Power Grid Vulnerability to Cascading Failures

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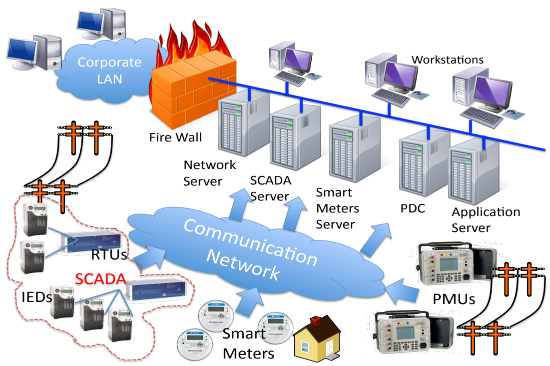
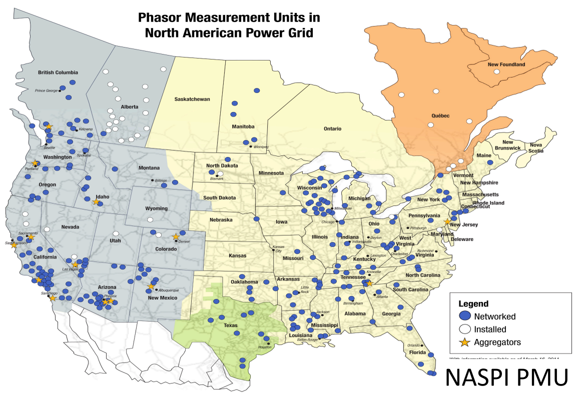
Outline

1. What is the problem of cascading failures
2. Review of the literature
3. The Proposed Stochastic Model for Cascading Failures
4. Vulnerability Measures
5. Some Experiment Results
6. Conclusion

Problem Statement

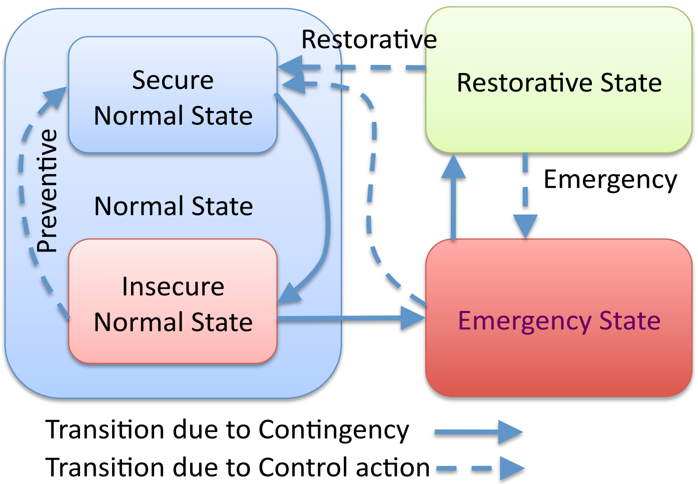
Enhanced Monitoring & Measuring Architecture





NASPI PMU Map

Enhanced System State Awareness



Cascading Failure in Power Grids is a severe security threat. However, an accurate model and metrics to evaluate the risk of failure and the time margin to perform corrective action are missing.

Background

- The electric power grid inter-connectivity
  - enables long-distance transmission of power for more efficient system operation.
  - also allows the propagation of disturbances, even escalate in catastrophic blackouts.

- Many research efforts on the vulnerability of power grids
  - Static failure models
    - focus on topology vulnerability, such as network connectivity
    - do not describe the evolving process of cascading failures
    - works by Rosas-Casals, Valverde , etc.
    - some use realistic power grid networks; some use inappropriate network topologies.

  - Dynamic overload failure models
    - 3 components: triggering events, flow re-dispatch, additional line trips
    - Graphic theory approaches (Motter : graph network)
    - Models based on realistic power flows (Dobson: linear OPF)
    - Stochastic models (Dobson: branching process, Hines: critical slowing down)

On the Graphic Approaches

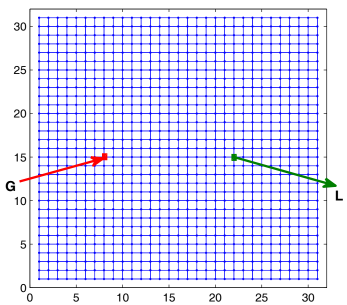
- Critique of Graph Theoretic Models
  - more suitable for communication or traffic networks
  - the information packets and vehicles can freely switch their route
  - the flows preferentially choose shorter paths
  - The nodes in a network can easily switch its role as a sender or receiver

- The flows in a power grid network
  - dominated by Kirchhoff's Voltage/Current laws and Ohm's law

