Optimal Design of Wireless Sensor Network under Communication and Energy Budget Constraints

Zhengyuan Zhu
The University of North Carolina at Chapel Hill

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Joint work with XuanLong Nguyen, Jun Yang, and Yi Zhang
Wireless Sensor Network

- Wiki definition: spatially distributed autonomous devices using sensors to cooperatively monitor physical or environmental conditions, such as temperature, sound, vibration, pressure, motion or pollutants, at different locations.

- Increasingly popular for monitoring spatial-temporal phenomena
  - Volcano Monitoring, Forest Ecological Research, Soil Moisture Modeling, etc.

- Challenges in design: communication cost, energy
Redwood sensor network

From: Dawson et al., in review
Duke Forest, Eno Division
A typical WSN

A node, usually with multiple attached sensors

Gateway or base station, typically more powerful than nodes, and sometimes tethered, serving as the sink for data collection

Messages need to be relayed because nodes have limited communication ranges

⇒ multi-hop routing
Typical energy consumption

- CPU computing (Tmote Sky): 1.5mW
- Radio transmitting/receiving (Tmote Sky): 60mW
- Radio on (Tmote Sky): 1mW
- Humidity & temperature (Sensirion SHT1x): 3mW
- Gas flow (Sensirion ASF1400): 240mW
- Communication ≫ computation
- Sensing costs really depend on the types of measurements
Routing in WSN

- Huge body of literature See Al-Karaki & Kamal (2004), Akkaya & Younis (2005) for survey
- Diverse applications, numerous challenges
  - Ad hoc deployment
  - Ad hoc tasking
  - Limited energy
  - Fault tolerance
  - Network dynamics
  - Heterogeneity
- We consider: Planned deployment; Single, planned task.
Optimization of routing

• Optimizations of routing and of sampling are typically done separately

• Optimization of routing given a sampling plan:
  – Minimize the total energy consumption
  – Maximize the network lifetime
Optimization of Spatial Sampling Design

- Model-free sampling: randomization, blocking, balancing, etc.
- Model-based optimum design: optimize a criterion based on the assumed model.

We will focus on model-based approach.
Classical model-based ODE


- Information matrices, design measure, and approximate design.

- Choice of design criteria: preserve partial ordering, convex (D-, A-, E-, and G- optimal designs).

- Fedorov’s exchange type algorithms.
OSSD based on Gaussian process model

- For spatial prediction: McBratney et. al. (Comp. and Geo. 1981), Yfantis et. al. (Math Geo 1987), Zimmerman (2005), Zhu and Stein (2006), Smith and Zhu (200?).


Our approach

- Joint optimization of routing/sampling
  - What to sample and how to route are interconnected (Krause et al. 2006)

- Flexible multi-path routing
  - Always taking the shortest path drains out nodes on this path quickly (Chang & Tassiulas 2000)

- Optimal sampling: D-optimal design for parameter estimation
Modeling the Sensor Network

We model the sensor network \(\langle V, E, S, c_s, c_t, c_r, b \rangle\) as a directed acyclic graph with a single sink, the base station.

- \(V\) denotes the set of sensor nodes.
- \(E\) denotes the set of wireless link. Each edge \((u, v) \in E\) represents a direct wireless link from \(u\) to \(v\), i.e., \(u\) can directly communicate to \(v\).
- \((V, E)\) is a directed acyclic graph with a single sink \(S \in V\) denoting the base station.
Modeling the Cost

- $c_s : V \rightarrow \mathbb{R}$ sampling energy cost function. For each $u \in V$, $c_s(u)$ models the energy cost of taking a sample at $u$.

- $c_t : E \rightarrow \mathbb{R}$ transmission energy cost function. $c_t(u, v)$ models the expected total energy cost incurred by $u$ in transmitting a unit-size message to $v$.

- $c_r : E \rightarrow \mathbb{R}$ receiving energy cost function. $c_r(u, v)$ is the expected energy cost incurred by $v$ in receiving a unit-size message from $u$.

- $b : V \rightarrow \mathbb{R}$ is the energy budget function. $b(u)$ denote the total energy available at node $u$. 
Sampling and Routing Plans

- A sampling plan is a function $g : V \rightarrow \mathbb{N}$. For each node $u \in V$, $g(u)$ specifies the number of samples to be taken at $u$ (over time) and transmitted to the sink.

