ECONOMIC IMPACT OF EXTREME CLIMATE EVENTS: IMPLICATIONS FOR UNCERTAINTY QUANTIFICATION IN RISK ANALYSIS

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Hurricane Andrew (1992)

PHOTO: SCOTT OLSON/GETTY IMAGES

Hurricane Irene (2011)

Quote

John Tukey:

"*As I am sure almost every geophysicist knows, distributions of actual errors and fluctuations have much more straggling extreme values than would correspond to the magic bell-shaped distribution of Gauss and Laplace.*"

Outline

(1) **Motivation**

(2) **Ultimate Extreme Value Theory**

(3) **Penultimate Extreme Value Theory**

(4) **Damage Functions**

(5) **Economic Damage Caused by Hurricanes**

(6) **Return Levels Under Nonstationarity**

(1) Motivation

- Uncertainty Quantification

-- Tails of distributions

Most important for impacts

- "Risk, Decisions and Impacts"

-- SAMSI theme (erstwhile?)

"Risk of high impact, low probability events"

"Application of UQ to climate risk and decision analysis"

- **Extreme Value Theory**

-- Natural framework for statistical modeling of such events

- **Damage Functions**

-- Convert intensity of extreme climate event to economic damage (Functional form?)

- **Penultimate Approximations**

-- Refinement to Extreme Value Theory

-- Use to relate tail behavior of damage to that of underlying climate variable

(2) Ultimate Extreme Value Theory

- **"Ultimate" Extreme Value Theory**

  $X_1, X_2, . . ., X_n$ **independent with common cdf** F

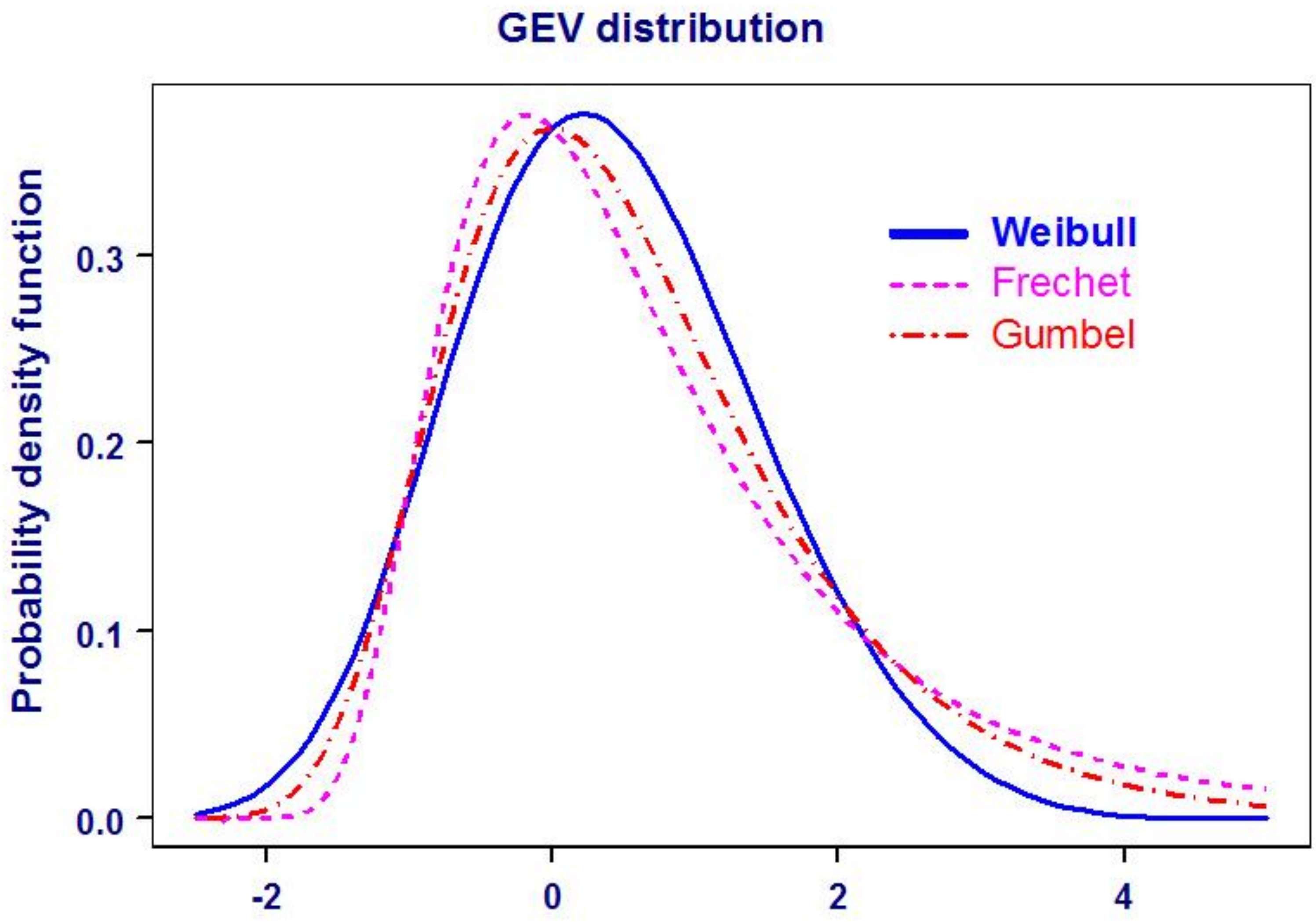
-- Suppose that there exist constants $a_n > 0$ **and** b_n **such that**

$$\Pr\{[\max(X_1, X_2, . . ., X_n) - b_n] / a_n \leq x\} \longrightarrow G(x) \text{ as } n \longrightarrow \infty$$

Then G **must a *generalized extreme value* (GEV) cdf; that is,**

$$G(x; \mu, \sigma, \xi) = \exp\left\{-[1 + \xi (x - \mu)/\sigma]^{-1/\xi}\right\}$$

μ **location,** $\sigma > 0$ **scale,** ξ **shape parameter**



GEV distribution
Probability density function
0.0
0.1
0.2
0.3
-2
0
2
4
Weibull
Frechet
Gumbel