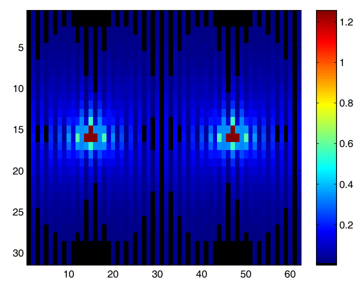
$$\sum_{(i,j)} U_{ij} = 0 \qquad \sum_{j} I_{ij} = 0 \qquad U_{ij} = I_{ij} Z_{ij}$$

  - the conservation of flows in a graph => equivalent to KCL
  - KVL and Ohm's law are unique for power gird flows
  - the nodes in a grid divided into 3 classes: generation/load/interconnection, switching roles are rare.

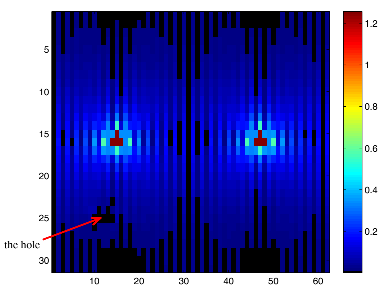
Visualized Examples



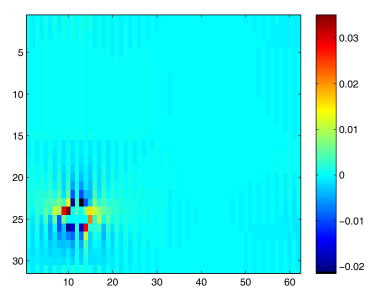
(a) the flattened grid



(b) the original flow distribution

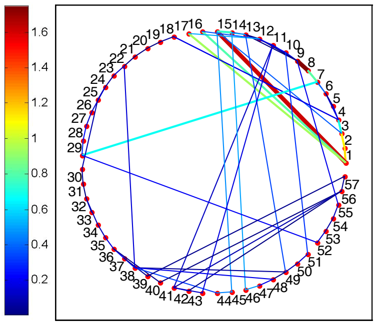


(c) flow redistribution after line trips

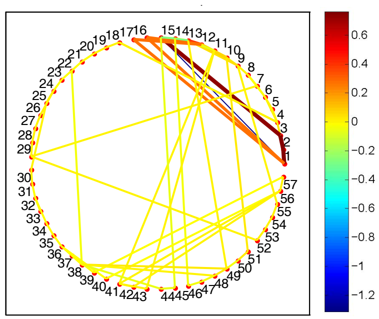


(d) changes in the flow magnitude

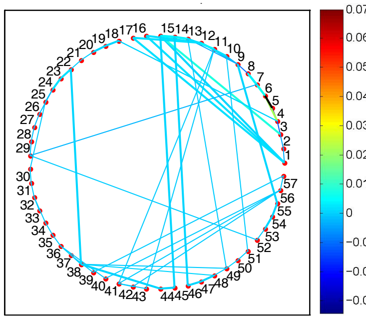
Fig. 1: Flow Redistribution in a Torus Grid Network



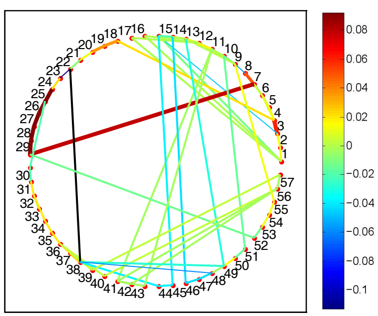
(a) the original flow distribution



(b) increasing the impedance of line 1-15 (rewire link)



(c) tripping line 5-6 (local link)



(d) tripping line 22-38 (rewire link)

Fig. 2: Flow Redistribution in the IEEE-57 System

The LOPF-models

- The models based on Linear Optimization:
  - DC power flow – consistent with power grid network.
  - determined optimization (LOPF) – inappropriate model for flow re-distribution after failures
    - the escalation of cascading failures: hundreds of lines tripped in a short period
    - there is usually not enough time for planning any optimized generation re-dispatch or load-shedding
    - self- contradictory settings in the model:

assuming necessary adjustment in generation and/or load shedding in order to avoid line overloads after line outages

↔

the task in fact is only partially finished: the line flows depressed slightly below the overload threshold and some chances left for additional overloads and line trips, so as to model the following stages.

Branching Process, etc.

- A few authors have considered stochastic modeling.
  - Dobson, Carreras and Newman (2005) [16] analyzed the line trips data of several blackouts in the US, and found that, the branching process model can provide a good fit for the cumulative number of line trips. ***Branching processes*** are useful to model population growth: each individual in one generation produces some random number of individuals in the next generation.
- Other works
  - Hines, et.al. : proposed that power grids may exhibit the ***critical slowing down*** phenomenon, which can be detected as a noticeable increase in the correlation of some phase angle or system frequency, that is a suitable risk indicator for the advent of a cascading blackouts.

