Avoiding Overages by Deferred Demand for PEV Charging on the Smart Grid

G. Kesidis
CSE and EE Depts, The Pennsylvania State University
kesidis@engr.psu.edu

In collaboration with:
S. Caron, CS Dept, Ecole Normale; G. Pang, IE Dept, PSU;
and T. Konstantopoulos, Math Dept, Uppsala

Outline

- Introduction and problem set-up.
- A model of consumer demand without deferment.
- Functional central limit theorem (FCLT) to derive a simple diffusion model of the undeferred load on the utility.
- A discrete-time load-deferment framework, potentially resulting in overages at the end of the deferment interval.
- A simple numerical study under idealized load deferment.
- Residual-demand/time slackness rule to decide whom to defer at each given point in time.
- Problem variations and related issues.

Introduction

- We consider the problem of management by a regional utility of its collective PEV/PHEV load, likely borne in the earlier morning hours.

- To avoid spikes in demand due to synchronization, the consumers independently and uniformly at random choose charging start-times.

- Given this, there may still be moderate overage periods which the utility will need to manage by deferring demand.

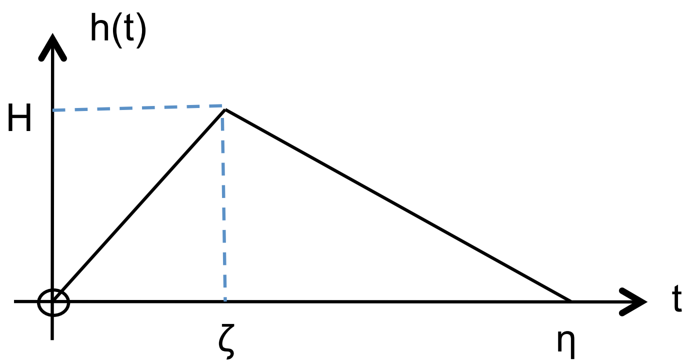
- With aggregate load characterized by a Gaussian process (FCLT), a regional utility can seek to minimize a weighted combination of
  - the probability of overage (negative social welfare), and
  - the (*e.g.*, day ahead) contractual cost of their supply from an electricity distribution system.

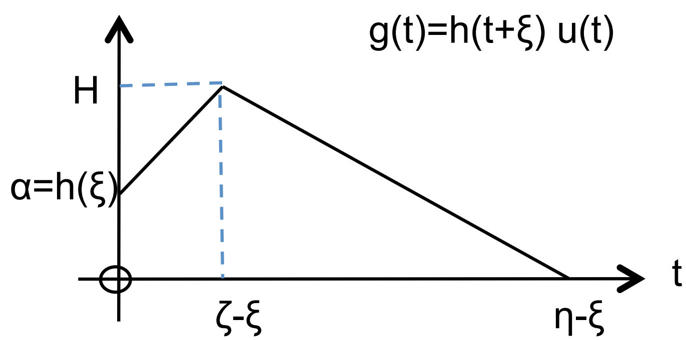
Problem Set-Up

- We consider a (finite) T-hour time period, *e.g.*, $T = 8$ hours from 10PM to 6AM, during which n consumers need to *automatically* schedule their electrical demand.

- A unimodal power consumption profile h is more typical of an initially empty, idealized Lithium-ion battery:
  - the current is constant and the voltage grows until a peak-power threshold at time $\zeta > 0$ is reached,
  - whereafter the voltage is constant and the current diminishes.

- The profile $g(t) = h(t+\xi)u(t)$, where u is the unit step, accounts for initial charge or available local supply at the consumer's premises.





Problem Set-Up (cont)

- Assume consumption profile parameters are mutually independent random variables over consumer index i.

- Also, assume that start-times $\xi_i \in [0, \eta_i]$ are chosen independently given η_i.

