Dynamic Multiscale Spatiotemporal Models for Poisson Data

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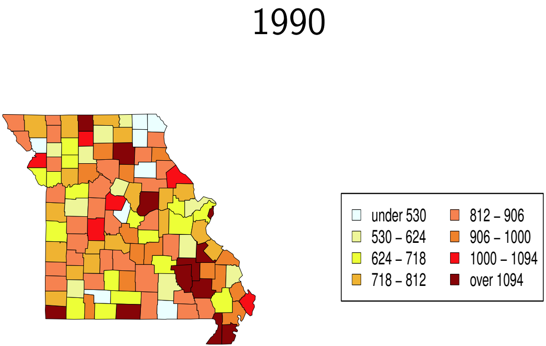
Outline

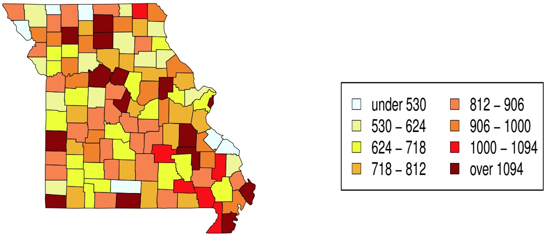
Outline

Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group



Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group

1991



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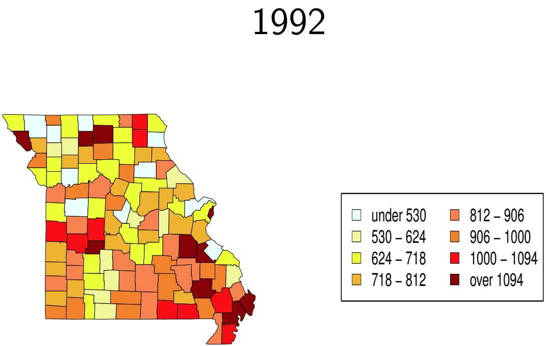

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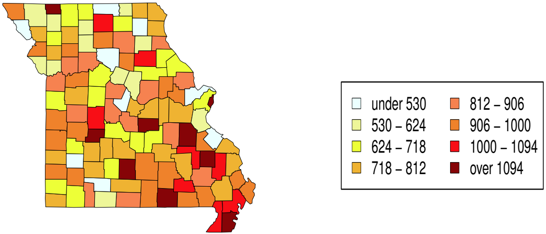
1993



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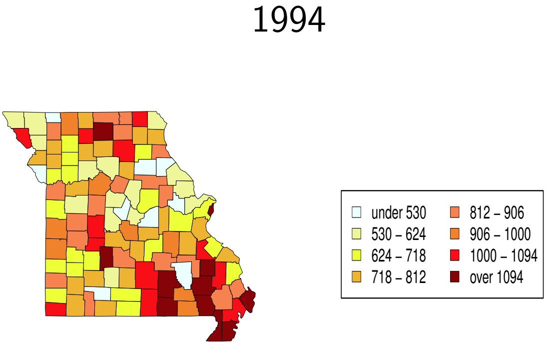

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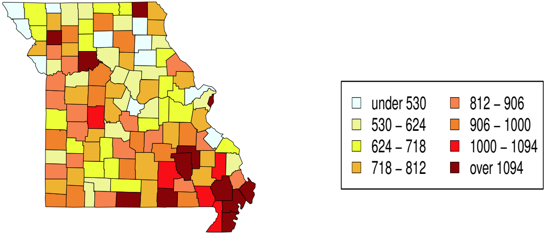
1995



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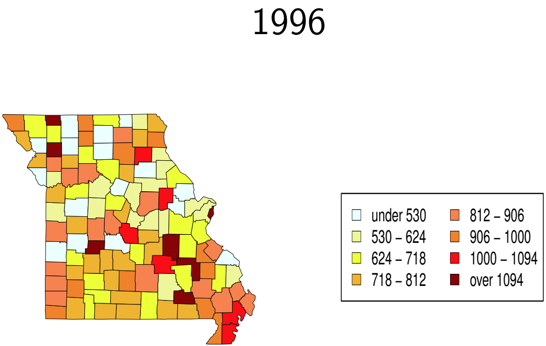

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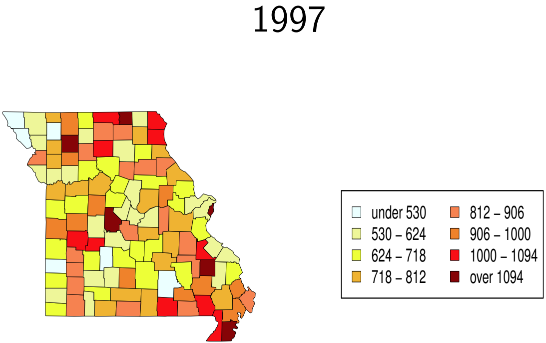

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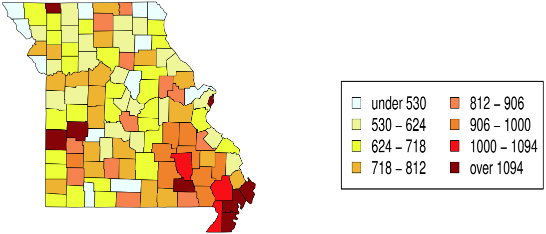
1998



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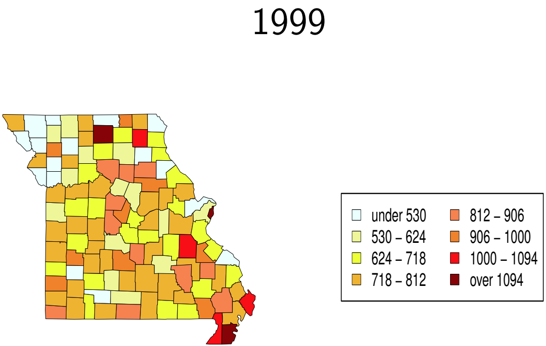