- Common Limitation: the models only describe the cascading process in state stages.
- Need a time measure for grid vulnerability.

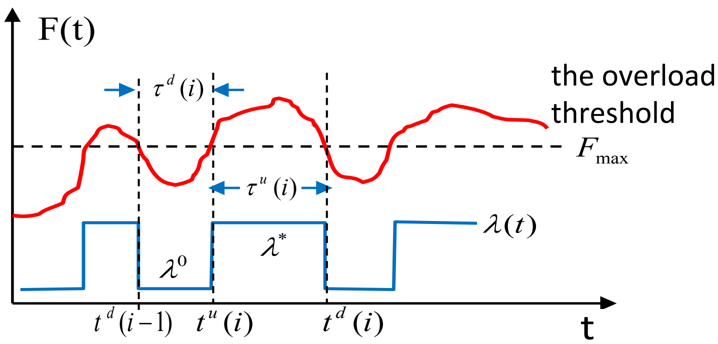
What we propose

A stochastic Markovian model for cascading failures in power grid

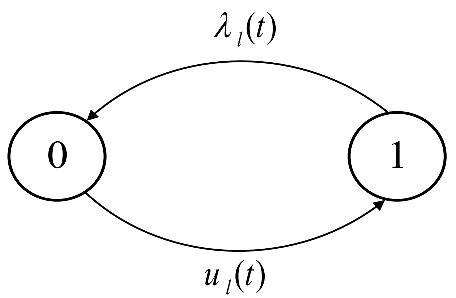
- Consider the limitations of other flow distribution models that are rooted in Kirchhoff' s and Ohm' s laws and in the energy management model of power dispatch
- with line-state transition probabilities derived from a stochastic model for the flow redistribution,
- which can potentially capture the progression of cascading failures and its time span.
- Define metrics that can be monitored to unveil the risk of failure and the time margin that is left to perform corrective action.

The Stochastic Model for Cascading Process

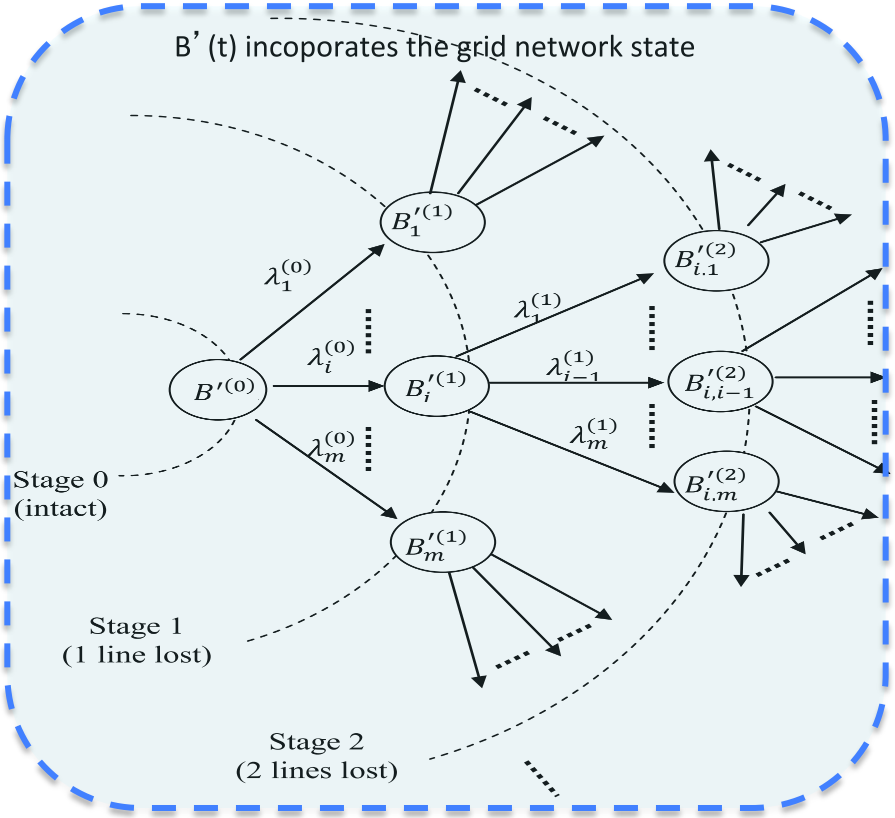
Model the the grid states as conditionally Markovian on the evolution process of line flows



