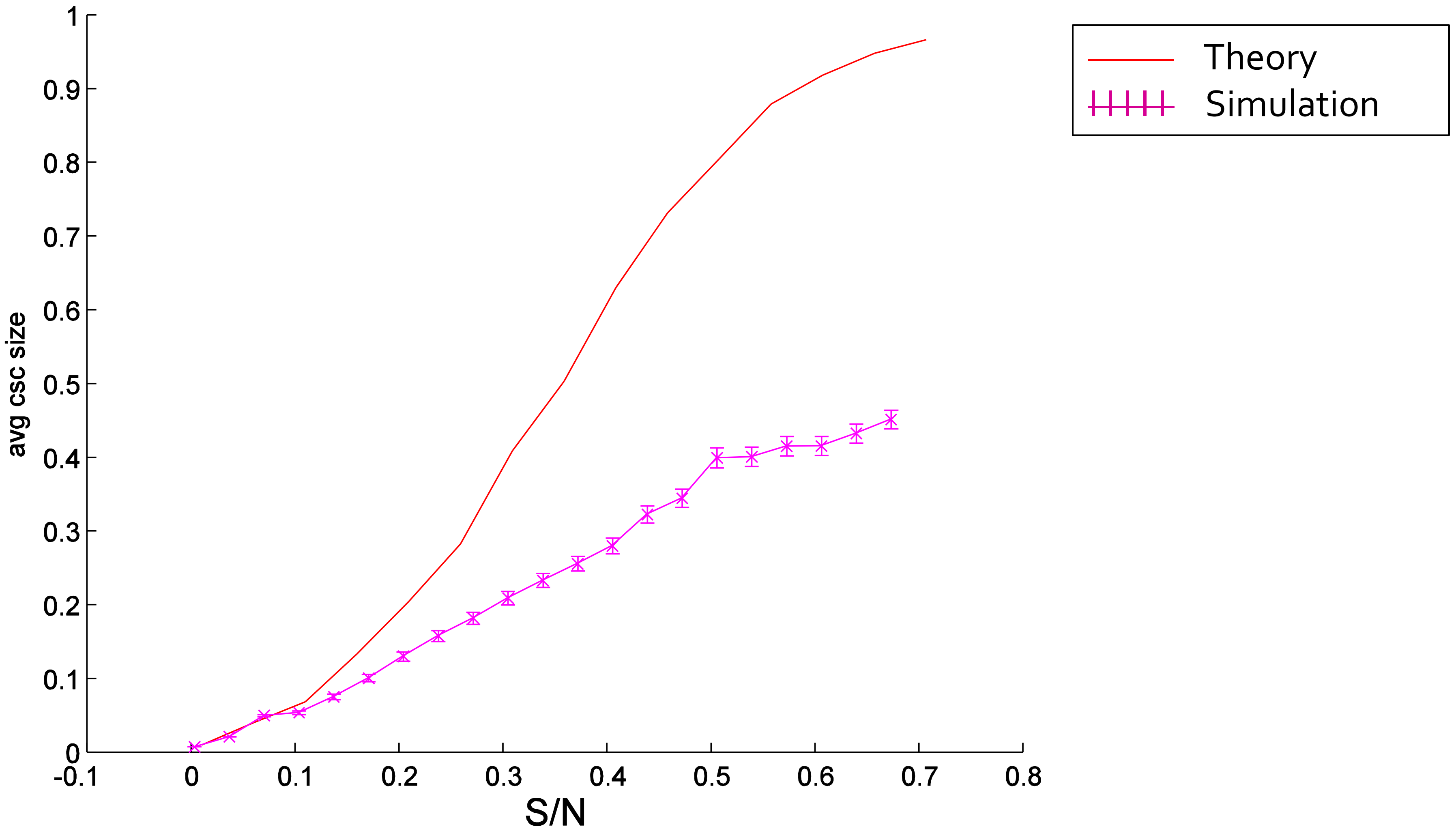
Simulations Versus Theory Results

- We received similar results from all models
- Illustrated with 8-bin Discrete time model on multiple network data sets
- Solid Lines represent Theory
- Symbols represent Simulation data