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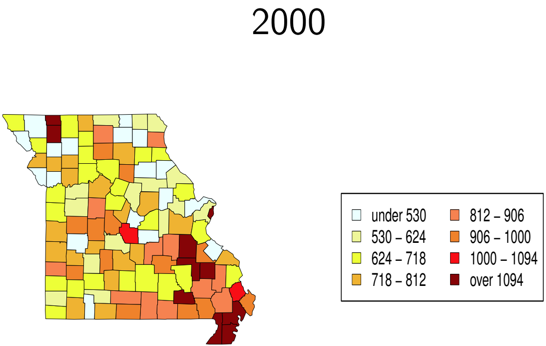

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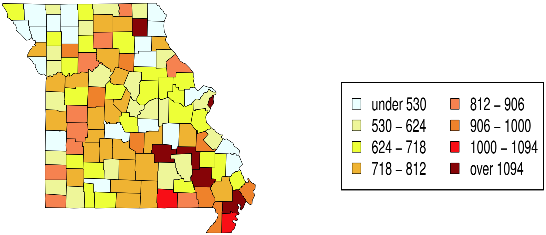
2001



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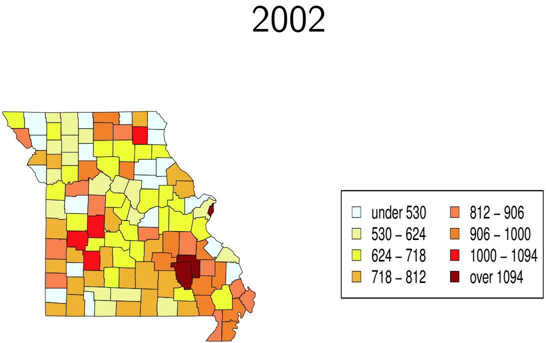

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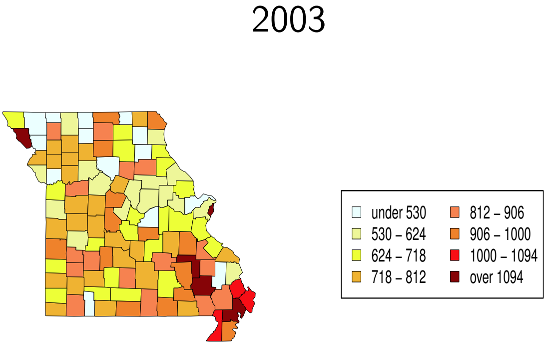

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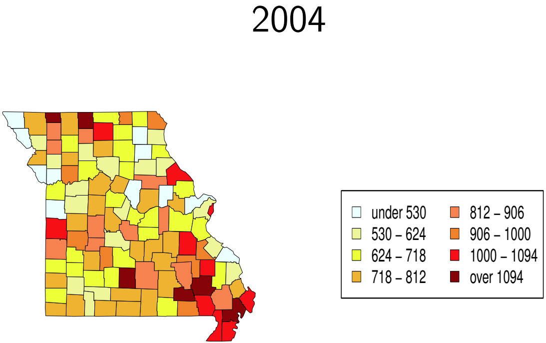

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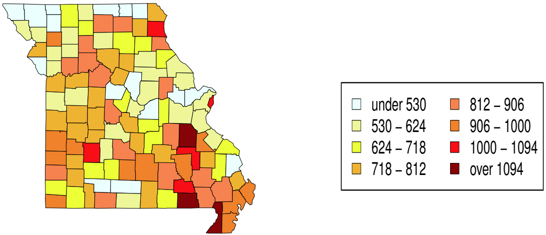
2005



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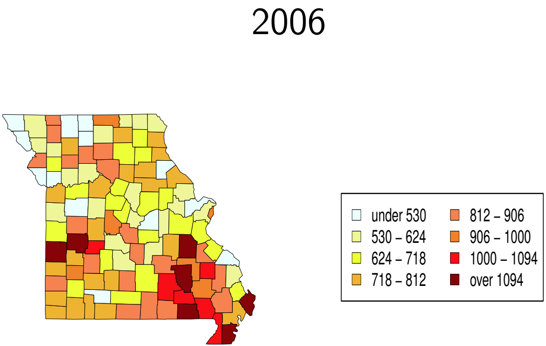

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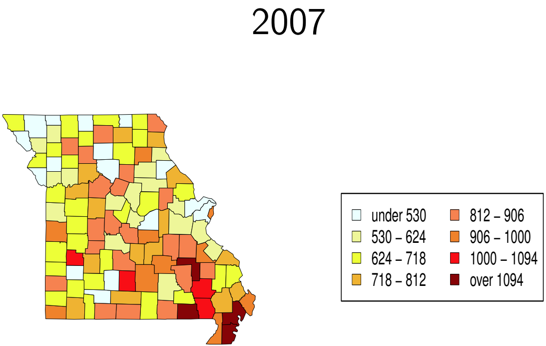

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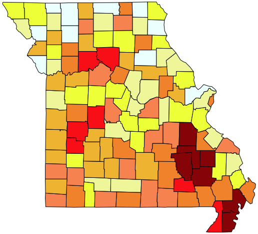
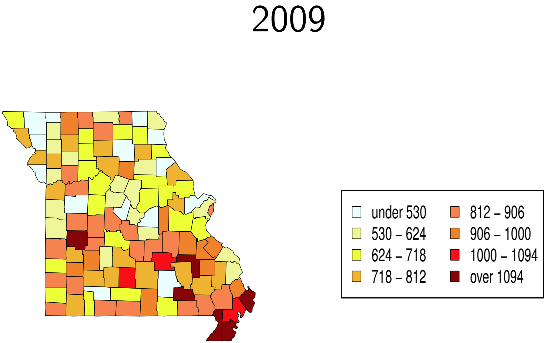
2008



under 530	812 – 906
530 – 624	906 – 1000
624 – 718	1000 – 1094
718 – 812	over 1094

Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



Introduction

- Here we are primarily concerned with Poisson spatiotemporal datasets available in the form of areal or regional data; that is, data available on a partition of the geographical domain of interest (Banerjee et al., 2004).
- Many methods have been recently proposed for the analysis of large point-referenced datasets (e.g., Banerjee et al., 2008; Paciorek and McLachlan, 2009; Lemos and Sansó, 2009).
- Here we propose a class of multiscale spatiotemporal models for Poisson areal data that lead to scalable, parallelizable, and computationally efficient inferential procedures.
- Related work on Gaussian dynamic spatiotemporal multiscale modeling: Berliner, Wikle and Milliff (1999), Johannesson, Cressie and Huang (2007), Ferreira, Bertolde and Holan (2010), Ferreira, Holan and Bertolde (2011).