The crossing intervals of line flow F(t) and the transition rates: λ0(t)-random contingency, λ*(t)-overload line tripping rate



Line State transition: line tripping vs. restoration

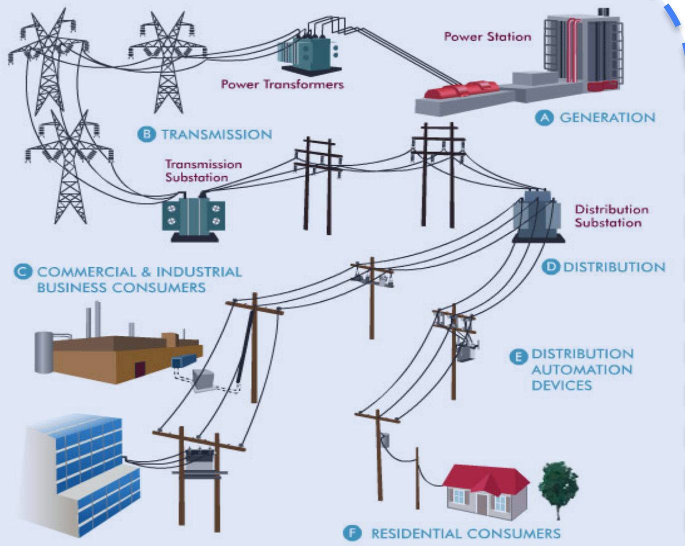


The Power Grid Network Model

$$\mu_P(t) = \begin{bmatrix} \mu_g(t) \\ -\mu_l(t) \end{bmatrix}$$
$$C_P(t) = \begin{bmatrix} \Sigma_{gg}(t) & \Sigma_{gl}(t) \\ \Sigma_{lg}(t) & \Sigma_{ll}(t) \end{bmatrix}$$

The statistics of generation and loads

We assume loads are independent spatially, but have a given autocorrelation function in time



$$B'(t) = A_t^T diag\{y_l(t)\} A_t = \widetilde{A}_t^T \widetilde{A}_t$$

$$F(t) = \sqrt{y_t}(\widetilde{A}_t^T)^\dagger P(t)$$

The power grid network: DC flow model

$$\mu_F(t) = \sqrt{y_t}(\widetilde{A}_t^T)^\dagger \mu_P(t)$$
$$C_F(t) = \sqrt{y_t}(\widetilde{A}_t^T)^\dagger C_P(t)(\widetilde{A}_t)^\dagger \sqrt{y_t}$$

The statistics of line flows

From which we can derive:

- using Rice's result (1958), obtain the probability distribution of level-crossing intervals
- the statistics of the crossing times give the statistics of the line state transition rate $\lambda_l(t)$
- we can then obtain the average lifetime
- some useful metrics: overload distances/probabilities, vulnerability measures, etc.

- The statistics of line flows

$$\mu_P(t) = \begin{bmatrix} \mu_g(t) \\ -\mu_l(t) \end{bmatrix}, \quad C_P(t) = \begin{bmatrix} \Sigma_{gg}(t) & \Sigma_{gl}(t) \\ \Sigma_{lg}(t) & \Sigma_{ll}(t) \end{bmatrix}, \quad \begin{aligned} \mu_F(t) &= \sqrt{y_t}(\widetilde{A}_t^T)^\dagger \mu_P(t) \\ C_F(t) &= \sqrt{y_t}(\widetilde{A}_t^T)^\dagger C_P(t)(\widetilde{A}_t)^\dagger \sqrt{y_t} \end{aligned}$$

- The overload probability $\rho_l(t) = \mathcal{Q}(a_l) + 1 - \mathcal{Q}(b_l) \approx \mathcal{Q}(a_l)$
- Useful distance metrics: with $\mathrm{a}_l = \frac{F_l^{\max} - \mu_{F_l(t)}}{\sigma_{F_l(t)}}$ and $b_l = \frac{-F_l^{\max} - \mu_{F_l(t)}}{\sigma_{F_l(t)}}$
  - Overload distance of one line a_l
  - overall overload distance of the system
    - Mahalanobis overload distance

$$D_m = \sqrt{(F^{\max} - \mu_{F(t)})^T C_F^\dagger(t)(F^{\max} - \mu_{F(t)})}$$

    - Euclidean overload distance

$$D_e = \sqrt{\sum_l a_l^2}$$

- The level crossing intervals
  - mean: $$\bar{\tau}_l^u = \frac{2\pi\rho_l e^{a_l^2/2}}{W}, \quad \bar{\tau}_l^d = \frac{2\pi(1-\rho_l)e^{a_l^2/2}}{W}$$

    with the equivalent bandwidth of the flow process: $W = \sqrt{\frac{-R''(0)}{R(0)}}$
  - PDF : $$f_T(\tau; a_l) \approx \begin{cases} \frac{\pi\tau}{2\bar{\tau}_l^2}\exp\left[-\frac{\pi}{4}\left(\frac{\tau}{\bar{\tau}_l}\right)^2\right], & a_l \gg 1 \\ \frac{1}{\bar{\tau}_l}\exp\left(-\frac{\tau}{\bar{\tau}_l}\right), & a_l \ll -1 \end{cases}$$