- **Excesses Over High Threshold**

-- X random variable

$Y = X - u$ "excess" over high threshold u, conditional on $X > u$

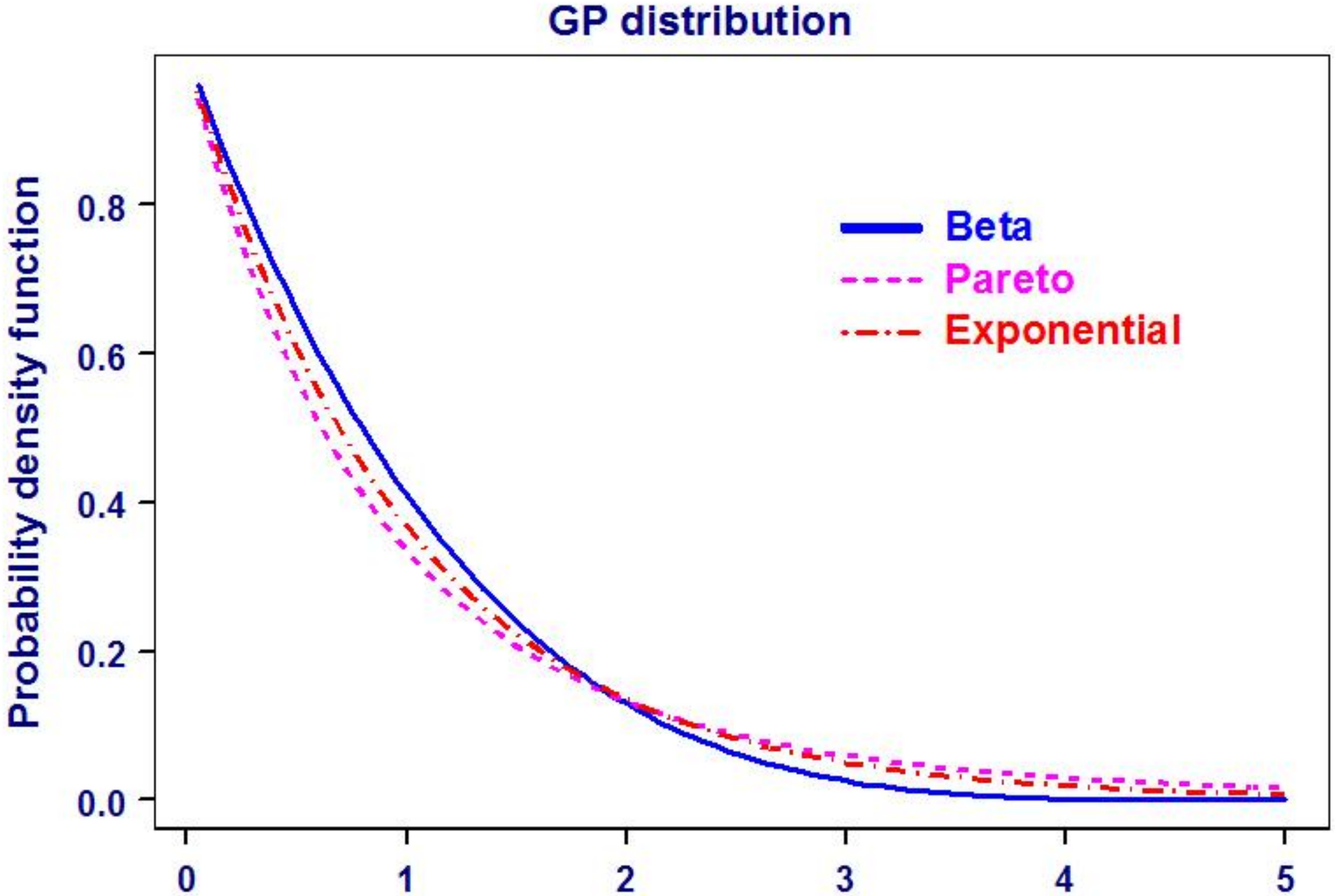
-- Consistent with GEV distribution for block maxima

Y has approx. *generalized Pareto* (GP) dist. for large u with cdf:

$$H(y;\, \sigma_u,\, \xi) = 1 - [1 + \xi\,(y\,/\,\sigma_u)]^{-1/\xi}\,,\quad y > 0$$

$\sigma_u > 0$ scale parameter (Depends on threshold u)

ξ shape parameter (Same interpretation as that of GEV dist.)


GP distribution
Probability density function
Beta
Pareto
Exponential
0.0
0.2
0.4
0.6
0.8
0
1
2
3
4
5

(3) Penultimate Extreme Value Theory

- **Penultimate Approximations**

-- Suppose cdf F in "domain of attraction" of Gumbel type (i. e., $\xi = 0$)

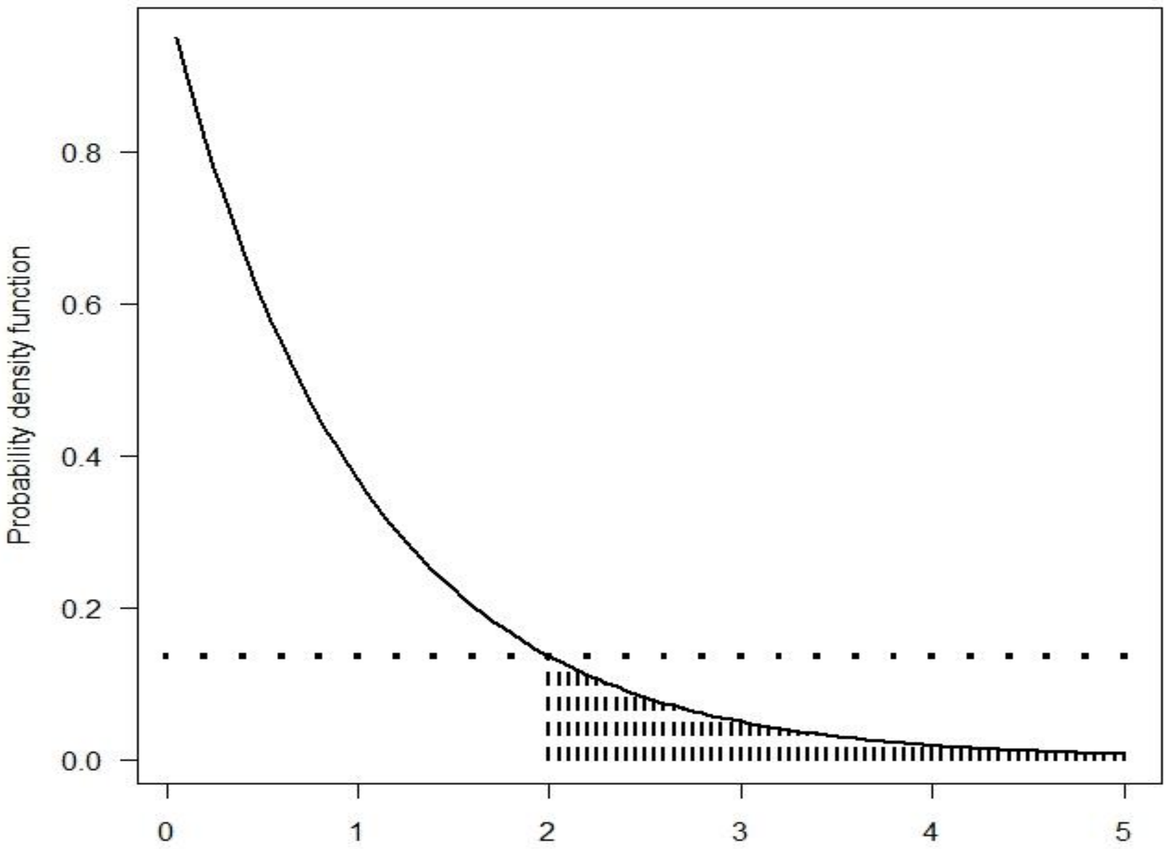
-- Still preferable in nearly all cases to use GEV as approximate distribution for maxima (i. e., act as if $\xi \neq 0$)