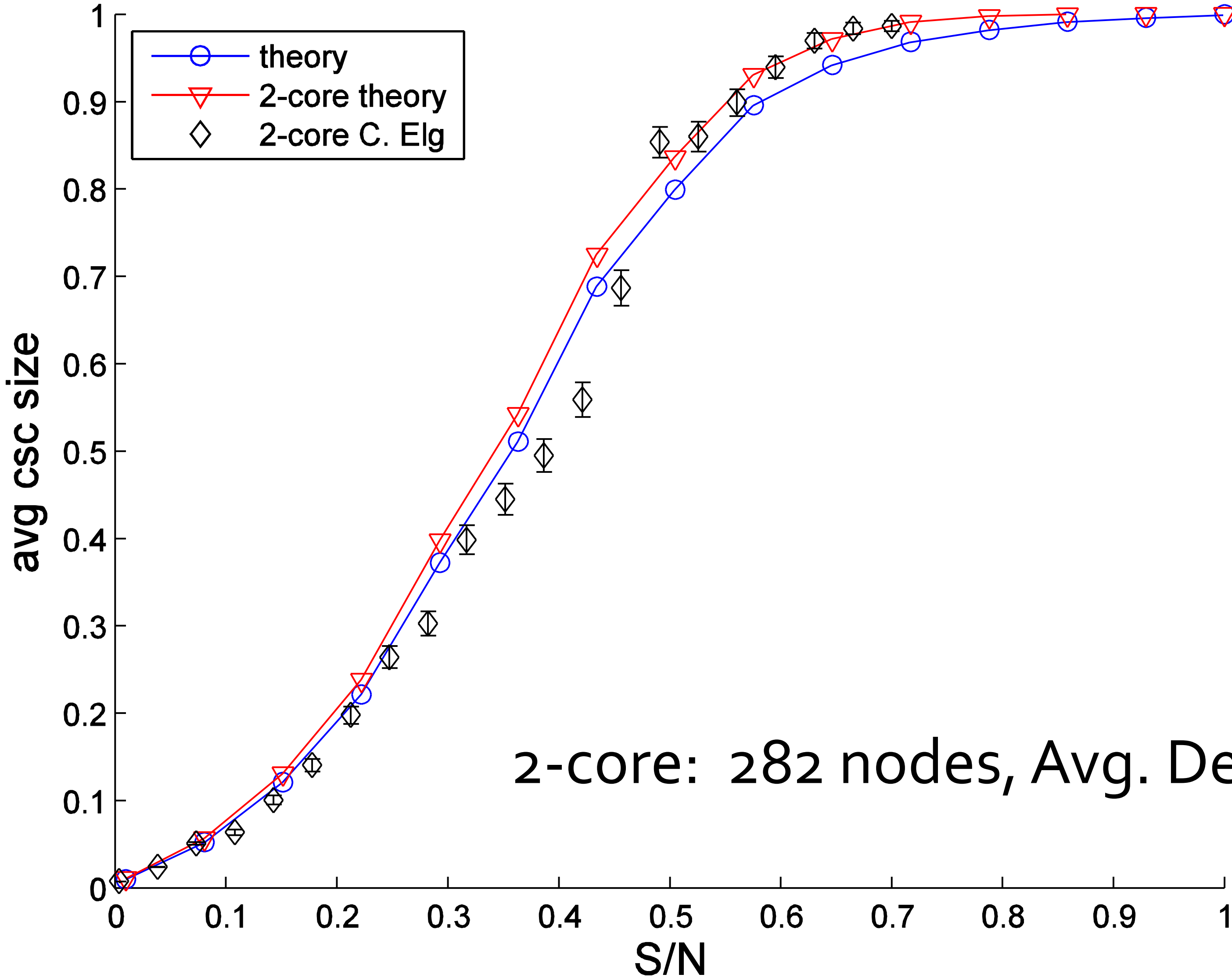
Manufactured Networks – PA, WS



C. elegans

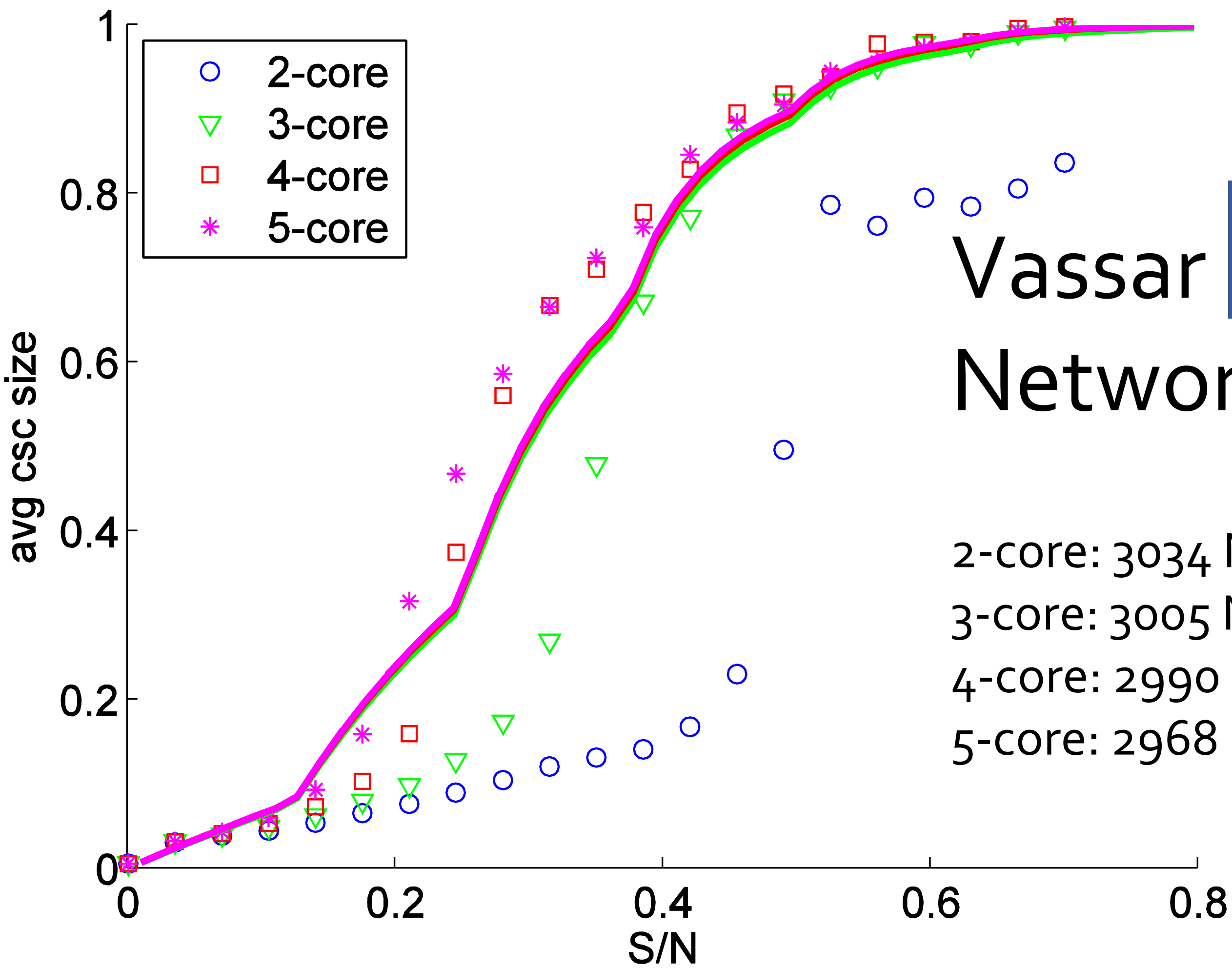

Theory
Simulation
avg csc size
S/N
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

C. elegans Two-Core



2-core: 282 nodes, Avg. Deg. ~15

Other Theory Explorations



Vassar facebook Network Subsets

2-core: 3034 Nodes, Avg. Deg ~79

3-core: 3005 Nodes, Avg. Deg. ~79

4-core: 2990 Nodes, Avg. Deg. ~80

5-core: 2968 Nodes, Avg. Deg. ~80

Future Directions

- What else may cause the theory to fail?
  - When is our all or nothing assumption violated?
  - What network structures cause this failure?
- What other network property may be influencing this result?

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