- The expected line states at the end of each interval

$$\alpha_l = E\left\{p\left\{s_l\left(t_l^d(i)\right) = 1\right\}\right\} \text{ and } \beta_l = E\left\{p\left\{s_l\left(t_l^u(i)\right) = 1\right\}\right\}$$

$$\alpha_l = \begin{cases} 1 - \lambda_l^* \bar{\tau}_l^u \exp\left(\frac{(\lambda_l^*\bar{\tau}_l^u)^2}{\pi}\right)\text{erfc}\left(\frac{\lambda_l^*\bar{\tau}_l^u}{\sqrt{\pi}}\right), & a_l \gg 1 \\ 1/\left(\lambda_l^*\bar{\tau}_l^u + 1\right), & a_l \ll -1 \\ c_1\left[1 - \lambda_l^*\bar{\tau}_l^u \exp\left(\frac{(\lambda_l^*\bar{\tau}_l^u)^2}{\pi}\right)\text{erfc}\left(\frac{\lambda_l^*\bar{\tau}_l^u}{\sqrt{\pi}}\right)\right] + \\ \quad + c_2\left[1/\left(\lambda_l^*\bar{\tau}_l^u + 1\right)\right], & a_l \text{ medium-value} \end{cases} ;$$

$$\begin{aligned} \beta_l = & \; E\left\{p\left\{s_l\left(t_l^u(i)\right) = 1\right\}\right\} \\ = & \begin{cases} 1 - \lambda_l^0 \bar{\tau}_l^d \exp\left(\frac{(\lambda_l^0\bar{\tau}_l^d)^2}{\pi}\right)\text{erfc}\left(\frac{\lambda_l^0\bar{\tau}_l^d}{\sqrt{\pi}}\right), & a_l \ll -1 \\ 1/\left(\lambda_l^0\bar{\tau}_l^d + 1\right), & a_l \gg 1 \\ 1 - \lambda_l^0\bar{\tau}_l^d, & a_l \text{ medium-value} \end{cases} \end{aligned}$$

- The expected life time of a line

$$\mathcal{T}_l = (\bar{\kappa}_l - 1)/\gamma_l + E\{\Delta t_l\}$$

  - Expected number of crossings after which he line gets tripped

$$\bar{\kappa}_l \; = \left[(1+\beta_l) - (\beta_l - \alpha_l)\rho_l\right] / (1 - \alpha_l \beta_l)$$

  - $E\{\Delta t_l\}$ is the mean duration of the *last* interval
    $E\{\Delta t_l\} = \left[\Delta T_l^* + \Delta T_l^0\right] / (1 - \alpha_l \beta_l)$
    with $\Delta T_l^* = (1-\alpha_l)\left[\beta_l + (1-\beta_l)\rho_l\right] / \lambda_l^*$
    and $\Delta T_l^0 = (1-\beta_l)\left[1 - (1-\alpha_l)\rho_l\right] / \lambda_l^0$