- For K parameter classes of profiles/flags, each occurring with prob. $p^{(k)} = n^{(k)}/n$, the total instantaneous power charging rate for class k is

$$D^{(k)}(t) \;=\; \sum_{i=1}^{n^{(k)}} g_i^{(k)}(t - \xi_i^{(k)}).$$

- As the total consumer/flag arrival rate $\lambda \to \infty$, with $\lambda = \sum_k \lambda^{(k)}$ in fixed proportions $\lambda^{(k)} = p^{(k)}\lambda$, the diffusion-scaled *energy* processes are

$$\begin{aligned}
\widehat{C}_\lambda^{(k)}(t) &= \frac{1}{\sqrt{\lambda}}\left(\int_0^t D^{(k)}(s)\mathrm{d}s - \lambda\mu^{(k)} t\right), \\
\widehat{C}_\lambda(t) &= \sum_{k=1}^{K} \widehat{C}_\lambda^{(k)}(t) = \frac{1}{\sqrt{\lambda}}\left(C_\lambda(t) - \lambda\mu t\right),
\end{aligned}$$

where $\mu^{(k)} := p^{(k)}\mathsf{E}\beta^{(k)} := p^{(k)}\mathsf{E}\int_0^\infty g^{(k)}(s)\mathrm{d}s$ and $\mu := \sum_{k=1}^K \mu^{(k)}$.

Diffusion Approximation of Undeferred Load

Theorem: (FCLT) If $\mathsf{E}H^{2+\delta}, \mathsf{E}\eta^{2+\delta} < \infty$ for some $\delta > 0$, then as $\lambda \to \infty$,

$$(\widehat{C}_\lambda^{(1)}, ..., \widehat{C}_\lambda^{(K)}, \widehat{C}_\lambda) \ \Rightarrow (\sigma_1 B^{(1)}, ..., \sigma_K B^{(K)}, \sigma B)$$

in $D([0,T], \mathbb{R}^{K+1})$ (endowed with the Skorohod J_1 product topology), where

- $(B^{(1)}, ..., B^{(K)})$ is a K-dimensional standard BM,
- B is a standard BM such that $\sigma B \stackrel{\mathrm{d}}{=} \sigma_1 B^{(1)} + \cdots + \sigma_K B^{(K)}$ with
  - $\sigma^2 = \sigma_1^2 + \cdots + \sigma_K^2$ and
  - $\sigma_k^2 = \mathrm{var}[\beta^{(k)}] + \bar{\lambda}_k^2 \mathsf{E}[\beta^{(k)}]^2, \quad k \in \{1, ..., K\}.$

Overages for Undeferred Load

- So, for large arrival rate λ, energy consumption up to time t is
$$C(t) \approx X(t) := \lambda\mu t + \sqrt{\lambda}\sigma B(t), \quad t \geq 0.$$

- Suppose we are given a threshold total charging rate L ($L > \lambda\mu$) and a fixed time window t_o, *e.g.*, each specified by a Day Ahead market-based Demand Response Program (DADRP).

- So, the energy consumption *overages* above L over $[0, T_o]$ are
$$C(t_o, L, T_o) \; := \; \sum_{i=1}^{\lceil T_o/t_o \rceil} [C(it_o) - C((i-1)t_o) - Lt_o]^+$$

- **Theorem:** When the demand is large, the expected energy consumption overages $\mathsf{E}C(t_o, L, T_o)$ over any time interval $[0, T_o]$,
$$\mathsf{E}C(t_o, L, T_o) \approx \frac{(\lambda\mu - L)T_o}{2}\Phi\left(\frac{(\lambda\mu - L)}{\sigma}\sqrt{t_o/\lambda}\right) + T_o\sigma\sqrt{\frac{\lambda}{2\pi t_o}}\exp\left(-\frac{(\lambda\mu - L)^2 t_o}{2\lambda\sigma^2}\right),$$
where $\Phi(\cdot)$ is the CDF of the standard normal distribution.

- Note that $\lambda\mu < L$, so the first term on RHS is < 0, but the second term will dominate and the sum is ≥ 0.

Load Deferral: Summary Framework

- Assume that the electricity pricing framework has two stages based on aggregate demand X (rather than on a per-consumer basis) at each epoch of duration t_o.

