Basic Epidemiology: SIR Models

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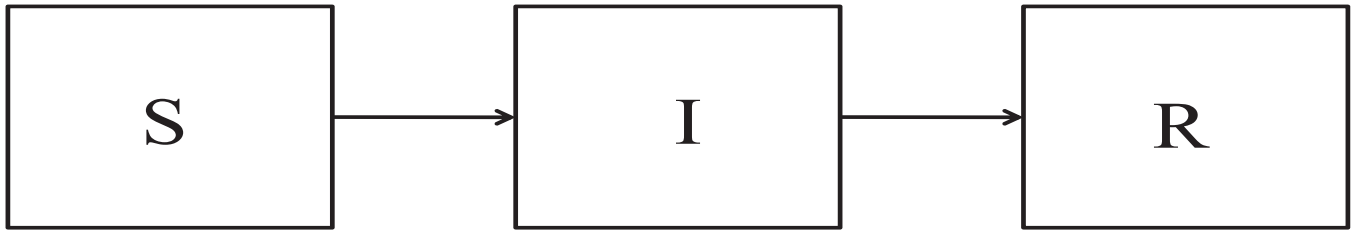
SIR Models

- Assume viruses spread through contact or close proximity between infected and healthy individuals.

- Omit a crucial point:

  How disease is spread through a population.

SIR Models: Population Classes



- Susceptibles (S): Individuals in the population who have not been infected, but are at risk.
- Infectives (I): Individuals who are infected and are capable of transmitting the disease.
- Removed (R): these are individuals who can no longer contract the disease because they have recovered with immunity, have been placed in isolation, or have died.

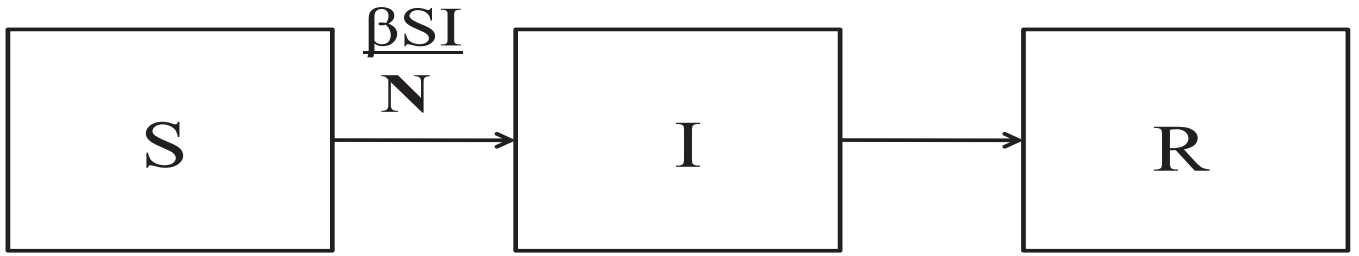
SIR Model: Population Assumptions

- Population is well-mixed

  Any two individuals are equally likely to encounter one another.

- Population is closed
  - No births or deaths
  - No migration
  - $N = S + I + R$, where N is the total number in the population.

SIR Model: Equations

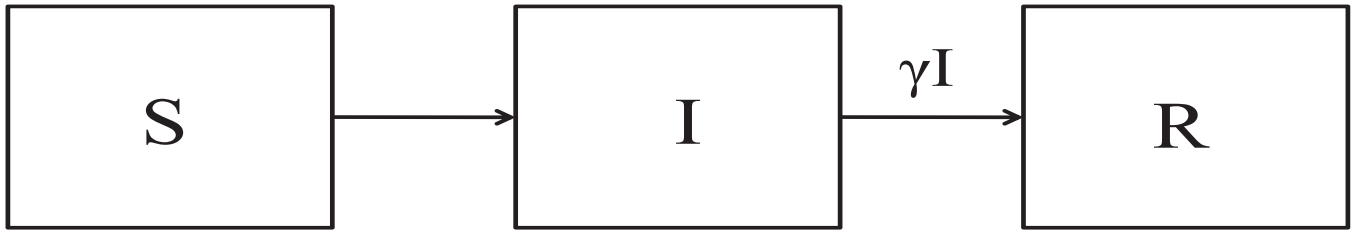


β Rate infected individual gives rise to new infections

$\frac{I}{N}$ Proportion of infected individuals in the population

$\beta S \frac{I}{N}$ Rate at which susceptible individuals encounter infected individuals and become infected.