-- Expression (as function of block size n) for shape parameter ξ_n

"Hazard rate" (or "failure rate"):

$$h_F(x) = F'(x) \,/\, [1 - F(x)]$$

Instantaneous rate of "failure" given "survived" until x



Hazard rate

-- Alternative expression: $h_F(x) = -[\ln(1 - F)]'(x)$

One choice of shape parameter (block size n):

$$\xi_n = (1/h_F)'(x)\,|_{x=u(n)}$$

Here $u(n)$ is "characteristic largest value" [or $(1 - 1/n)$th quantile of F]

$$u(n) = F^{-1}(1 - 1/n)$$

-- Because F assumed in domain of attraction of Gumbel,

$$\xi_n \longrightarrow 0 \text{ as block size } n \longrightarrow \infty$$

Von Mises condition: F is in domain of attraction of Gumbel if

$$(1/h_F)'(x) \longrightarrow 0 \text{ as } x \longrightarrow \infty$$

- ***Example***: **Normal Distribution (with zero mean & unit variance)**

-- **Fisher & Tippett (1928) proposed Weibull type of GEV as penultimate approximation**

Hazard rate: $h_F(x) \approx x$, **for large** x

[For standard normal dist., $1 - F(x) \approx F'(x) / x$, **for large** x**]**

Characteristic largest value: $u(n) \approx (2 \ln n)^{1/2}$, **for large** n

-- **Penultimate approximation is Weibull type with**

$$\xi_n \approx -1 / (2 \ln n)$$

For example: $\xi_{30} \approx -0.15$, $\xi_{100} \approx -0.11$

- *Example*: "Stretched Exponential" Distribution

-- Traditional form of Weibull distribution (unit scale, shape parameter c)

$$1 - F(x) = \exp(-x^{c}), \; x > 0, \; c > 0$$

Hazard rate: $h_F(x) = c\, x^{c-1}, \; x > 0$

Characteristic largest value: $u(n) = (\ln n)^{1/c}$

-- Penultimate approximation has shape parameter

$$\xi_n \approx (1 - c) \,/\, (c \ln n)$$

(i) $c > 1$ implies $\xi_n \uparrow 0$ as $n \longrightarrow \infty$ (i. e., Weibull type)

(ii) $c < 1$ implies $\xi_n \downarrow 0$ as $n \longrightarrow \infty$ (i. e., Fréchet type)

- **Simulation experiment**

-- Generated observations from stretched exponential distribution with shape parameter $c = 2/3$

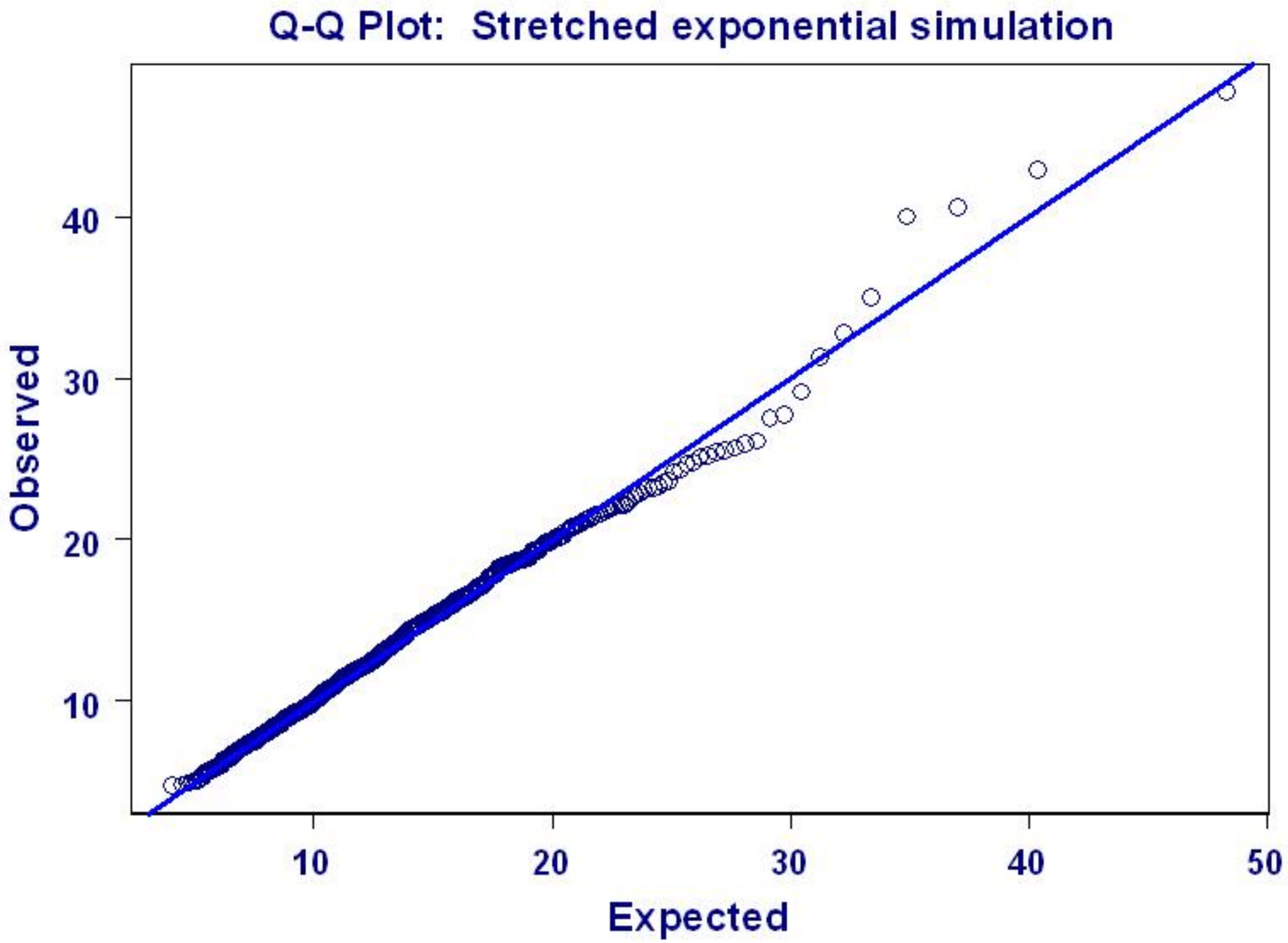
[Wilson & Toumi (2005): Hypothesized $c = 2/3$ for heavy prec.]

