Final Program Report
The Geometry and Statistics of Shape Spaces

1. Scientific Overview

Shapes are prevalent in the outside world and in science. They manifest themselves in live animals, plants, landscapes, or in man-made materials, like cars, planes, building, bridges, and they are designed from aesthetic as well as efficacy considerations. Internal organs of humans or other animals also have a commonly accepted, well-defined shape, and their study is an old science called anatomy. For the human mind, there is an intuitive notion of what shapes are, why they differ or look alike, or when they present abnormalities with respect to ordinary observations. Sculpture is the art of rendering existing shapes, or creating new ones, and the fact that artists are still able to provide unambiguous instances of subjects through distorted or schematic representations is a strong indication of the robustness of the human shape recognition engine.

However, an analytical description of a shape is much less obvious, and humans are much less efficient for this task, as if the understanding and recognition of forms works without an accurate extraction of their constituting components, which is probably the case. We can recognize a squash from an eggplant or a pepper using a simple outline, and even provide a series of discriminative features we can distinguish, but it is much harder to instantiate a verbal description of any of them, accurate enough, say for a painter to reproduce it.

It is therefore not surprising that, for mathematics, shape description remains mostly a challenge. The last fifty years of research in computer vision has shown a amazingly large variety of points of view and techniques designed for this purpose: 2D or 3D sets they delineate (via either volume or boundary), moment-based features, medial axes or surfaces, null sets of polynomials, configurations of points of interest (landmarks), to name but a few. Yet, it does not seem that any of these methods has emerged as ideal, neither conceptually nor computationally, for describing shapes.

Beyond the shape characterization issue, the more ambitious program which has interested a large group of researchers during the last two decades, starting with the seminal work of David Kendall, is the study of shapes spaces and their statistics. Here shapes are not only considered individually, but they are seen as variables, belonging to some generally infinite dimensional space which possesses a specific geometry. The theoretical study of such spaces, the definition of computationally feasible algorithmic and statistical procedures has been the subject of a still growing line of work. For example, Kendall's original contribution focused on collections of landmarks modulo the action of rotation and scale. It has since been extended to the actions of other groups and to plane curves instead of points. Other examples build shape spaces using the medial axis representation. The last few years has seen the emergence and the development of several new techniques, building infinite dimensional Riemannian metrics on curves and other shape representations, involving several groups over the world. Within applied mathematics, the analysis of shape spaces arises at a nodal point in which geometry, statistics and numerical analysis each have a fundamental contribution.

The SAMSI summer program in Shape Spaces sought to bring this area to the attention of statisticians, computer scientists and mathematical scientists, whose expertise is critical for the full development of a mathematical and statistical theory of shape which has direct application to a diverse array of problems in computer imaging, medical imaging, and the physical and biological sciences. The program included a mixture of tutorials, research presentations, and working group activities on the subject. The goal was to provide an entry point into the field to interested students and faculty, and to allow researchers who are specialists in the area to exchange recent results and information.

2. Program Scope, Timing, And Activities

The program took place July 7-13, 2007 (from Saturday through Friday). The meeting consisted of two days of tutorials, three days of conferences on shape spaces and two days for working group meetings.

Tutorials at Radisson RTP (Saturday-Sunday July 7-8):

Two days of tutorials were held to provide participants with the foundations for infinite dimensional shape spaces, statistics and analysis on these spaces, and a survey of the application of these methods to medical imaging. All tutorials were taught by experts listed for each area.

July 7: Differential geometry and curvature in infinite dimensional spaces with application to shape spaces by Peter Michor (Universitat Wien) and David Mumford (Brown University) and Diffeomorphisms as an infinite dimensional Lie group and the Euler-Poincare reduction by Laurent Younes (Johns Hopkins University).

July 8: Probability measures and statistics on function spaces and nonlinear infinite dimensional spaces by Alain Trouve (Ecole Normale Superieure de Cachan), Numerical methods for shape analysis by Stephen Marland jointly prepared with Robert McLachlan (Massey University), and Shapes in medical imaging: Computational Anatomy by Michael Miller(Johns Hopkins University).

Workshop: Geometry and Statistics of Shape Spaces at Radisson RTP (Monday – Wednesday, July 9-11) .

The workshop featured talks by invited principal lecturers of approximately 45 minutes, shorter invited talks by New Researchers of approximately 20-25 minutes, and a poster session to which all participants were invited to contribute.

Principal Lecturers:
Daniel Cremers (University of Bonn), James Damon (University of North Carolina at Chapel Hill), Peter Giblin (The University of Liverpool), John Kent (University of Leeds), Benjamin Kimia (Brown University), Hamid Krim (North Carolina State University), Huiling Le (University of Nottingham), Robert McCann (University of Toronto), Steve Pizer (University of North Carolina at Chapel Hill), Anuj Srivastava

(Florida State University), Paul Thompson(University of California, Los Angeles), Keith Worsley (McGill University).

New Researcher Lecturers:
Yan Cao (University of Texas, Dallas), Pedro Felzenszwalb (University of Chicago), Tom Fletcher (University of Utah), Kathryn Leonard (California Institute of Technology), Namrata Vaswani (Iowa State University), Stéphanie Allassonnière (LAGA, Universite Paris 13), Guillaume Charpiat (Max Planck Institute), Joan Glaunes (University Paris 5)
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These lectures presented recent developments related to the geometric and statistical properties associated to shapes and the application of these results to a wide variety of imaging problems. The lecture emphasized open problems and new directions arising from the results presented.

Working Groups at SAMSI (Thursday-Friday July12-13):
Four Working Groups were formed to discuss open problems, and promising areas for investigation. The group meetings consisted of open discussion sessions, one for each half day. This allowed participants to be involved with more than one group if desired. The working groups spanned the following topics:
Topic 1: The geometry of shape spaces.
Topic 2: Probabilistic models of shapes.
Topic 3: Applications of 2D shape analysis.
Topic 4: Applications of 3D shape analysis

3. Organization and Program Leadership

Program Leaders: Darryl Holm (Imperial College, London), Peter Michor (University of Vienna), Michael Miller (Johns Hopkins University), David Mumford (Brown University), Tilak Ratnanather (Johns Hopkins University), Alain Trouve (Ecole Normale Superieure de Cachan) and Laurent Younes (Johns Hopkins University); Directorate Liaison – James Damon (SAMSI).

4. Program Participants

Invitations were extended to approximately 90 participants, and with additional interest, the final number of participants was 118 from areas of statistics, mathematics, computer science, and the health sciences including 44 new researchers. Of these, approximately 30 participated in Working Groups.

5. Program Funding

The workshop was partially supported by the NSF award 0456253, FRG: The Geometry, Mechanics and Statistics of the Infinite-dimensional Manifold of Shapes. One of the commitments of this Focused Research Group was to

contribute to the organization of an international workshop on the group themes.

6. Program Outcome

The goal of the Program was to introduce researchers to these new ideas for studying shapes in imaging problems via statistical and geometric properties defined using shape spaces. It served to assist in the identification of new mathematical and statistical directions for the analysis of shape stimulated by ongoing problems for various imaging modalities.