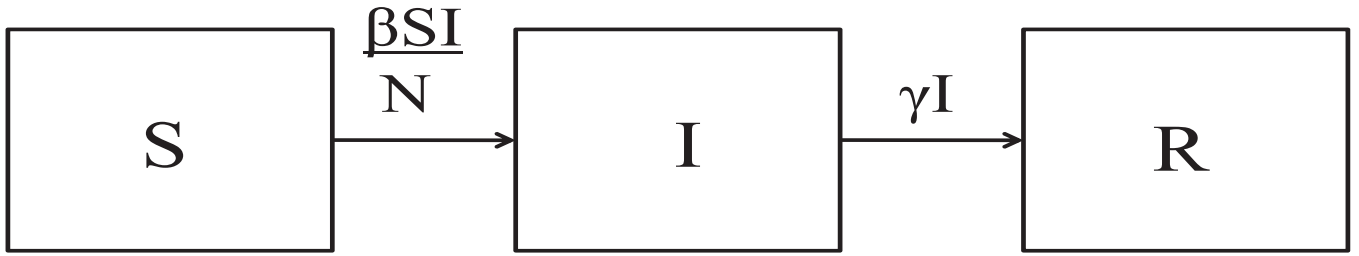
SIR Model: Equations



γ Rate of recovery once infected

γI Rate at which infected individuals are removed from the infective class.

SIR Model: Equations



$$
\begin{aligned}
\frac{dS}{dt} &= -\frac{\beta SI}{N} \\
\frac{dI}{dt} &= \frac{\beta SI}{N} - \gamma I \\
\frac{dR}{dt} &= \gamma I
\end{aligned}
$$

SIR Model: Equilibrium

$$\begin{aligned} \frac{dR}{dt} &= \gamma I \\ \Rightarrow I &= 0 \\ S + R &= N \end{aligned}$$

Consider two specific times $S + R = N$

- $S = N$, $R = 0$

  Before disease begins spreading

- $S = 0$, $R = N$

  After disease has moved through entire population

SIR Model: Basic Reproductive Number R_0

R_0 Average number of secondary infections that occur when one infective is introduced into a completely susceptible host population

When the disease begins spreading,

$$S_0 = N$$

For the disease to begin spreading,

$$\frac{dI}{dt} > 0$$

SIR Model: Basic Reproductive Number R_0

Consider when $S_0 = N$ and $\frac{dI}{dt} > 0$:

$$\begin{aligned} \frac{dI}{dt} &> 0 \\ \frac{\beta S_0 I}{N} - \gamma I &> 0 \\ \left(\frac{\beta S_0}{N} - \gamma\right) I &> 0 \\ (\beta - \gamma) I &> 0 \end{aligned}$$