-- Determine maximum of sequence of length $n = 100$

Penultimate approximation gives: $\xi_{100} \approx 0.11$

-- Fitted GEV distribution (1000 replications):

Obtained estimate of $\xi \approx 0.12$


Q-Q Plot: Stretched exponential simulation
Observed
10
20
30
40
10
20
30
40
50
Expected

(4) Damage Functions

- **Relation to Economic Damage**

-- Relate damage L to climate variable V

(e. g., wind speed)

Power transformation typically assumed

$$L \propto V^{b}, \; b > 0$$

(e. g., $b = 3$ argued on physical basis for hurricane damage as function of intensity)

-- **Assume stretched exponential distribution for climate variable (at least for upper tail) with shape parameter *c***

Implies stretched exponential distribution for upper tail of damage with shape parameter: $c^* = c / b$

-- **Exploit penultimate approximation**

Wind speed (measure of hurricane intensity):

Common to fit with stretched exponential distribution (Typical values of *c*: roughly 2 or 3)

But heavy tail observed for damage (Could be explained by penultimate approximation)

(5) Economic Damage Caused by Hurricanes

- **Data**

-- Data set for time period 1900 – 2005

Pielke et al. (2008): sciencepolicy.colorado.edu/

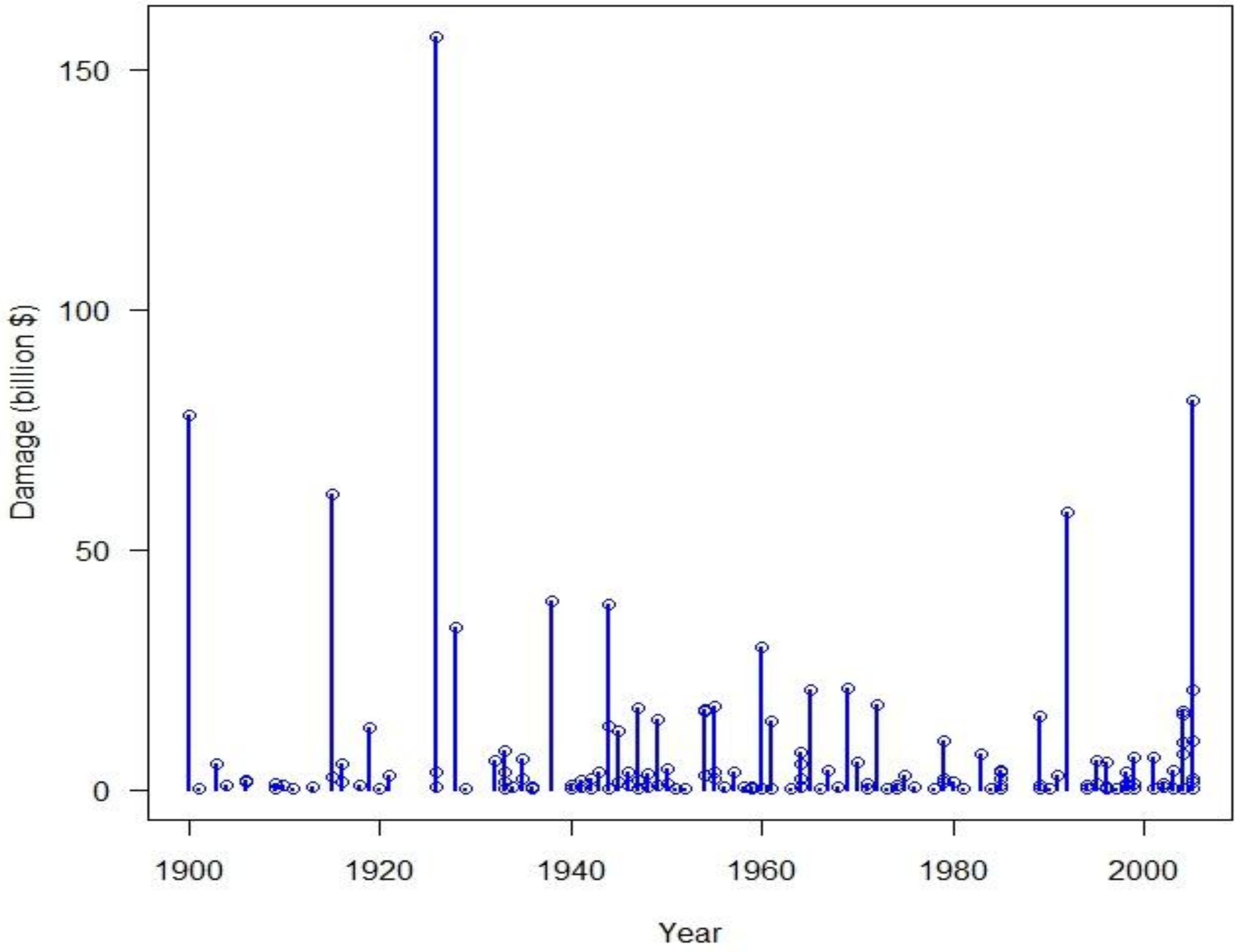
publications/special/normalized_hurricane_damages.html