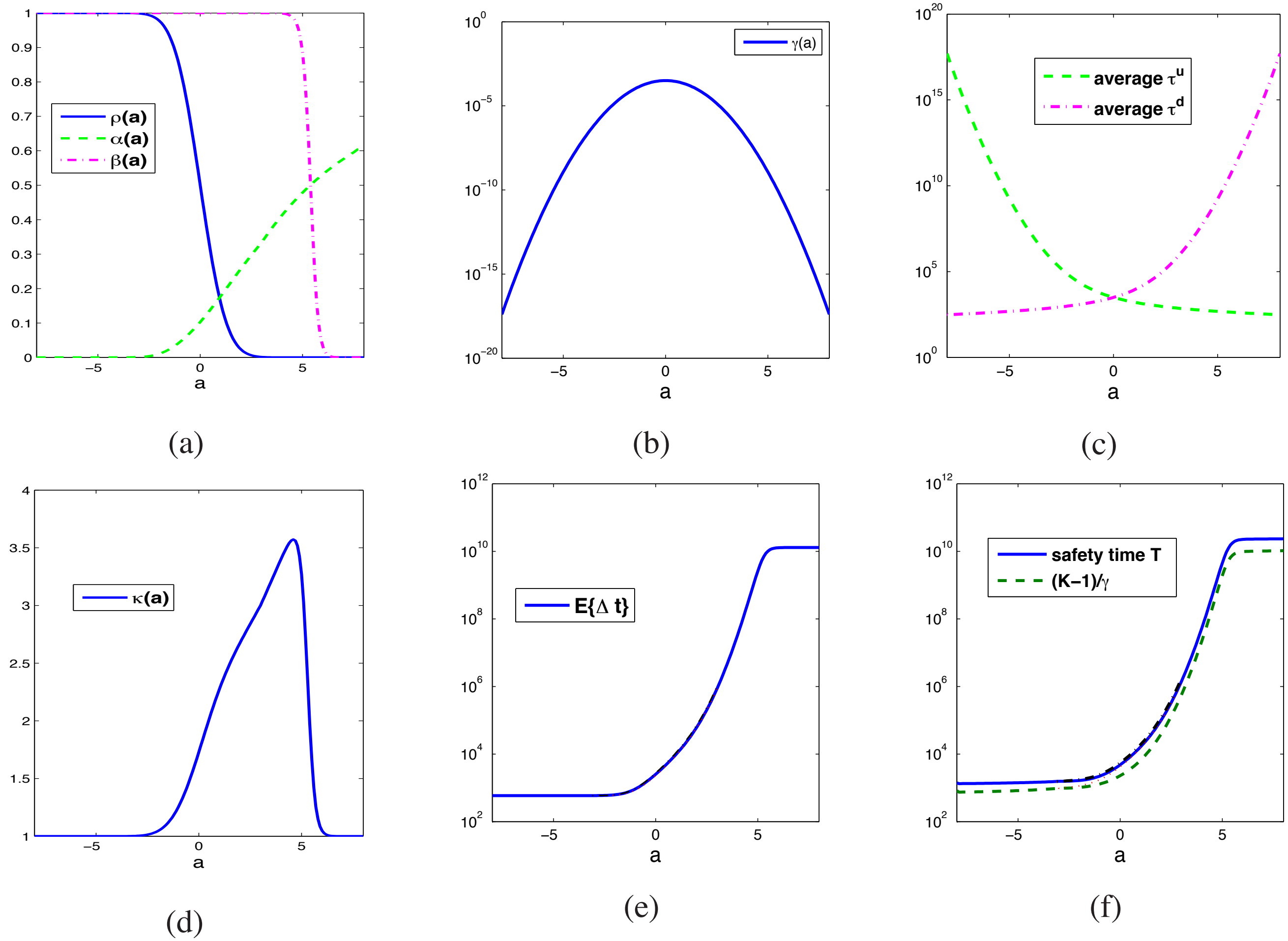


Fig. 4: The quantities versus the normalized distance to the overload threshold a: (a) the overloading probability $\rho(a)$, the line non-trip probabilities $\alpha(a)$ and $\beta(a)$; (b) the average crossing rate $\gamma(a)$; (c) the average interval length $\bar{\tau}^u(a)$ and $\bar{\tau}^d(a)$; (d) the expected number of crossing before tripping $\bar{\kappa}(a)$; (e) the average time before tripping in the last interval $E\{\Delta t\}$; (f) the safety time $\mathcal{T}(a)$.

Extension to non-stationary flow process

- If the operation settings of the whole process is available, we can then evaluate the safety time of a line based on the worst flow condition:

$$\mathcal{T}_l^*(t_0) = (\kappa_l^* - 1)/\gamma_l^* + E\{\Delta t_l^*\}\ , \text{ with } \ a_l^{\min}(t_0) = \min_{t\in[t_0,\infty)} \frac{F_l^{\max} - \mu_{F_l}(t)}{\sigma_{F_l}(t)}$$

- Assume a cyclo-stationary process for the grid loads whose statistical properties vary cyclically with time.

Lemma : For a cyclostationary flow process $F_l(t)$ with a cyclic period of T^D on the l-th line, given its smallest average number of crossings κ_l^* before tripping and the shortest duration of the last interval $E\{\Delta t_l^*\}$ evaluated at the worst-case condition during T^D, we can estimate its safety time as

$$\mathcal{T}_l^{\min} = T^D(\kappa_l^* - 1)/\gamma_l^D + E\{\Delta t_l^*\}.$$

where $\gamma_l^D = \int_{t_0}^{t_0+T^D} \gamma_l(t)dt$ is time invariant and called the average daily interval crossing rate.