SIR Model: Basic Reproductive Number R_0

Thus, the basic reproductive number is

$$R_0 = \frac{\beta}{\gamma}$$

where

β represents the rate an infected individual gives rise to new infections

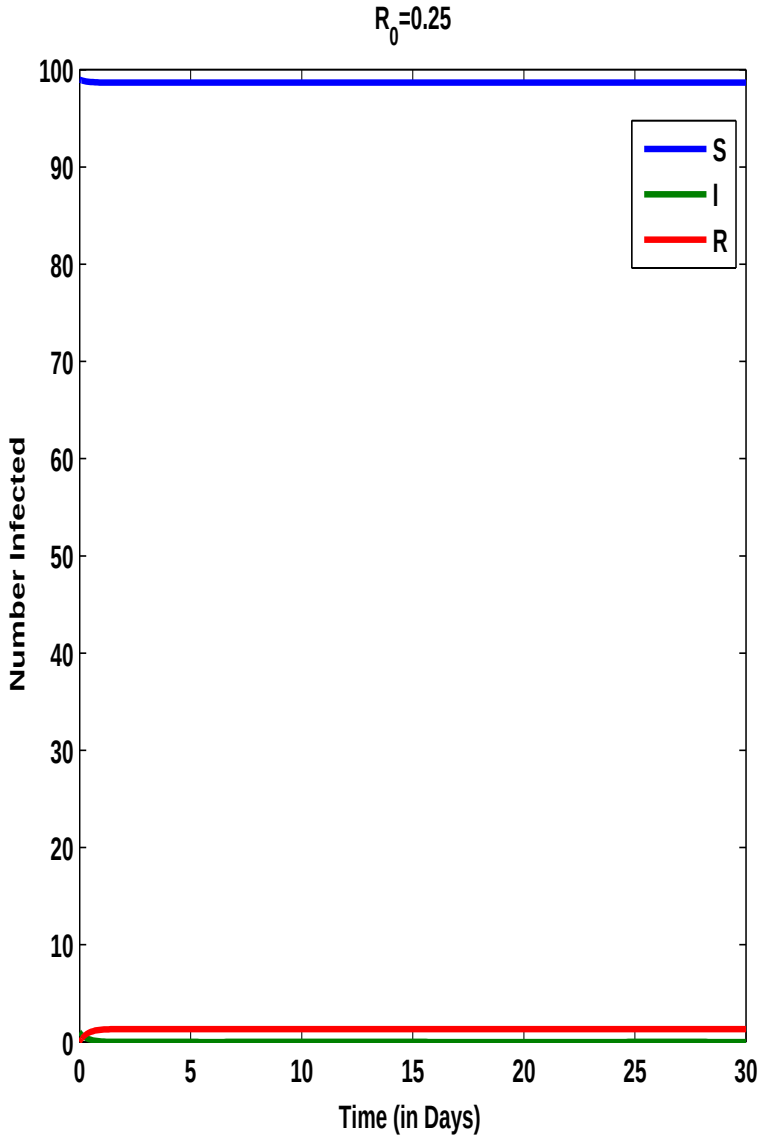
$\frac{1}{\gamma}$ represents the infectious period

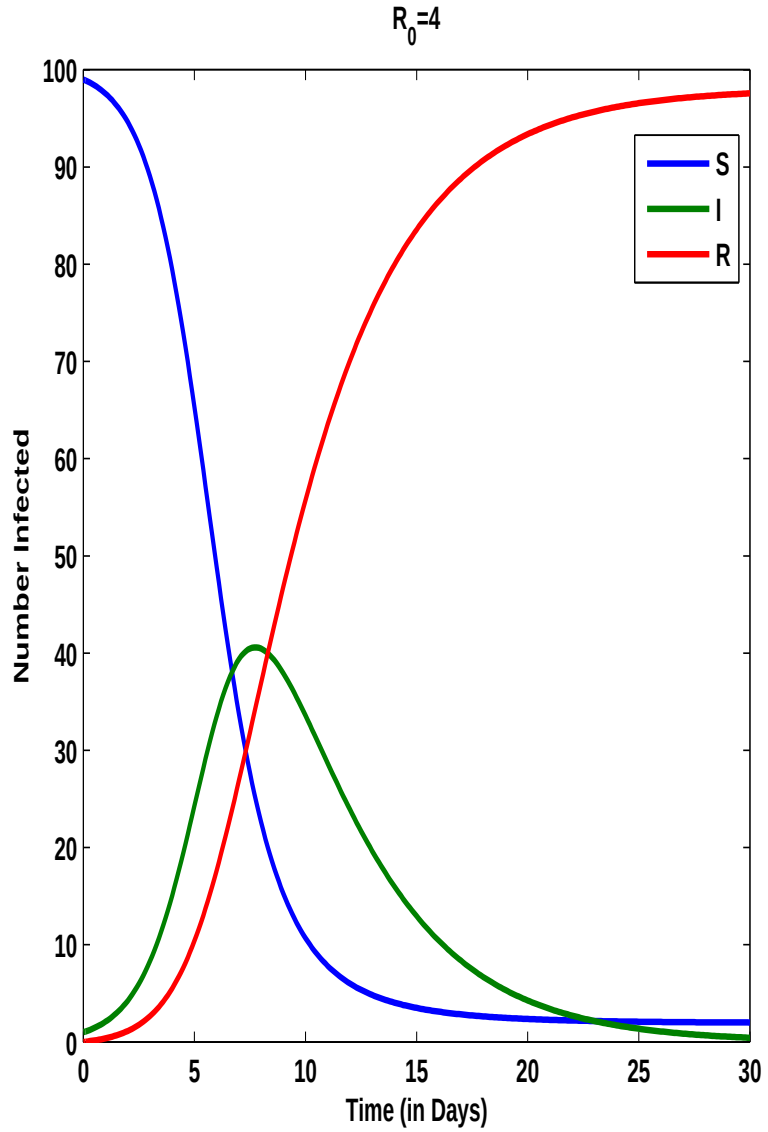
SIR Model: Basic Reproductive Number R_0