-- Adjusted for inflation & changes in societal vulnerability (US$ 2005)

Increased population along coast

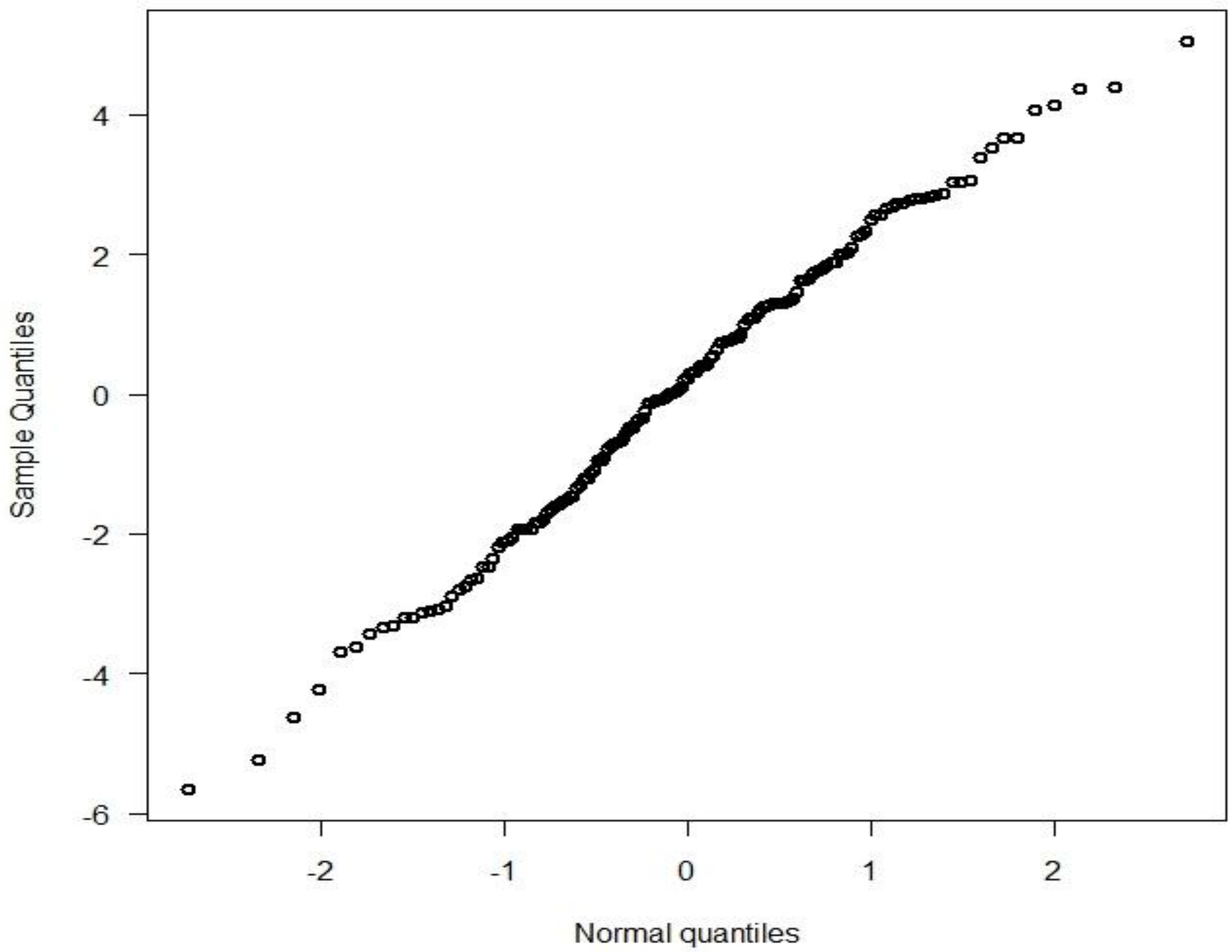
Increased wealth along coast

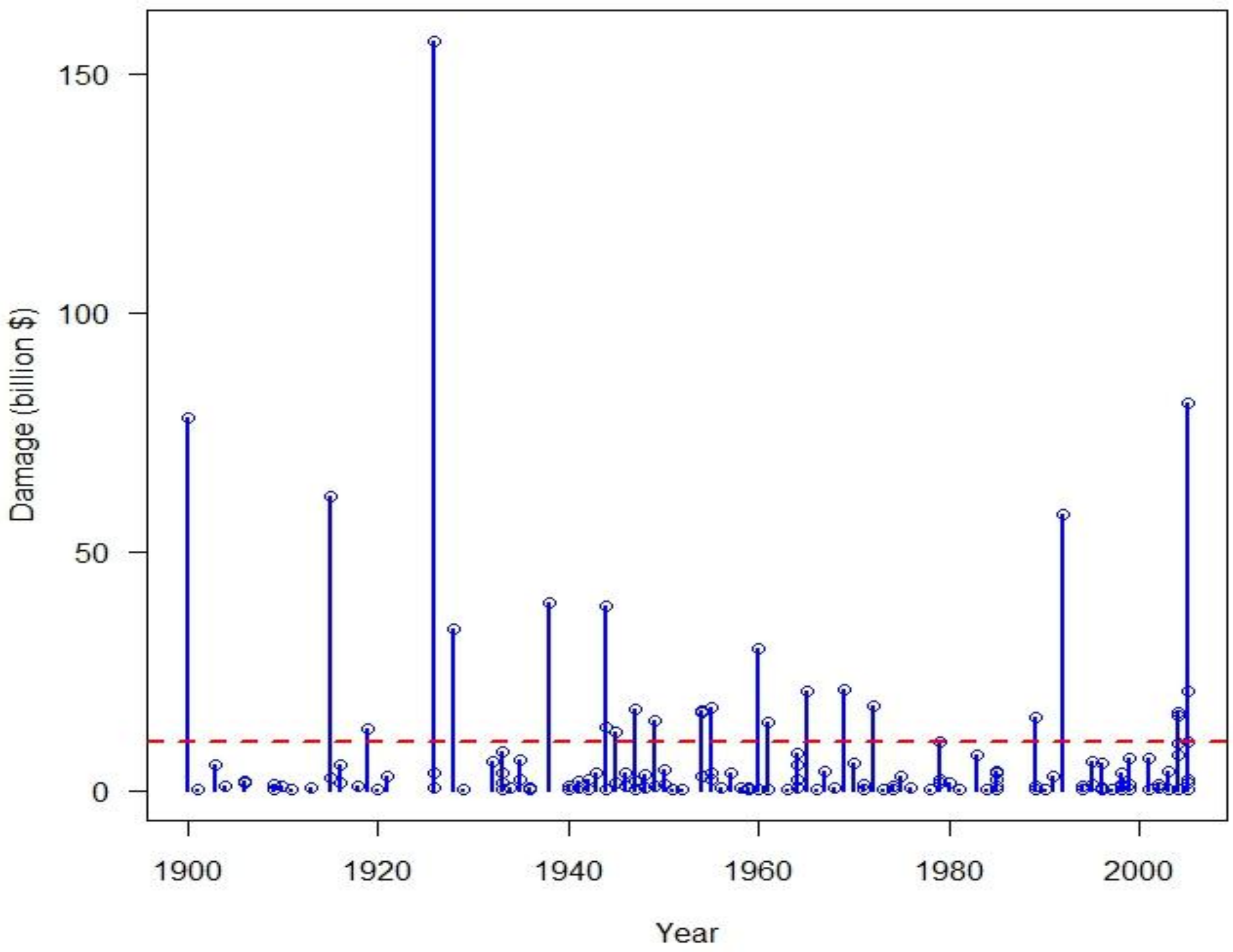
-- Exclude events with damage < $0.1 billion