Vulnerability metrics in terms of cascading failures

- The proposed model computes ***the expected life time*** of a line $\mathcal{T}_l(a_l)$, monotonically increasing with the line's overload distance a_l

- The most critical lines in a network in terms of ***the minimum safety time*** can also be identified as the line with the smallest overload distance a_l

$$l^* = \arg \min_l \mathcal{T}_l = \arg \min_l a_l$$

- 4 affecting factors:

Electrical betweenness

most graphic vulnerability metrics only considers partially (1), i.e., the network topology

(1)
network condition: connecting topology + line impedances

(3)
line capacities (or overload threshold)

the criterion of of *high traffic or high flow increase* after 1 trip based on LODF (line outage distribution factor)

$$\min_l \mathcal{T}_l|_k = \min_l \{\mathcal{T}_l |\ s_k(0){=}0, s_{i\neq k}(0){=}1\}$$

(2)
operating condition: generation dispatch and load settings

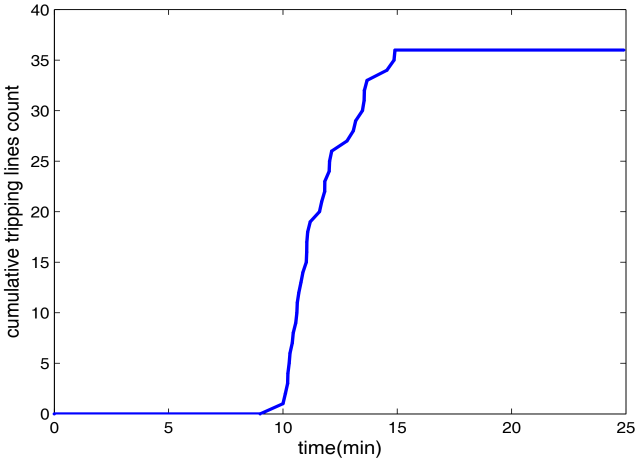
(4)
statistics of the line flows derived from the statistics of injected power