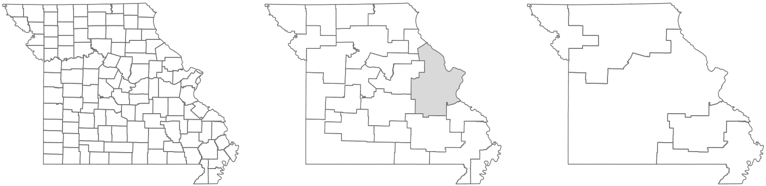
Data Structure

The region of interest is divided in geographic subregions or blocks, and the data are number of occurrences of event of interest over these subregions.

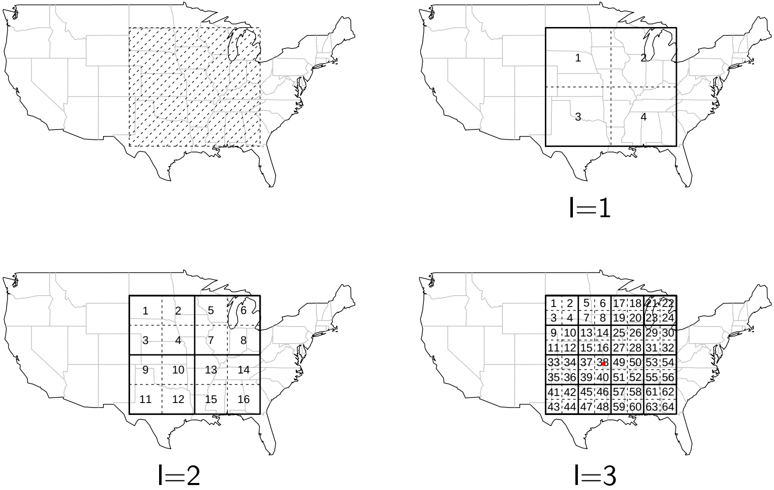
Moreover, there is a nested multiscale structure that aggregates subregions at one resolution level onto coarser resolution levels.

Our framework can also handle spatiotemporal point process data.

Multiscale structure for the state of Missouri (Ferreira et al. 2011)



USA Tornado alley and multiscale structure



Outline

Poisson multiscale factorization

At each time point we decompose the data into empirical spatiotemporal multiscale coefficients using the spatial multiscale modeling framework of Kolaczyk and Huang (2001). See also Chapter 9 of Ferreira and Lee (2007).

Interest lies in an inhomogeneous Poisson process with rate $\{\lambda(s) : s \in \mathcal{S}\}$ on a domain $\mathcal{S} \subset \mathbb{R}^k$.

Because of measurement, resources or confidentiality restrictions, data are available only up to a given scale of resolution L on a partition of the domain $\mathcal{S}$. Denote this partition by $\{B_{L1}, \ldots, B_{L,n_L}\}$, with $B_{Lj} \in \mathcal{S}$, $j = 1, \ldots, n_L$, $B_{Li} \cap B_{Lj} = \emptyset$, $i \neq j$, and $\cup_{j=1}^{n_L} B_{Lj} = \mathcal{S}$.

For each subregion B_{Lj} there is a count y_{Lj} of the number of occurrences of the event of interest.

Moreover, the expected number of counts on B_{Lj} is $\mu_{Lj} = E(y_{Lj}) = \int_{B_{Lj}} \lambda(s)ds$, $j = 1, \ldots, n_L$.

Further, similarly to Kolaczyk and Huang (2001) we assume that $y_{L1}, \ldots, y_{L,n_L}$ are conditionally independent given $\mu_{L1}, \ldots, \mu_{L,n_L}$.

In what follows, the latent spatial process $\lambda(s)$ is constructed in such a way that this latent process will be spatially correlated. Therefore, this spatial dependence will transfer to the counts $y_{L1}, \ldots, y_{L,n_L}$ and lead their marginal distribution to contain spatial dependence.

In addition to the mean process at the Lth resolution level, we are also interested in the process at aggregated coarser scales.

At the lth scale of resolution, the domain $\mathcal{S}$ is partitioned in n_l subregions $B_{l1}, \ldots, B_{ln_l}$, $l = 1, \ldots, L-1$.

Moreover, the partition at level l is assumed to be a refinement of the partition at level $l+1$; that is, $B_{lj} = \cup_{(l+1,j') \in D_{lj}} B_{l+1,j'}$, where D_{lj} is the set of descendants of subregion j at level l, and $D_{lj} \cap D_{li} = \emptyset, i \neq j$.

Additionally, let $A_l(L,j)$ be the ancestral at resolution level l of subregion (L,j).

Finally, denote by d_{lj} the number of descendants of subregion (l,j).

The aggregated counts at the lth level of resolution are recursively defined as

$$y_{lj} = \mathbf{1}'_{d_{lj}} y_{D_{lj}}$$

with corresponding aggregated mean process

$$\mu_{lj} = \mathbf{1}'_{d_{lj}} \boldsymbol{\mu}_{D_{lj}}$$

The mean μ_{lj} may be written as

$$\mu_{lj} = \lambda_{lj} e_{lj}$$

where λ_{lj} is the relative risk and e_{lj} is either known or unknown up to a low-dimensional parameter vector. Similarly to the observed counts, e_{lj} may be aggregated as

$$e_{lj} = \mathbf{1}'_{d_{lj}} e_{D_{lj}}$$

and in that case the aggregation for the mean process implies that the relative risk process is aggregated as

$$\lambda_{lj} = e_{lj}^{-1} \boldsymbol{\lambda}'_{D_{lj}} e_{D_{lj}}$$

Because of conditional independence, the likelihood function admits the multiscale factorization (Kolaczyk and Huang, 2001)

$$\prod_{j=1}^{n_L} p(y_{Lj}|\mu_{Lj}) = \prod_{j=1}^{n_1} p(y_{1j}|\mu_{1j}) \prod_{l=1}^{L-1} \prod_{j=1}^{n_l} p(\mathbf{y}_{D_{lj}}|y_{lj}, \boldsymbol{\omega}_{lj}), \qquad (1)$$

where $y_{1j}|\mu_{1j} \sim \text{Poisson}(\mu_{1j})$.