150
100
50
0
Damage (billion $)
1900
1920
1940
1960
1980
2000
Year

Log-Transformed Hurricane Damage





150
100
50
0
Damage (billion $)
1900
1920
1940
1960
1980
2000
Year

- **Extreme High Damage**

-- Excess $Y = L - u$ over high threshold u

Model Y with GP distribution

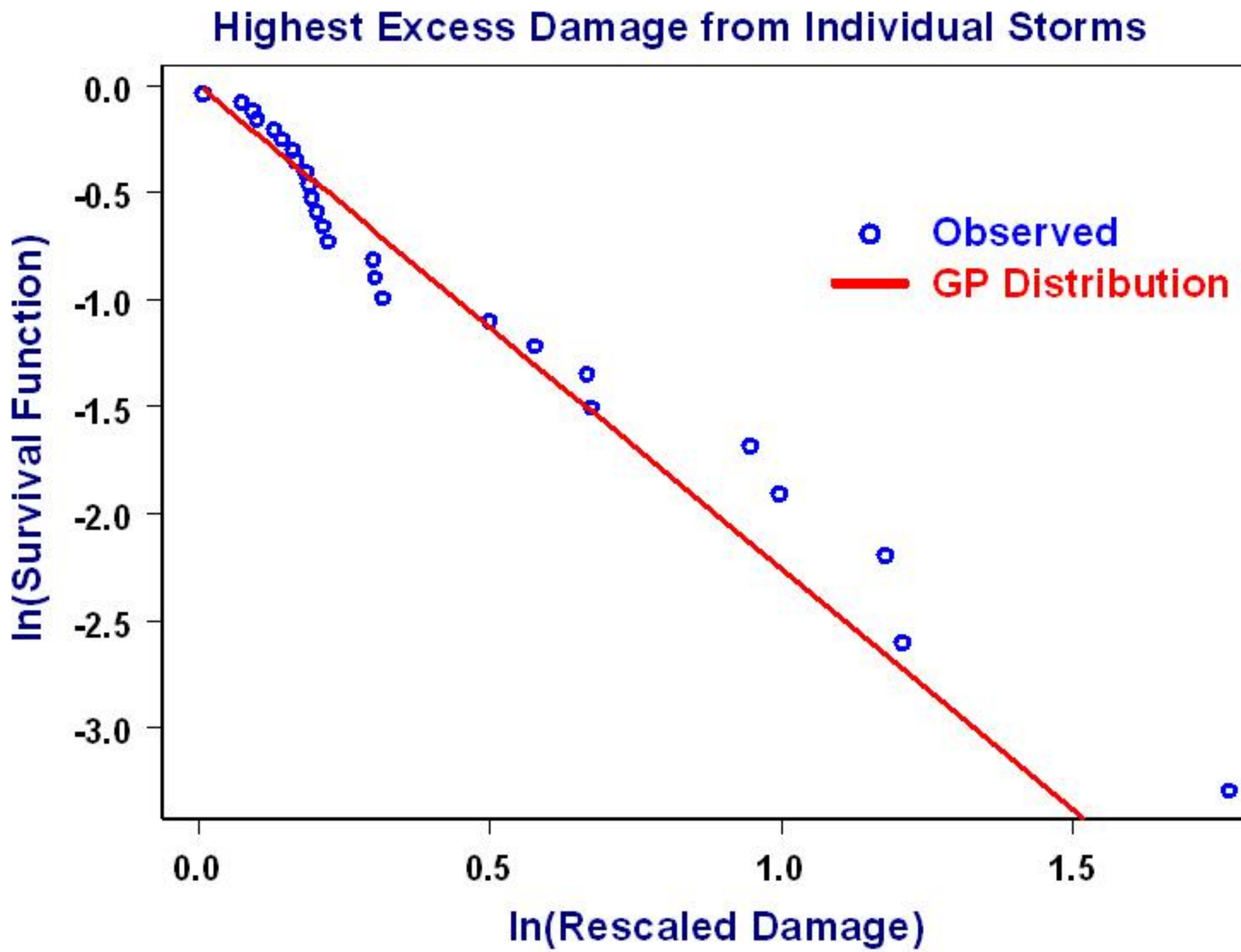
Log-transformed "survival function":

$$\ln[1 - H(y;\, \sigma_u,\, \xi)] = -\,(1/\xi)\,\ln[1 + \xi\,(y\,/\,\sigma_u)]$$

-- Excess in damage over threshold u = \$10 billion (26 storms)

-- Estimated shape parameter ξ of GP distribution ≈ 0.44

90% confidence interval (Profile likelihood): $0.07 < \xi < 1.06$



Highest Excess Damage from Individual Storms
ln(Survival Function)
ln(Rescaled Damage)
Observed
GP Distribution
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
0.0
0.5
1.0
1.5

