Basic Epidemiology: SIR Models

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SIR Models

- Assume viruses spread through contact or close proximity between infected and healthy individuals.

- Omit a crucial point:
  How disease is spread through a population.
**SIR Models: Population Classes**

- **Susceptibles (S):** Individuals in the population who have not been infected, but are at risk.
- **Infectives (I):** Individuals who are infected and are capable of transmitting the disease.
- **Removed (R):** These are individuals who can no longer contract the disease because they have recovered with immunity, have been placed in isolation, or have died.
**SIR Model: Population Assumptions**

- Population is well-mixed
  
  Any two individuals are equally likely to encounter one another.

- Population is closed
  - No births or deaths
  - No migration
  - \( N = S + I + R \), where \( N \) is the total number in the population.
**SIR Model: Equations**

\[ \frac{\beta S I}{N} \]

- \( \beta \): Rate infected individual gives rise to new infections
- \( \frac{I}{N} \): Proportion of infected individuals in the population
- \( \beta S \frac{I}{N} \): Rate at which susceptible individuals encounter infected individuals and become infected.
**SIR Model: Equations**

\[ \gamma \quad \text{Rate of recovery once infected} \]

\[ \gamma I \quad \text{Rate at which infected individuals are removed from the infective class.} \]
SIR Model: Equations

\[
\begin{align*}
\frac{dS}{dt} &= -\frac{\beta SI}{N} \\
\frac{dI}{dt} &= \frac{\beta SI}{N} - \gamma I \\
\frac{dR}{dt} &= \gamma I
\end{align*}
\]
Consider two specific times $S + R = N$

- $S = N, \; R = 0$
  
  Before disease begins spreading

- $S = 0, \; R = N$
  
  After disease has moved through entire population
$R_0$  Average number of secondary infections that occur when one infective is introduced into a completely susceptible host population

When the disease begins spreading,

$$S_0 = N$$

For the disease to begin spreading,

$$\frac{dI}{dt} > 0$$
Consider when $S_0 = N$ and $\frac{dI}{dt} > 0$:

$$\frac{dI}{dt} > 0$$

$$\frac{\beta S_0 I}{N} - \gamma I > 0$$

$$\left(\frac{\beta S_0}{N} - \gamma\right) I > 0$$

$$(\beta - \gamma)I > 0$$
Thus, the basic reproductive number is

$$R_0 = \frac{\beta}{\gamma}$$

where

\(\beta\) represents the rate an infected individual gives rise to new infections

\(\frac{1}{\gamma}\) represents the infectious period
**SIR Model: Basic Reproductive Number** \( R_0 \)

\[
R_0 = \frac{\beta}{\gamma}
\]

- \( R_0 < 1 \): The infection dies out and there is no epidemic
- \( R_0 > 1 \): The infection will be established in the population. Infection peaks and then disappears
**SIR Model: Plots for** $R_0 = 0.25 \text{ and } R_0 = 4$

*with 1% Infected Initially*
**SIR Model: Plots for** $R_0 = 0.25$ and $R_0 = 4$

*with 20% Infected Initially*
**SIR Model: Plots for Various $R_0$**

- $R_0 = 0.5$
- $R_0 = 1$
- $R_0 = 1.5$
- $R_0 = 3$
- $R_0 = 10$
- $R_0 = 20$
References