- Given $\langle V, E, S, c_s, c_t, c_r, b \rangle$ and $g$, a valid routing plan is a function $f : E \rightarrow \mathbb{N}$ satisfying the following constraints:

  \[ \forall u \in V \setminus \{S\} : g(u) + \sum_w f_{w,u} = \sum_v f_{u,v} \quad (1) \]

  \[ c_s(u)g(u) + \sum_w c_r(w, u)f_{w,u} + \sum_v c_t(u, v)f_{u,v} \leq b(u) \quad (2) \]
Linear Model for Sensor Data

Let $Y(u, t)$ be the observation at sensor node $u \in V$ and time $t \in \{1, 2, \ldots, n\}$ such that

$$Y(u, t) = x^T(u)\beta + \epsilon(u, t),$$

- $x(u) = (x_1(u), \ldots, x_q(u))^T$ are known spatially varying covariates
- $\epsilon(u, t)$s are i.i.d. with mean zero and constant variance $\sigma^2$.

We are interested in estimating $\beta$. 
D-optimal Design for Linear Models

Under sampling design $g$, the covariance matrix of the least square estimator of $\beta$ is given by $\sigma^2 M(g)^{-1}$, with the information matrix $M(g)$ given by

$$M(g) = \sum_{u \in V} x(u)x^T(u)g(u).$$

Define the utility of the sampling design $g$ as

$$\Phi_D(g) = -\log |M(g)|,$$

D-optimal design minimizes $\Phi_D(g)$ over all design $g$. 
D-optimal Design under Constraints

D-optimal design under communication and energy budget constraints:

\[ \text{Min}_g \quad \Phi_D(g) \]

s.t. \[ g(u) + \sum_w f_{w,u} = \sum_v f_{u,v} \]

\[ c_s(u)g(u) + \sum_w c_r(w, u)f_{w,u} + \sum_v c_t(u, v)f_{u,v} \leq b(u) \]

- Standard exchange type algorithm won’t work
Semidefinite Programming (SDP)

- Minimize a linear objective function over the intersection of the cone of positive semidefinite matrices (Boyd and Vandenberghe 2004)

- D-optimal design with constraints can be solved efficiently via semidefinite programming and a primal-dual path-following method

- The dual formulation reveals interesting geometry of the optimal network design
D-optimal Design without communication cost

\[
\min_g \quad \log \det(M(g))^{-1} \\
\text{subject to} \quad M(g) = \sum_{u \in V} g_u x_u x_u^T \succ 0, \\
c^T g \leq C \\
g_u \geq 0, u \in V.
\]

- \(c_u\) denotes the sampling cost at node \(u\)
- \(x_u\) denotes covariate vector corresponding to node \(u\)
Dual formulation

$$\max_{\tilde{W}} \quad \log \det \tilde{W}$$

subject to

$$\tilde{W} = \tilde{W}^T \succ 0$$

$$x_i^T \tilde{W} x_i \leq c_i.$$

- Suppose $c$ is a uniform cost vector (otherwise replacing $x_i$ by $x_i/\sqrt{c_i}$) $c_1 = \ldots = c_n$

- Dual formulation has a simple geometric interpretation: finding a minimum volume ellipsoid $x^T \tilde{W} x = c_1$ that contains all points $x_1, \ldots, x_n$.

- Sampling locations are nodes $u$ for which $x_u$ lie on the ellipsoid’s boundary (Titterington 1975)
With communication constraints

\[
\begin{align*}
\min_g \quad & \log \det(M(g))^{-1} \\
\text{subject to} \quad & M(g) = \sum_{u \in V} g_u x_u x_u^T \succ 0, \\
& b^T f + c^T g \leq A \\
& g_u + \sum_{v \neq u} f_{vu} - \sum_{w \neq u} f_{uw} = 0, \ u \in V - \{S\} \\
& f_{uv} \geq 0 \quad u, v \in V \\
& g_u \geq 0 \quad u \in V.
\end{align*}
\]