- If the aggregate demand exceeds a threshold (L), we assume that the smart grid is capable of deferring the overage $(X - L)$ to the next epoch.

- To do so, the grid needs to be able to predict that an overage will occur at the start of the epoch, ascertain which consumers to defer, and be able to pause the power supply to those consumers (discussed later).

- Note that the charging of Lithium-ion car batteries can be paused without significant adverse effects on their performance or longevity.

Load Deferral: Summary Framework

- Discretizing time by t_o seconds, we can obtain an approximate i.i.d. Gaussian process for the new power demand P_j for epoch $j \in \mathbb{Z}^+$, *i.e.*, at time jt_o,

$$P_j \stackrel{\mathsf{d}}{=} \mathsf{N}(\lambda\mu t_o, \lambda\sigma^2 t_o)$$

- Assuming idealized deferment of the excess aggregate load, where the carry-over load is the backlog of a discrete-time GI/D/1 queue X with i.i.d. arrivals P_j and deterministic service Lt_o,

$$X_j = (X_{j-1} + P_j - Lt_o)^+, \quad j \in \mathbb{Z}^+ \text{ with } X_0 = 0.$$

Load Deferral: Residual Service Demand

- Let $J = T_o/t_o$ and note that the residual demand over the final (post deferment) epoch $[T_o, T]$ is

$$X_J \stackrel{\mathrm{d}}{=} \max\{Y_0, Y_1, ..., Y_J\}$$

  where $Y_j = \sum_{i=1}^{j}(P_i - Lt_o)$ for $j \geq 1$, and $Y_0 = 0$.

- By the reflection principle,

$$\mathsf{P}(X_J > x) = 2\mathsf{P}(Y_J > x) = 2\mathsf{P}(\mathsf{N}([\lambda\mu - L]T_o, \lambda\sigma^2 T_o) > x).$$

- So, we assume the average total overages charged by the utility will be

$$\pi\Omega := \pi\mathsf{E}(X_J - L(T - T_o))^+$$

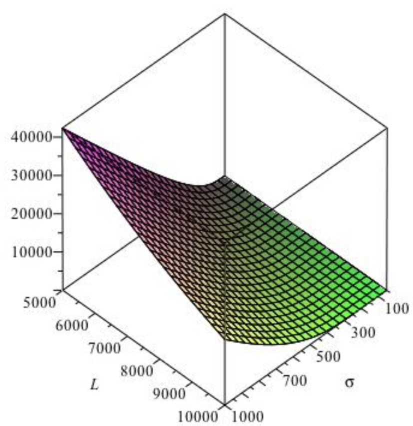
  where π \$/kWh is the overage billing rate.

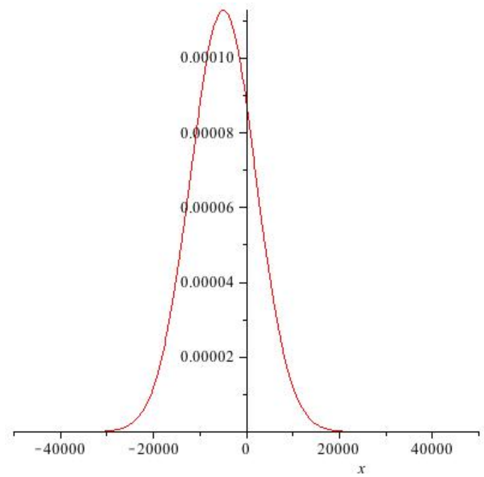
- One may expect that the *contract* between the electrical distribution system and the utility will depend on the π, L, and $\phi \leq \pi$ (the cost per kWh consumed below L) will itself cost the utility in a manner increasing in L.

- Also, one would expect that ϕ would be an increasing function of L, but π may be decreasing in L.

Load Deferral: Numerical Example

- The following figure on left depicts the overages Ω evaluated numerically as a function of L and σ when: $\lambda = 1000$ consumers/h (hour), $T_o = 5$h and $T = 8$h (so that $n = \lambda T_o = 5000$ consumers and $\eta_{\mathsf{max}} = T - T_o = 3$h), $t_o = 0.05$h (*i.e.*, 3 minutes), and $\mu = 5$ kWh/consumer.