Further, $\mathbf{y}_{D_{lj}}|y_{lj}, \boldsymbol{\omega}_{lj} \sim \text{Multinomial}(y_{lj}, \boldsymbol{\omega}_{lj})$, where y_{lj} plays the role of the sample size parameter of the multinomial distribution, and $\boldsymbol{\omega}_{lj} = \boldsymbol{\mu}_{D_{lj}}/\mu_{lj}$ is the vector of probabilities.

$\boldsymbol{\omega}_{lj}$ describes how the counts at subregion (l, j) are expected to be distributed among its descendants D_{lj}, and connects coarser to finer resolution levels.

In analogy to wavelet analysis we refer to $\boldsymbol{\omega}_{lj}$ as a spatial multiscale coefficient.

Outline

Multiscale spatiotemporal model

The model for the number of counts at the finest resolution level L at time t is

$$y_{tLj}|\mu_{tLj} \sim \mathrm{Poisson}(\mu_{tLj}) \tag{2}$$

Further, we assume that $\mu_{tLj} = \lambda_{tLj} e_{tLj}$ where λ_{tLj} is the risk on subregion (L, j) at time t and e_{tLj} is either known or unknown up to a low-dimensional parameter vector.

Examples of known e_{tLj} include the case when $e_{tLj} = 1$ and the case when e_{tLj} is the known population size of subregion (l, j) at time t.

An example of e_{tLj} unknown up to a low-dimensional parameter vector is $e_{tLj} = \exp(x_t' \boldsymbol{\beta}_{Lj})$, where x_t is a known vector of regressors common to all regions at time t and $\boldsymbol{\beta}_{Lj} = \boldsymbol{\beta}_{A_1(L,j)}$.

It follows from Equation (1) that the multiscale factorization at time t for the Poisson model is

$$\prod_{j=1}^{n_L} p(y_{tLj}|\mu_{tLj}) = \prod_{j=1}^{n_1} p(y_{t1j}|\mu_{t1j}) \prod_{l=1}^{L-1} \prod_{j=1}^{n_l} p(\mathbf{y}_{t,D_{lj}}|y_{tlj}, \boldsymbol{\omega}_{tlj}), \quad (3)$$

where $y_{t1j}|\mu_{t1j} \sim \text{Poisson}(\mu_{t1j})$ and

$\mathbf{y}_{t,D_{lj}}|y_{tlj}, \boldsymbol{\omega}_{tlj} \sim \text{Multinomial}(y_{tlj}, \boldsymbol{\omega}_{tlj})$, with $\boldsymbol{\omega}_{tlj} = \boldsymbol{\mu}_{t,D_{lj}}/\mu_{tlj}$.

The parameter $\boldsymbol{\omega}_{tlj}$ is the spatiotemporal multiscale coefficient which represents the vector of probabilities associated with how the counts in y_{tlj} are distributed for each descendant in D_{lj}.

Let $\boldsymbol{\omega}^e_{tlj} = y_{tD_{lj}}/y_{tlj}$ be an estimator of $\boldsymbol{\omega}_{tlj}$. We refer to $\boldsymbol{\omega}^e_{tlj}$ as an empirical spatiotemporal multiscale coefficient.

Beta evolution for coarse level risk (Smith and Miller, 1986)

The beta temporal evolution for λ_{t1j} is defined as

$$\lambda_{t1j} = \lambda_{t-1,1j}\gamma_j^{-1}\eta_{tj}, \tag{4}$$

where

$$\eta_{tj}|\mathcal{D}_{t-1}, \gamma_j \sim Beta(\gamma_j a_{t-1,j}, (1-\gamma_j)a_{t-1,j}),$$

$0 < \gamma_j \leq 1$ is a discount factor parameter, and $a_{t-1,j} > 0$.

Dirichlet evolution for multiscale coefficients

The stochastic temporal evolution for $\boldsymbol{\omega}_{tlj}$ is defined as

$$\boldsymbol{\omega}_{tlj} = \frac{1}{S_{t-1,lj}} \boldsymbol{\phi}_{t-1,lj} \odot \boldsymbol{\omega}_{t-1,lj}. \quad (5)$$

with $\boldsymbol{\phi}_{t-1,lj} = (\phi_{t-1,lj1}, \ldots, \phi_{t-1,lj,d_{lj}})'$, where

$\phi_{t-1,lj1}, \ldots, \phi_{t-1,lj,d_{lj}}$ are i.i.d. $Beta(\delta_{lj} c_{t-1,lji}, (1-\delta_{lj}) c_{t-1,lji})$,

$S_{t-1,lj} = \boldsymbol{\phi}'_{t-1,lj} \boldsymbol{\omega}_{t-1,lj}$, $0 < \delta_{lj} \leq 1$ is a discount factor parameter, and $c_{t-1,lji} > 0$, $i = 1, \ldots, d_{lj}$.

Initial conditions and priors for the discount factors

$$\lambda_{01j}|\mathcal{D}_0 \sim \text{Gamma}(a_{0j}, b_{0j})$$

$$\boldsymbol{\omega}_{0lj}|\mathcal{D}_0 \sim \text{Dirichlet}(c_{0lj})$$

$$\gamma_j \sim Beta(a_\gamma, b_\gamma)$$

$$\delta_{lj} \sim Beta(a_\delta, b_\delta)$$

Outline

Posterior factorization

Theorem

Consider the multiscale spatiotemporal model for Poisson data defined by Equations (2), (3), (4), and (5). Given the discount factor parameters $\gamma_j, j = 1, \ldots, n_1$, *and* δ_{lj}, $l = 1, \ldots, L-1$, $j = 1, \ldots, n_l$, *the vectors* $\boldsymbol{\lambda}_{1:T,11}, \ldots, \boldsymbol{\lambda}_{1:T,1n_1}$, $\boldsymbol{\omega}_{1:T,11}, \ldots, \boldsymbol{\omega}_{1:T,1n_1}$, …, $\boldsymbol{\omega}_{1:T,L-1,1}, \ldots, \boldsymbol{\omega}_{1:T,L-1,n_{L-1}}$ *are conditionally independent a posteriori.*

Filtering for $\lambda_{1:T,1j}$

Theorem

Assume the initial distribution $\lambda_{01j}|\mathcal{D}_0 \sim Gamma(a_{0j}, b_{0j})$ and consider the Observation Equation (3) and the beta evolution for λ_{t1j} given by Equation (4). Then, for $t = 1, \ldots, T$:

(i) *Posterior for $\lambda_{t-1,1j}$:*
$\lambda_{t-1,1j}|\mathcal{D}_{t-1}, \gamma_j, \boldsymbol{\beta}_j \sim Gamma(a_{t-1,j}, b_{t-1,j})$.