- \(c_u\) denotes sampling cost vector at node \(u\)
- \(b_{uv}\) denotes the cost of transmitting from \(u\) to \(v\)
Dual formulation

\[
\begin{align*}
\max_{\tilde{W}, \alpha} \quad & \log \det \tilde{W} \\
\text{subject to} \quad & \tilde{W} = \tilde{W}^T \succ 0 \\
& \mathbf{x}_u^T \tilde{W} \mathbf{x}_u \leq c_u + \alpha_u, \ u \in V \\
& \alpha_u - \alpha_v \leq b_{uv}, \ u \neq v; u \in V - \{S\}; v \in V \\
& \alpha_S = 0.
\end{align*}
\]

• By Lagrange duality, a location \( u \) takes sample only if

\[
\mathbf{x}_u^T \tilde{W} \mathbf{x}_u = c_u + \alpha_u, \ u \in V.
\]

• An edge \( uv \) admits a flow from \( u \rightarrow v \) only if

\[
\alpha_u - \alpha_v = b_{uv}.
\]
Geometry of the optimal design

- Optimal sampling design without communication costs (i.e. $b_{uv} \equiv 0$):
  - Sampling at extreme points of a minimum volume ellipsoid, whose radius is driven by the sampling cost vector

- Optimal network design without sampling costs (i.e., $c_u \equiv 0$):
  - let $\mathcal{A}$ be the $|V|$-dim polytope bounded by hyperplanes of the form
    \[ \alpha_u - \alpha_v = b_{uv}, \quad u \in V - \{S\}, \quad v \in V \]
    \[ \alpha_S = 0; \quad \alpha_u \geq 0, \quad u \in V. \]
– Find a minimum volume ellipsoid $x^T \tilde{W} x = 1$ that contains all points $x_u/\alpha_u$ for some $\alpha \in \partial \mathcal{A}$

– Sampling only at locations $u$ where $x_u/\sqrt{\alpha_u}$ lies on the ellipsoid boundary

– Routing *only* at locations lying in some shortest path (made of cost vector $b$) from the sampling nodes to the base station $S$
Geometry of the optimal design

• General OSD with communication constraints:
  – let $\mathcal{A}$ be the same $|V|$-dim polytope bounded by hyperplanes
  – Find a minimum volume ellipsoid $\mathbf{x}^T \tilde{\mathbf{W}} \mathbf{x} = 1$ that contains all points $\mathbf{x}_u / \sqrt{c_u + \alpha_u}$ for some $\alpha$ being a boundary point of $\mathcal{A}$
  – Sampling only locations $u$ where $\mathbf{x}_u / \sqrt{c_u + \alpha_u}$ lies on the ellipsoidal boundary
  – Routing only at locations lying in some shortest paths (from the sampling nodes) to the base station $S$, 
Optimization algorithms

- Design solver based on a MATLAB package, SDPT3, for solving semidefinite-quadratic-linear programming.

- Takes following network parameters as input: node locations, covariates in the model for each node, per-node energy budget, total energy budget of all nodes, sampling cost, pair-wise connectedness, and communication costs.

- Output both the D-optimal sampling design and routing plan.
Simulation results for linear models

- All designs on $9 \times 9$ grids $[-4, -3, \ldots, 4]^2$. Base station is located at $0 \times 0$.

- Communication range is 3. $c_s = 0.2$, $c_t = c_r$ ranging from 0 to 1.8. Total energy budget is 25 times per node energy budget.

- Covariates:
  - Case 1. $q = 1, x_1 = 1$ (maxize # of sample under communication constraints)
  - Case 2. $q = 2, x_1 = 1, x_2 = x$;
  - Case 3. $q = 2, x_1 = 1, x_2 = \sqrt{x^2 + y^2}$;
  - Case 4. $q = 3, x_1 = 1, x_2 = x, x_3 = y$;
Uniform Design
Case 1, $c_r = c_t = 0.1$
Case 2, $c_r = c_t = 0.1$
Case 3, $c_r = c_t = 0.1$
Case 4, $c_r = c_t = 0.1$
Case 4, \( c_r = c_t = 0.6 \)
Comparison of Designs

| \(- \log |M(g)|\) | Unif | A1C1 | A4C1 | A4C2 |
|-----------------|------|------|------|------|
| C=0             | -25.24 | -23.12 | -25.03 | -25.39 |
| C=0.1           | -21.62 | -21.09 | -23.50 | -23.34 |
| C=0.6           | -17.74 | -17.14 | -19.21 | -19.22 |
Space-time design

Consider the model

\[ Y(u, t) = \mu(u, t) + \epsilon(u, t), \]

where \( \mu(u, t) = \sum_i x_i(u, t)\beta_i, \)

\( x_i(u, t), i = 1, 2, \ldots, q \) are covariates depending on both space and time.