$$R_0 = \frac{\beta}{\gamma}$$

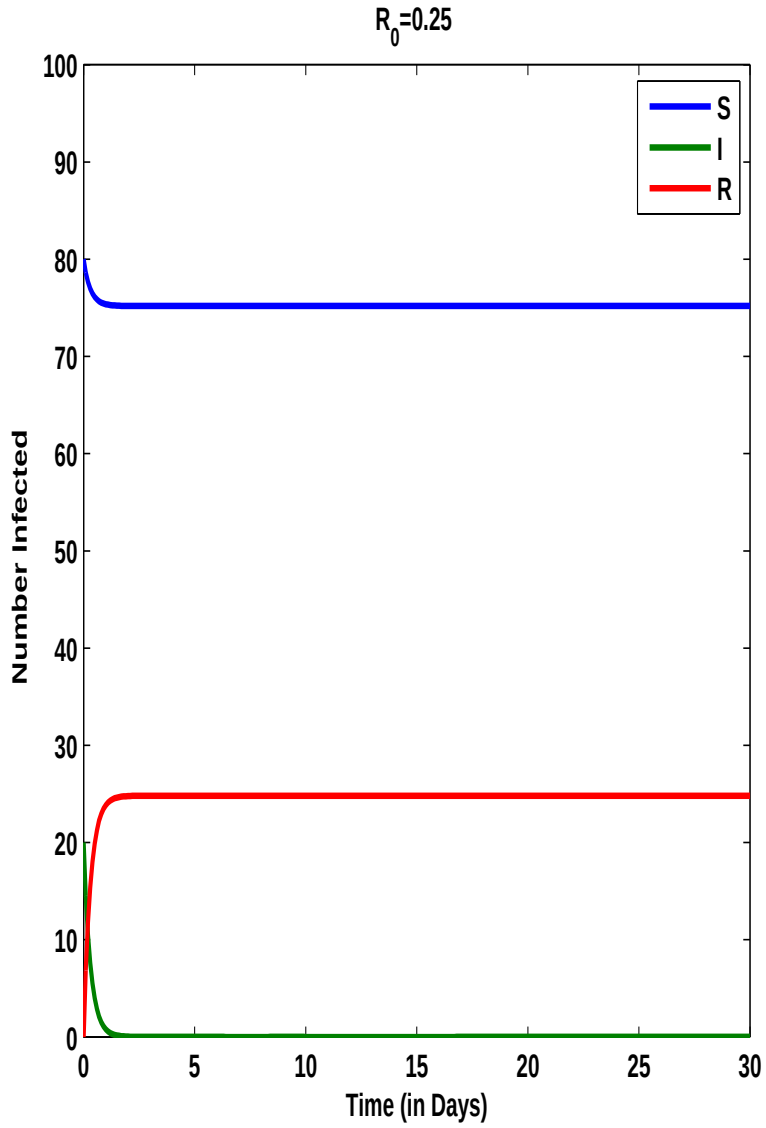
- $R_0 < 1$: The infection dies out and there is no epidemic
- $R_0 > 1$: The infection will be established in the population. Infection peaks and then disappears