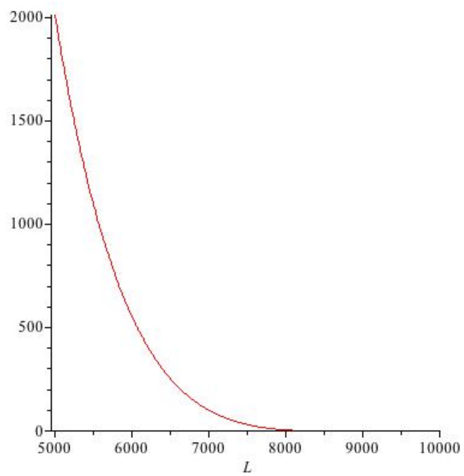




- The distribution of X_J is shown in the figure on right for $L = 6000$ $(\lambda\mu)$ and $\sigma = 100$.

Load Deferral: Numerical Example (cont)

- Following intuition, overage Ω is decreasing in L and increasing in σ.



- Note that if the overage charging rate $\pi(L)$ is increasing in L (again, such that $\pi(L) > \phi(L)$ where ϕ is increasing), then the total overage charges $\pi(L)\Omega(L)$ may be unimodal with a unique minimum L.

Load Deferral: Choosing Which User to Defer

- Simply, if there is $w\%$ overage, then $w\%$ of consumers could be chosen uniformly at random for deferment ($\Rightarrow$ higher $\mathsf{var}(X_J)$ than for idealized deferment above).

- Alternatively, deferment could be based on a heuristic score S of each consumer's *residual* power-demand profile and the remaining time $T - t$.

- For example, for a consumer that started at time τ with initial charge zero, a large score

$$S\left(h(t-\tau),\ \int_{t-\tau}^{\sigma} h(r)\mathrm{d}r,\ \frac{\eta-(t-\tau)}{T-t}\right)$$

  would correspond to a larger probability of deferment where S would be

  - an increasing function of its first argument (current/instantaneous power demand) and
  - a decreasing function of the last two arguments (residual energy demand, and residual charge time as a fraction of remaining time).

- See, *e.g.*, the decentralized "time/slackness" scheduling heuristic of [CK-SGC-2010].

Future Work and Problem Variations

- It would be more realistic to expect that the grid will only know the instantaneous power demand $X(jt_o)$ at the start of the j^{th} interval, rather than the overages in that interval *a priori*.
  - Using the reflection principle, the grid can compute the risk of overage for each epoch j, $\rho_j = \mathsf{P}(\max_{jt_o \leq t < (j+1)t_o} X(t) > L \mid X(jt_o))$.
  - So, if $\rho_j > \rho^{\max}$, the grid could defer some of the consumers so that the overage risk in the interval is reduced to the maximum allowable, $\rho^{\max}$.

- We have extended this model to accommodate additional variable (short-term predictable) power supply (*e.g.*, from wind farms) modulating the overage threshold L upward.

- Can employ a large deviations principle to model aggregate demand.

- There is some research on the use of "excessively charged" car batteries as *sources* of energy to avoid overages by the aggregate demand.

Future Work and Problem Variations: Policing the Consumer

- A demand deferment policy may need to rely on trusting the residual demand reported by the consumer, unless that was somehow securely metered by the grid.
- Again note that smart metering could be used to detect if a consumer misstates their load, does not obey a request to defer, or simply does not disclose their demand to the grid for purposes of "smart" deferment.
- Given simple applied cryptographic techniques (which may be in play in any case to ensure privacy), the utility can accept load attestations from the consumers in a non-repudiatable fashion (*i.e.*, using accompanying digital signatures)
- The grid can easily police whether the consumers start at random times based on sampling their starting times night after night, obtain the resulting empirical distribution, and comparing to a uniform via, e.g., Kolmogorov-Smirnov.