(ii) *Prior for λ_{t1j}:*
$\lambda_{t1j}|\mathcal{D}_{t-1}, \gamma_j, \boldsymbol{\beta}_j \sim Gamma(a_{t|t-1,j}, b_{t|t-1,j})$,
where $a_{t|t-1,j} = \gamma_j a_{t-1,j}$ *and* $b_{t|t-1,j} = \gamma_j b_{t-1,j}$.

(iii) *Posterior for λ_{t1j}:*
$\lambda_{t1j}|\mathcal{D}_t, \gamma_j, \boldsymbol{\beta}_j \sim Gamma(a_{tj}, b_{tj})$,
where $a_{tj} = \gamma_j a_{t-1,j} + y_{t1j}$ *and* $b_{tj} = \gamma_j b_{t-1,j} + e_{t1j}$.

Filtering for $\boldsymbol{\omega}_{1:T,lj}$

Theorem

Assume the initial distribution $\boldsymbol{\omega}_{0lj}|\mathcal{D}_0 \sim$ *Dirichlet*(c_{0lj})*, and consider the Observation Equation (3) and the Dirichlet temporal evolution for the spatiotemporal multiscale coefficient* $\boldsymbol{\omega}_{tlj}$ *given by Equation (5). Then, for* $t = 1, \ldots, T$*:*

(i) *Posterior for* $\boldsymbol{\omega}_{t-1,lj}$*:* $\boldsymbol{\omega}_{t-1,lj}|\mathcal{D}_{t-1}, \delta_{lj} \sim$ *Dirichlet*$(\mathbf{c}_{t-1,lj})$.

(ii) *Prior for* $\boldsymbol{\omega}_{tlj}$*:* $\boldsymbol{\omega}_{tlj}|\mathcal{D}_{t-1}, \delta_{lj} \sim$ *Dirichlet*$(\mathbf{c}_{t|t-1,lj})$,
where $\mathbf{c}_{t|t-1,lj} = \delta_{lj}\mathbf{c}_{t-1,lj}$.

(iii) *Posterior for* $\boldsymbol{\omega}_{tlj}$*:* $\boldsymbol{\omega}_{tlj}|\mathcal{D}_t, \delta_{lj} \sim$ *Dirichlet*$(\mathbf{c}_{tlj})$,
where $\mathbf{c}_{tlj} = \delta_{lj}\mathbf{c}_{t-1,lj} + y_{tlj}\boldsymbol{\omega}^e_{tlj}$.

Smoothing for $\lambda_{1:T,1j}$

Proposition

Assume the Observation Equation (3) and the beta evolution for λ_{t1j} given by Equation (4). Then, the conditional smoothing distribution of $\lambda_{t-1,1j}$ given λ_{t1j} is equal to

$$p(\lambda_{t-1,1j}|\mathcal{D}_T, \lambda_{t1j}, \gamma_j)$$

$$= \frac{b_{t-1,j}^{(1-\gamma_j)a_{t-1,j}}}{\Gamma((1-\gamma_j)a_{t-1,j})}(\lambda_{t-1,1j} - \gamma_j\lambda_{t1j})^{(1-\gamma_j)a_{t-1,j}-1}$$

$$\times \exp\{-b_{t-1,j}(\lambda_{t-1,1j} - \gamma_j\lambda_{t1j})\},$$

where $\lambda_{t-1,1j} - \gamma_j\lambda_{t1j} > 0$.

Smoothing for $\boldsymbol{\omega}_{1:T,lj}$

Proposition

Consider the Observation Equation (3) and the Dirichlet evolution for the spatiotemporal multiscale coefficient $\boldsymbol{\omega}_{tlj}$ given by Equation (5). Then,

(i) $\boldsymbol{\omega}_{t-1,lj} | \mathcal{D}_T, S_{t-1,lj}, \boldsymbol{\omega}_{tlj}, \delta_{lj} \sim$ *Mod-Dirichlet*$((1-\delta_{lj})\mathbf{c}_{t-1,lj}, \mathbf{0}, S_{t-1,lj}\boldsymbol{\omega}_{tlj})$

(ii) $S_{t-1,lj} | \mathcal{D}_T, \boldsymbol{\omega}_{tlj}, \delta_{lj} \sim \mathrm{Beta}(\delta_{lj}\tilde{c}_{t-1,lj}, (1-\delta_{lj})\tilde{c}_{t-1,lj})$

where $\tilde{c}_{t-1,lj} = \sum_{i=1}^{d_{lj}} c_{t-1,lji}$ and Mod-Dirichlet denotes a modified Dirichlet distribution.

Likelihood function for the discount factor γ_j and the regression coefficients $\boldsymbol{\beta}_j$

$$p(\mathbf{y}_{1:T,1j}|\gamma_j, \boldsymbol{\beta}_j, \mathcal{D}_0)$$

$$= \prod_{t=\tau_1}^{T} p\left(y_{t1j}|\mathcal{D}_{t-1}, \gamma_j, \boldsymbol{\beta}_j\right)$$

$$= \prod_{t=\tau_1}^{T} \int_0^\infty p\left(y_{t1j}|\gamma_j, \boldsymbol{\beta}_j, \lambda_{t1j}\right) p\left(\lambda_{t1j}|\mathcal{D}_{t-1}, \gamma_j, \boldsymbol{\beta}_j\right) d\lambda_{t1j}$$

$$= \prod_{t=\tau_1}^{T} \left\{ \frac{\Gamma(\gamma_j a_{t-1,j}+y_{t1j})}{\Gamma(\gamma_j a_{t-1,j})\Gamma(y_{t1j}+1)} \left(\frac{\gamma_j b_{t-1,j}}{e^{x'_{tj}\boldsymbol{\beta}_j}}\right)^{\gamma_j a_{t-1,j}} \left(1 + \frac{\gamma_j b_{t-1,j}}{e^{x'_{tj}\boldsymbol{\beta}_j}}\right)^{-(\gamma_j a_{t-1,j}+y_{t1j})} \right\}.$$