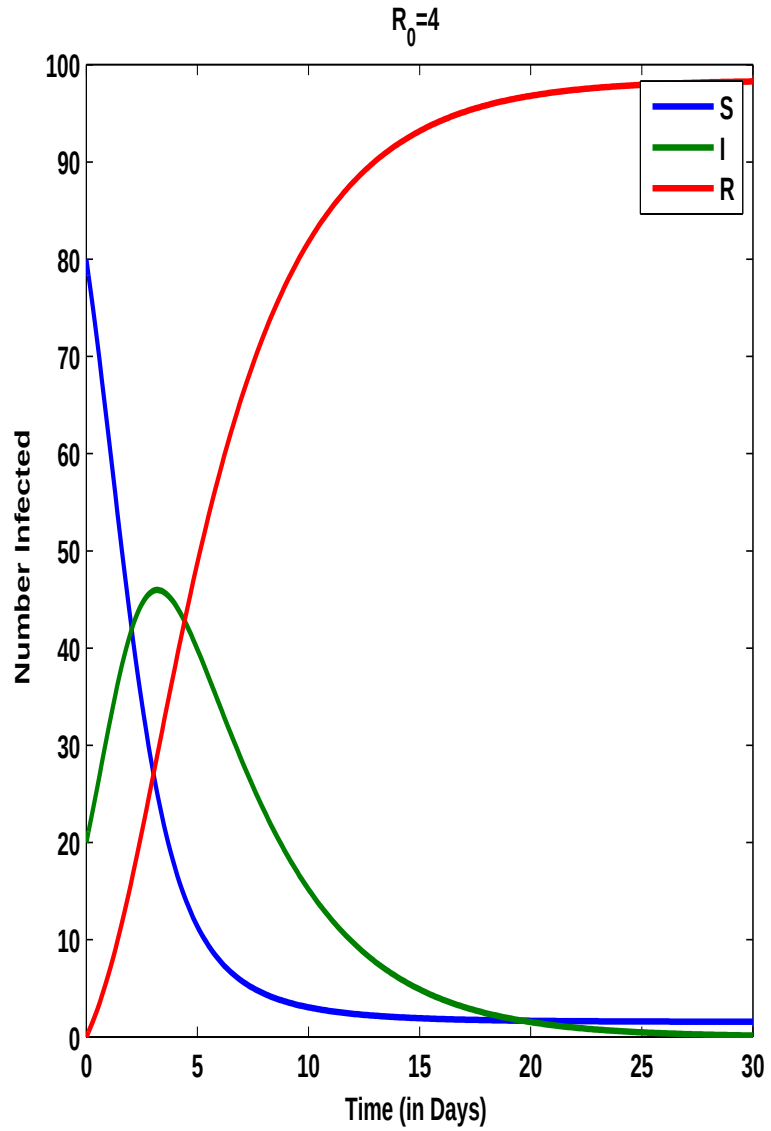
SIR Model: Plots for $R_0 = 0.25$ and $R_0 = 4$ with 1% Infected Initially