Let \( x(u, t) = (x_1(u, t), x_2(u, t), \ldots, x_q(u, t))^T, \)

\( M(u, t) = x(u, t)x^T(u, t), \)

then the information matrix is given by

\[ M(G) = \sum_{(u,t) \in G} M(u, t), \]

where \( G \) is the set of all nodes and time where we take the observation.
Space-time design

The optimization problem becomes

$$\text{Min}_G - \log |M(G)|$$

subject to (1) and (2).

which is in general intractable.
Two-step algorithm

• Step 1: Let $M(u) = T^{-1} \int_0^T M(u, t) dt$, and $M^*(g) = \sum_u M(u) g(u)$. Solve problem (3) for $g(u)$, with $M(g)$ replaced by $M^*(g)$.

• Step 2: For each node $u$ we solve the problem

\[
\min_{g(u, t)} - \log \left| \sum_t M(u, t) g(u, t) \right|
\]

subject to $\sum g(u, t) \leq g(u)$, \hspace{1cm} (4)

where $g(u, t)$ is an indicator variable, with $g(u, t) = 1$ indicating that a sample is taken at node $u$ and time $t$. 
Non-linear Models for Sensor Data

\[ Y(u, t) = \eta(x(u), t; \beta) + \epsilon(u, t), \]

where \( \eta() \) is a nonlinear function.

- Local optimal design at \( \beta_0 \): let
  \[ f(x(u), t; \beta) = \frac{\partial \eta(x(u), t; \beta)}{\partial \beta}, \]
  and define
  \[ M(u, t) = f(x(u), t; \beta) f^T(x(u), t; \beta)|_{\beta=\beta_0}. \]

- In practice, can take adaptive sequential design approach, minimax approach or Bayesian approach.
Simulation study for nonlinear model

Consider the following nonlinear model

\[ \eta_1 = \frac{W}{((\beta_1 + x(u)\beta_2)t + 1)^\theta}, \]

- Motivated by the soil moisture model.
- \( x(u) = \sin(\phi(u)) \), where \( \phi(u) \) is the slope.
- \( \sin(\phi(u)) \) is randomly generated in the example.
Simulated $\sin(\phi(u))$
\[ c_r = c_t = 0.1 \]
\[ c_r = c_t = 0.6 \]
Multiple Measurements

\[ Y_i(u, t) = x_i^T(u, t)\beta_i + \epsilon_i(u, t), \]

where \( i = 1, \ldots, r \) represent \( r \) measurements,
\[
\text{Var}(\epsilon_i(u, t)) = \sigma_i^2, \quad \text{Cov}(\epsilon_i(u, t), \epsilon_j(u, t)) = \sigma_{ij}, \quad \text{Cov}(\epsilon_i(u, t), \epsilon_j(u', t')) = 0. \]

Let \( X(u) = \text{diag}(x_1^T(u), \ldots, x_r^T(u))^T, \)
\[
M(g) = \sum_u X(u)^T \Sigma^{-1} X(u) g(u)
\]

- Can use the same SDP algorithm to solve.
- Generalization to space-time and nonlinear models similar to univariate case.
Next Step

• Algorithms for selecting $n$ sensor locations from $N$ candidate locations
  – Extra constraint $\sum_b u < B$ forces sparsity in design. Can lead to more efficient algorithm to solve the combinatorial problem.

• Space-time design formulation is a combinatorial optimization problem generally intractable
  – Can it be relaxed and solved as an SDP?

• Generalization to models with spatial and/or temporal correlations: Can not be solved by SDP. Need to deal with the constraints in EX, SAA, GA.