Likelihood function for the discount factor δ_{lj}

$$p(\mathbf{y}_{1:T,D_{lj}}|\mathcal{D}_0, y_{1:T,lj}, \delta_{lj})$$

$$= \prod_{t=\tau_2}^{T} p(\mathbf{y}_{t,D_{lj}}|\mathcal{D}_{t-1}, y_{tlj}, \delta_{lj})$$

$$= \prod_{t=\tau_2}^{T} \int p(\mathbf{y}_{t,D_{lj}}|y_{tlj}, \boldsymbol{\omega}_{tlj}) p(\boldsymbol{\omega}_{tlj}|\mathcal{D}_{t-1}, \delta_{lj}) d\boldsymbol{\omega}_{tlj}$$

$$= \prod_{t=\tau_2}^{T} \left\{ \frac{\Gamma(\sum_i \delta_{lj} c_{t-1,lji})\Gamma(y_{tlj}+1)}{\Gamma(\sum_i \delta_{lj} c_{t-1,lji}+y_{tlj})} \prod_{i=1}^{d_{lj}} \frac{\Gamma(\delta_{lj} c_{t-1,lji}+y_{tlj}\omega^e_{tlji})}{\Gamma(y_{tlj}\omega^e_{tlji}+1)\Gamma(\delta_{lj} c_{t-1,lji})} \right\}.$$

Outline

Priors

In all applications, we have used the same default prior distributions:

- For $\lambda_{01j}|\mathcal{D}_0$, we assume $a_{0j} = b_{0j} = 0.01$,
- For $\boldsymbol{\omega}_{0lj}|\mathcal{D}_0$, we assume $c_{0lj} = 0.01\mathbf{1}_{d_{lj}}$,
- For γ_j, we assume $a_\gamma = b_\gamma = 1$,
- For δ_{lj}, we assume $a_\delta = b_\delta = 1$.

Simulated dataset

Coarsest level: 9 subregions

Intermediate level: 36 subregions

Finest level: 72 subregions

$T = 200$

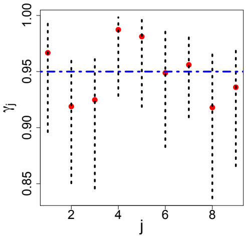
At time 0, $\mu_{0Lj} = 30 + 5 \times \text{longitude}_j + 8 \times \text{latitude}_j$.

At time 0, $\boldsymbol{\omega}_{0lj} = d_{lj}^{-1}\mathbf{1}_{d_{lj}}$.

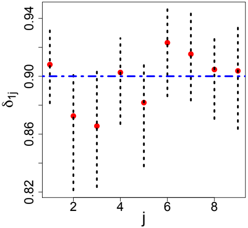
$\gamma_j = 0.95$, $j = 1, \ldots, n_1$.

$\delta_{lj} = 0.90$, $l = 1, 2$, $j = 1, \ldots, n_l$.

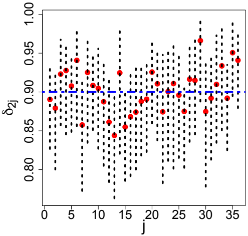
Posterior mean (dot) and 95% credible interval (vertical dashed line) for the discount factors



(a) γ_j

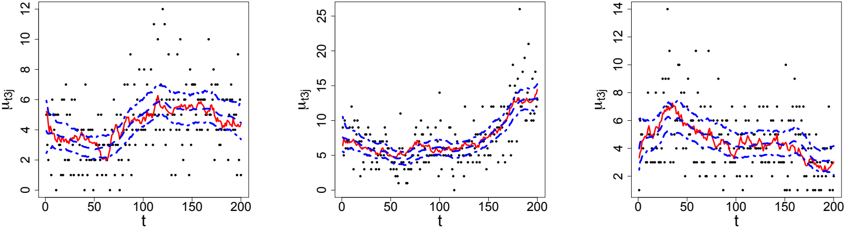


(b) δ_{1j}



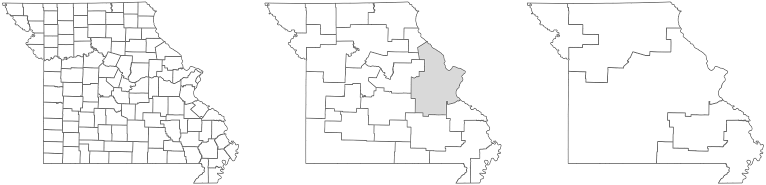
(c) δ_{2j}

Time series plots for $\boldsymbol{\mu}_{1:T,3j}$

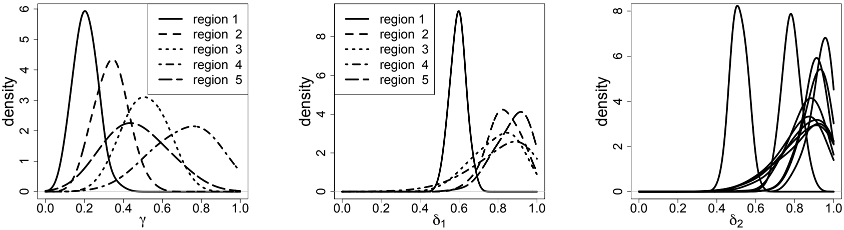


(a) $j = 6$ (b) $j = 25$ (c) $j = 63$

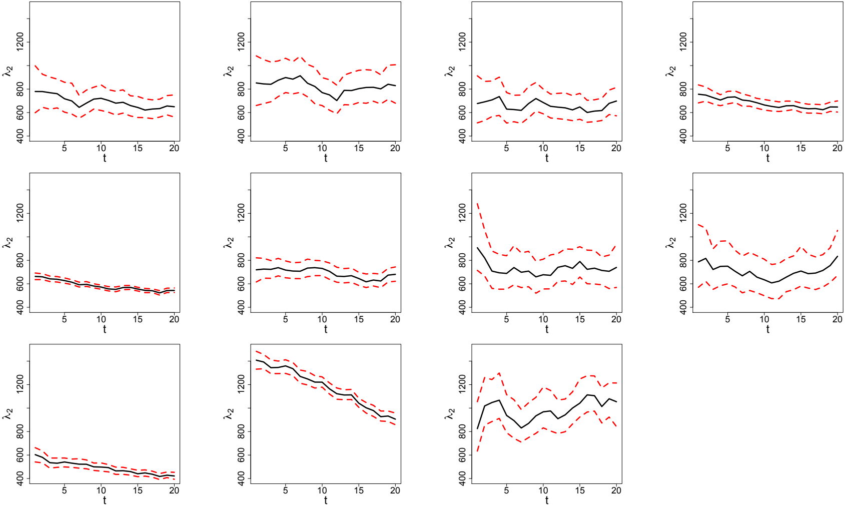
Mortality in Missouri for 45 to 64 years old age group