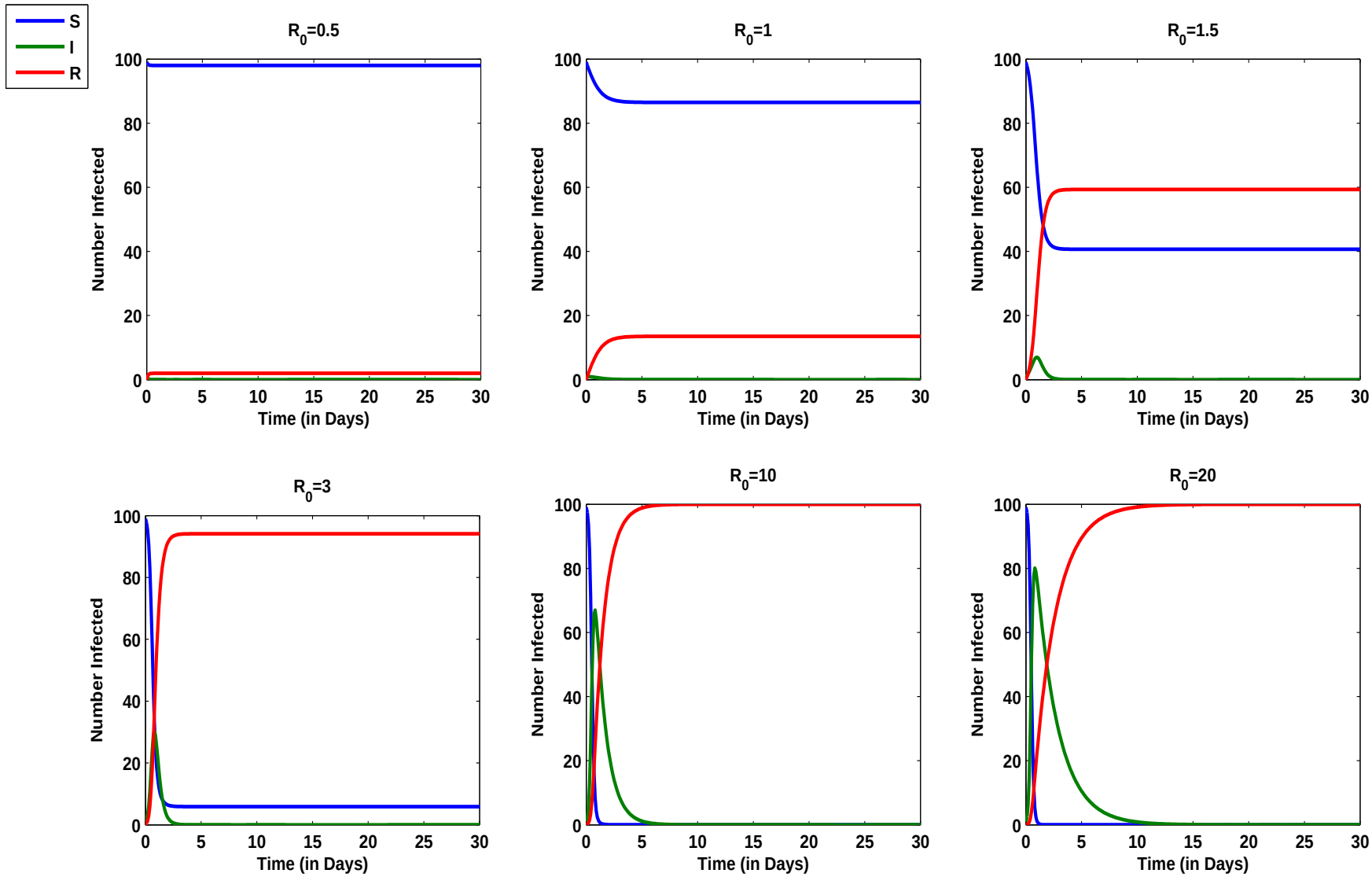


SIR Model: Plots for $R_0 = 0.25$ and $R_0 = 4$ with 20% Infected Initially





SIR Model: Plots for Various R_0



References

- *Mathematical Models in Biology* by Leah Edelstein-Keshet, SIAM, 2005.

- "The Mathematics of Infectious Disease" by Herbert W. Hethcote, SIAM Review, Vol. 42, No. 4, p. 599-653, 2000.

- *The Geographic Spread of Infectious Disease: Models and Applications* by Lisa Sattenspiel with contributions by Alun Lloyd, Princeton, 2009.