- **Hurricane damage function**

-- **Hurricane intensity measured by wind speed (at landfall) *V***

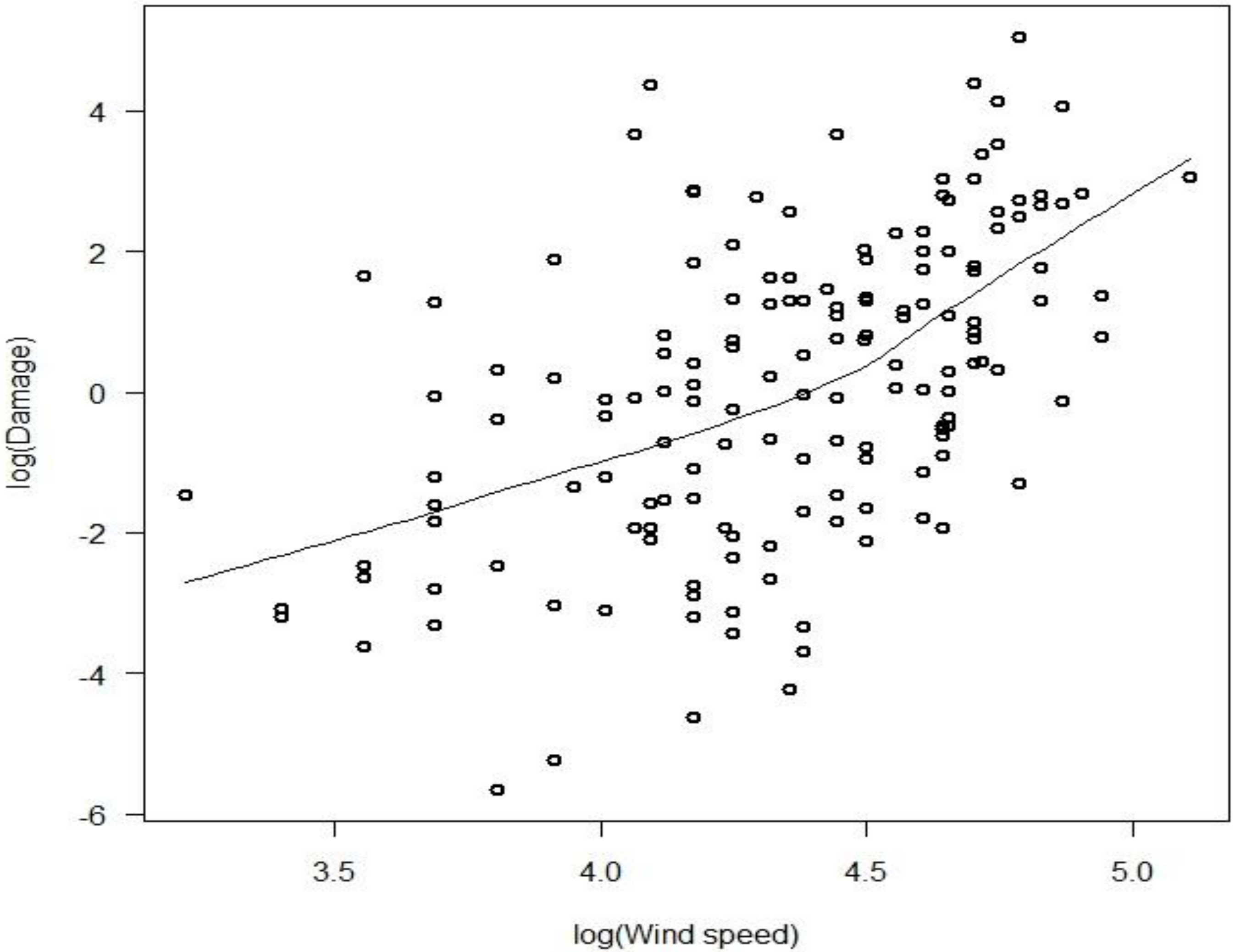
Power transformation implies log-log linearity

So regress ln *L* vs. ln *V* (with slope *b*):

	Estimated *b*	(Std. error)
All Data	2.43	(0.33)
$V > 90$ kts	4.91	(1.72)

-- **Both estimates *not* inconsistent with $b = 3$**

***Note*: Nordhaus (2010) advocated $b = 9$**



4
2
0
-2
-4
-6
log(Damage)
3.5
4.0
4.5
5.0
log(Wind speed)

- Distribution of wind speed (upper tail)

-- Fit of stretched exponential (with shape parameter *c*) to wind speed *V*

	Estimated *c*	(Standard error)
All Data	3.32	(0.21)
V > 90 kts	1.76	(0.17)

Consistent with typical estimate of *c* for wind speed (2 or 3)

- **Inferred Upper Tail of Damage**

-- **Penultimate approximation**

Need to convert threshold *u* to block size *n* (Set *n* = 30 or *n* = 100)

c	b	ξ_{30}	ξ_{100}
3	3	0	0
2	3	0.15	0.11
3	5	0.20	0.15
2	5	0.44	0.33
3	9	0.59	0.43
2	9	1.03	0.76

- **Damage Function via Extreme Value Theory**

-- **Fit GP distribution to excesses in damage**

Introduce V or $\ln V$ as covariate:

Express log-transformed scale parameter $\ln \sigma_u$ as linear function of V or $\ln V$