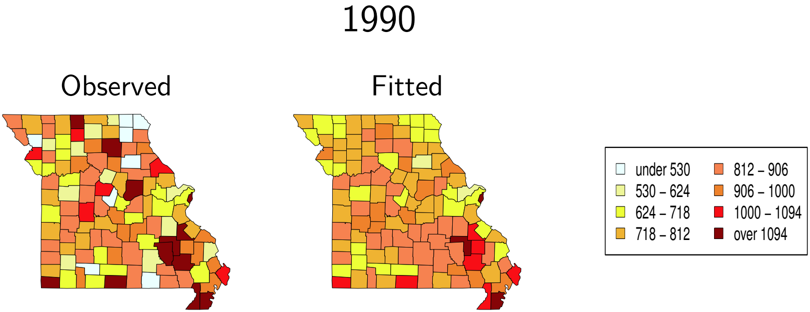
Posterior densities for discount factors



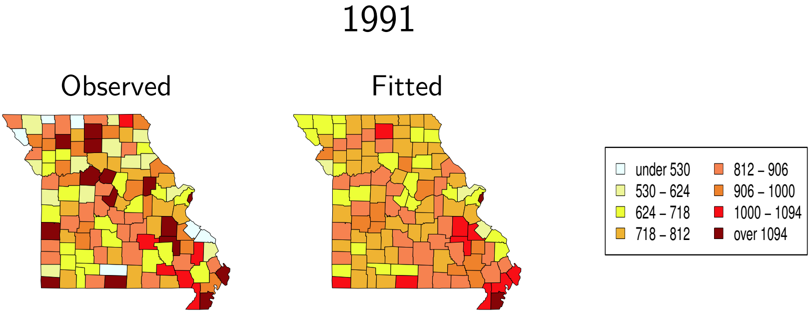
Estimated risk level for counties within the Saint Louis metropolitan area



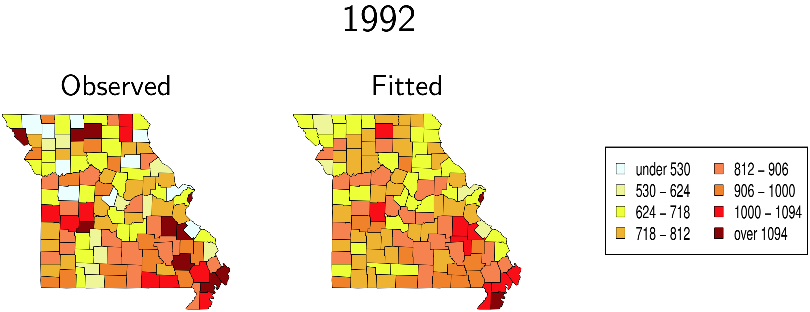
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group



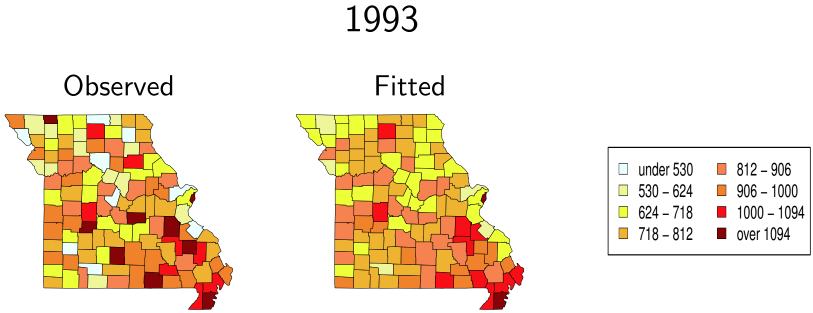
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group



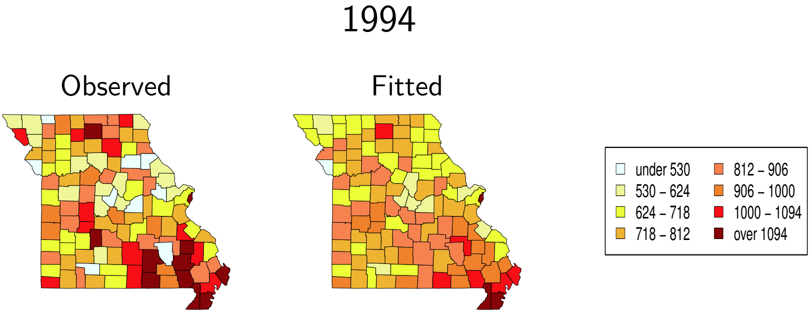
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