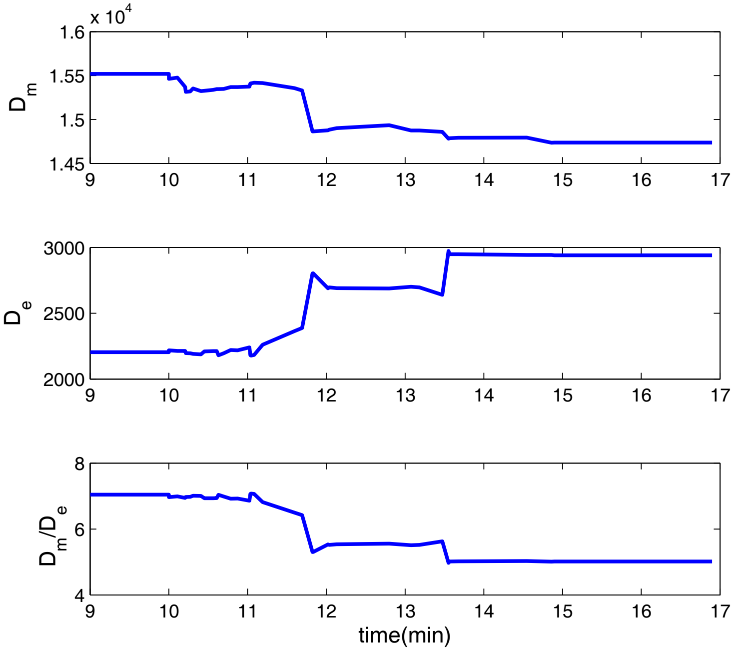
Experiment with the IEEE 300

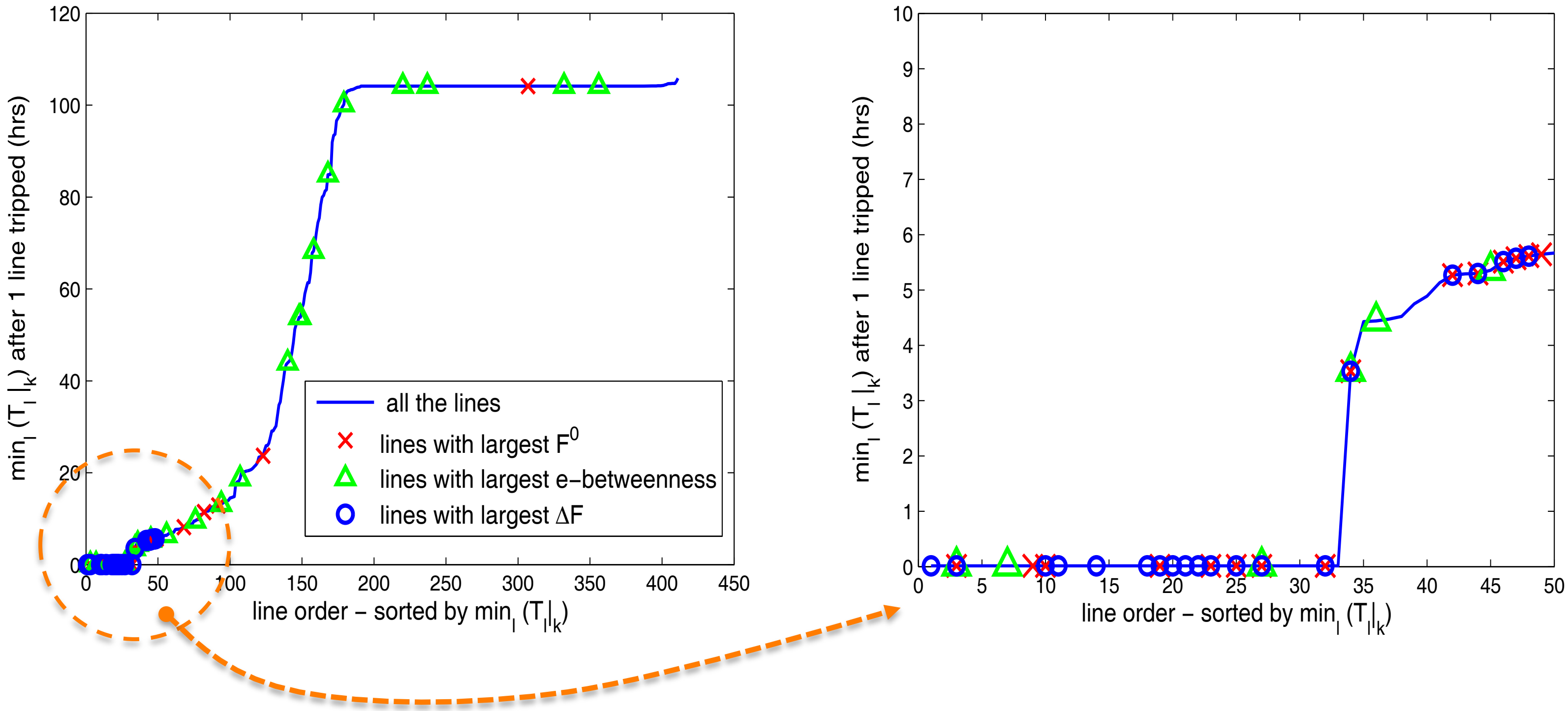
- *T = 30 mins,   W = 10^{-5} Hz*

$$\lambda^* = 1.92 \cdot 10^{-2} \quad \lambda^0 = 7.70 \cdot 10^{-11}$$

- The experiments with the IEEE 300-bus system have shown that:
  - the evolution process of cascading failures with the cumulative line trips increasing exponentially versus time, and a pattern compatible with historical records from some realistic power grids.
  - The line flows in a power grid are correlated, which means that power network flows are less prone to generate cascading failures compared to independent line flows.
  - The experiments also showcase how the (N – 1) contingencies affect the system's minimum safety time, therefore one is able to identify the critical set of lines in the system whose tripping might kindle cascading failures.







NOTE: Accurate evaluation of failure risk in a power grid depends on the correct information about load and generation setting and changes, the line flow process, the overload status, …. , which ask for accurate and timely system awareness, i.e., an efficient, fast, and resilient State Estimation function.

Conclusions

- The proposed stochastic cascading model based on Markov transition:
  - takes into account the uncertainty in the load settings, the generation and line flows;
  - correctly captures the stochastic process of the evolution of cascading failures in a power grid with regard to real time signal (i.e., instead of stage numbers);
  - able to indicate which parts of the system are under most stresses therefore most likely to break down in the next time interval.
  - useful to identify and predict the critical paths of the possible cascading failures, given some steady initial condition, with a probabilistic model that allows to explore selectively the future beyond single failures.

- The introduced metrics can unveil the risk of failure and the time margin that is left to perform corrective action.

- The proposed model can be extended to include the dynamics of generations and loads during the cascading process evolution and a full AC network model.

Some Extra References:

- HICSS 2012 conference paper, "A Markov-Transition Model for Cascading Failures in Power Grids " (to appear).
- Full paper "A Stochastic Approach to Studying Cascading Failures in Electric Power Grids", available on line: *http://arxiv.org/submit/265679,* 2011.
- We are going to put the software in the public domain.
- We are focusing right now on state estimation by network diffusion, and want to use this Markovian model to provide a Bayesian method for Grid-State estimation.

Questions?

Thank You!