-- **Covariate far from statistically significant**

(6) Return Levels Under Nonstationarity

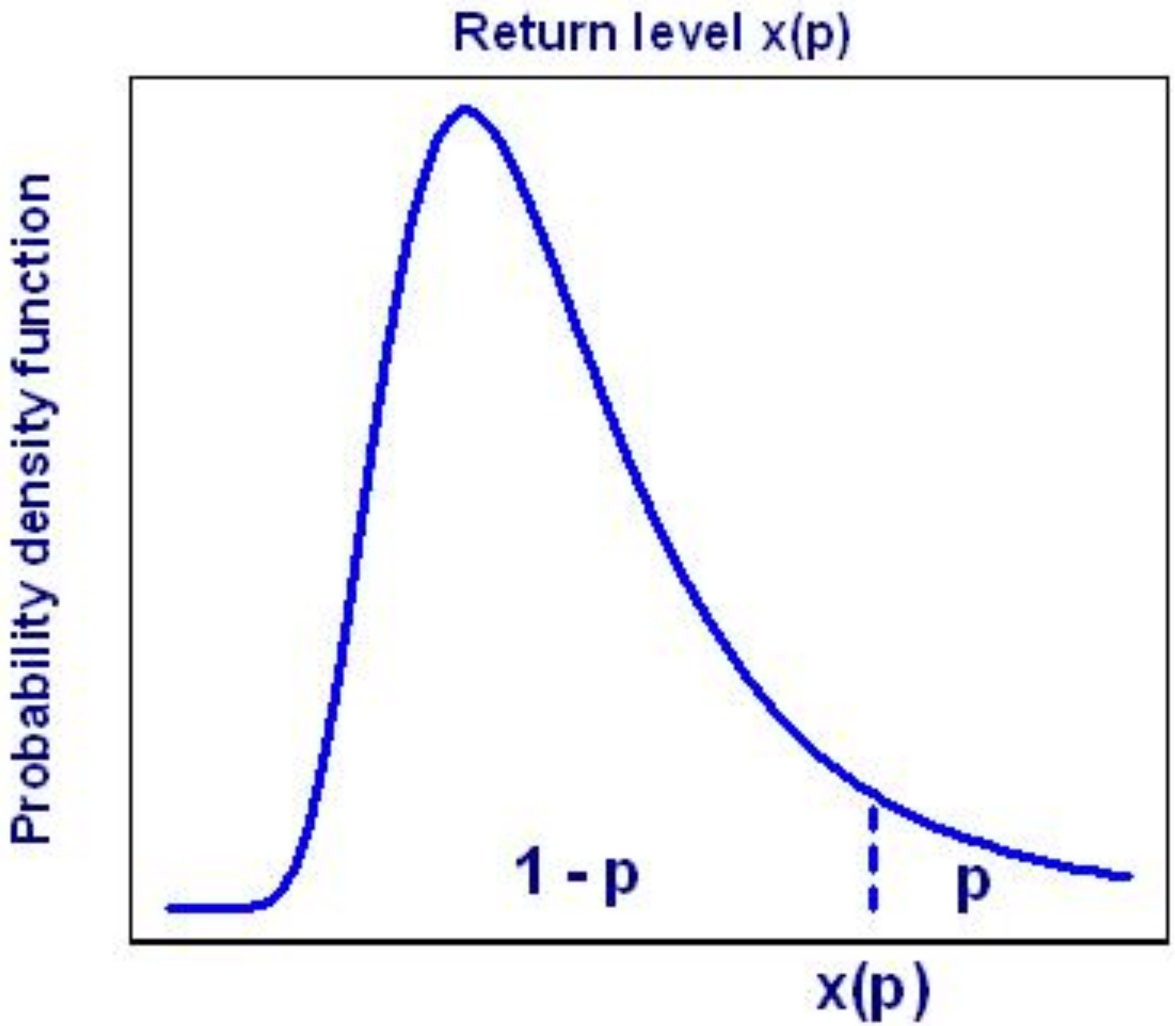
- **Return Level Under Stationary**

-- "Return level" with ($1/p$)-yr "return period"

($1 - p$)th quantile of relevant cdf (say F)

$$x(p) = F^{-1}(1 - p), \quad 0 < p < 1$$

(e. g., $p = 0.01$ corresponds to 100-yr return period)


Return level x(p)
Probability density function
1 - p
p
x(p)

- **Interpretation Under Stationarity**

(i) Expected waiting time (Assume temporal independence)

Waiting time W has geometric distribution:

$$\Pr\{W = k\} = (1 - p)^{k-1} p, \quad k = 1, 2, \ldots$$

So $E(W) = 1/p$

(ii) Length of time T_p for which expected number of events $= 1$

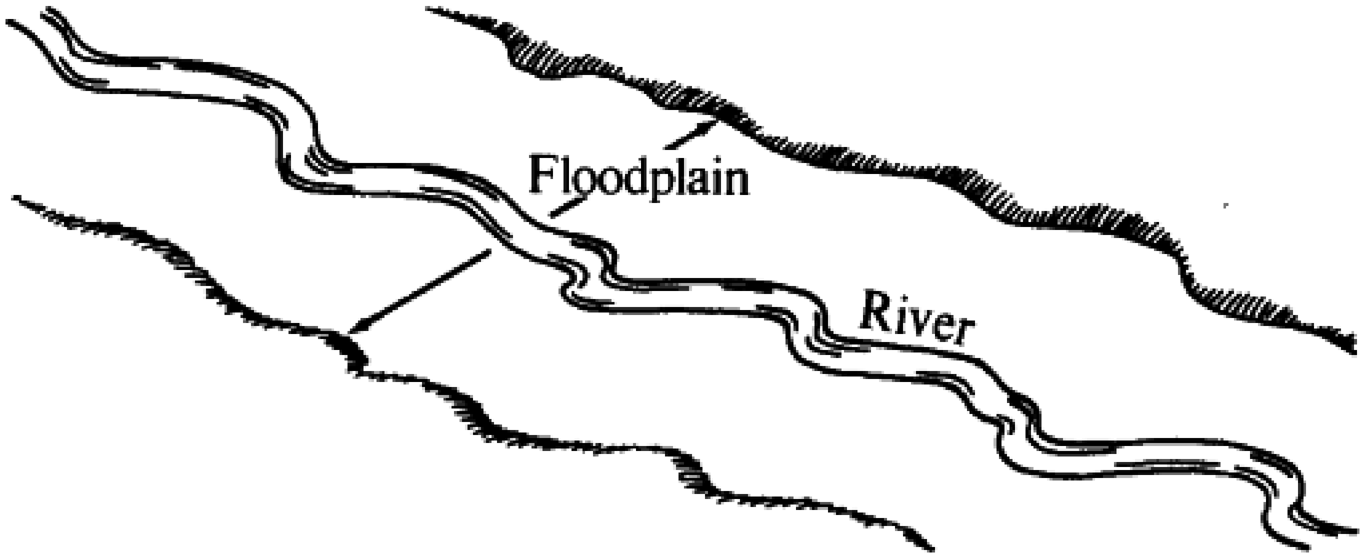
$$1 = \text{Expected no. events} = T_p \, p$$

So $T_p = 1/p$

- **Nonstationarity**

-- **Could use conditional quantiles**

But cannot necessarily change flood plain from year to year



Resources

- **Statistics of Weather and Climate Extremes**

-- www.isse.ucar.edu/extremevalues/extreme.html

- **Extremes Toolkit**

-- www.isse.ucar.edu/extremevalues/evtk.html