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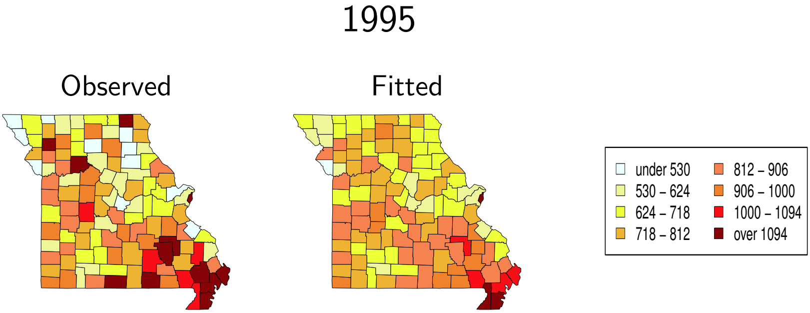
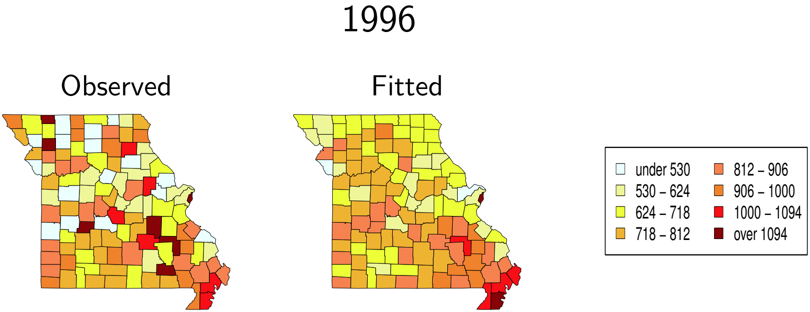


Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



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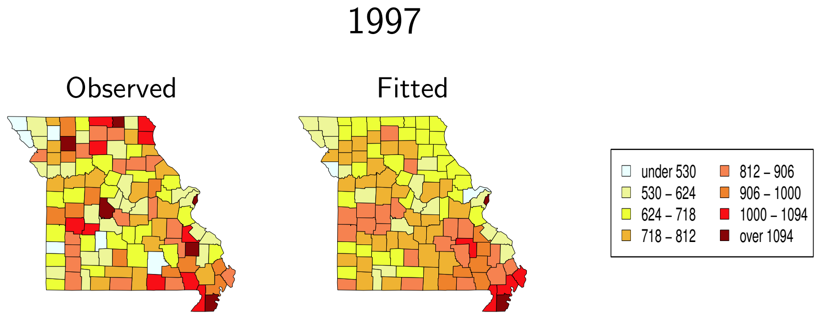
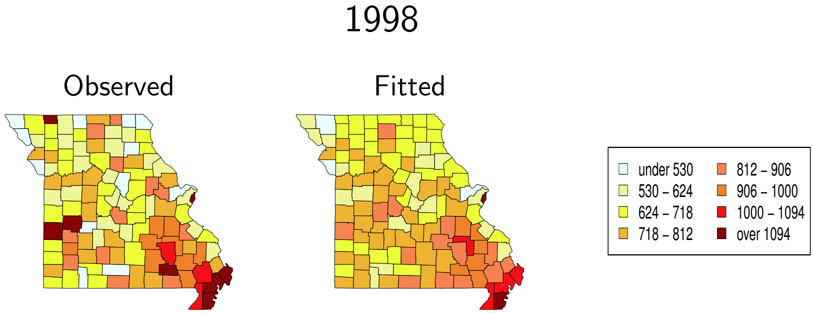
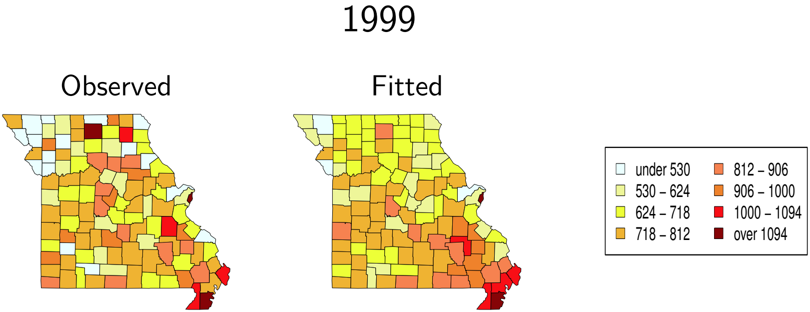


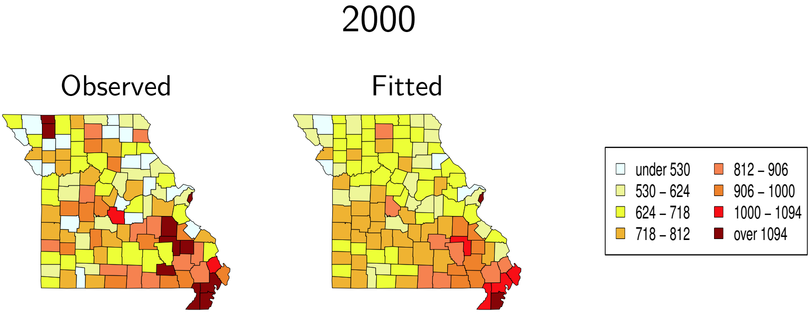
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



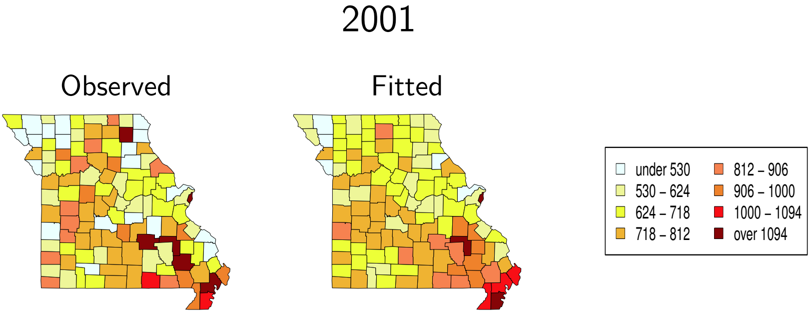
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



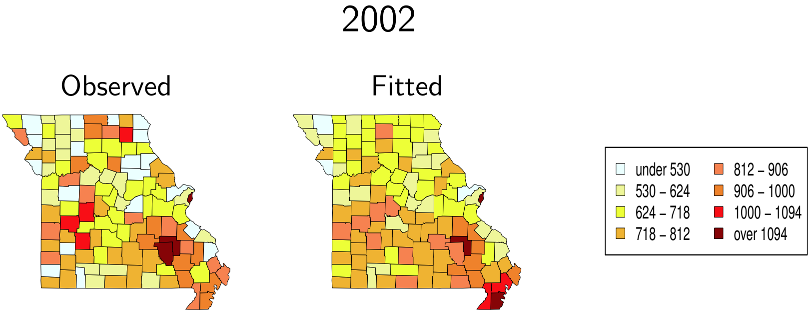
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



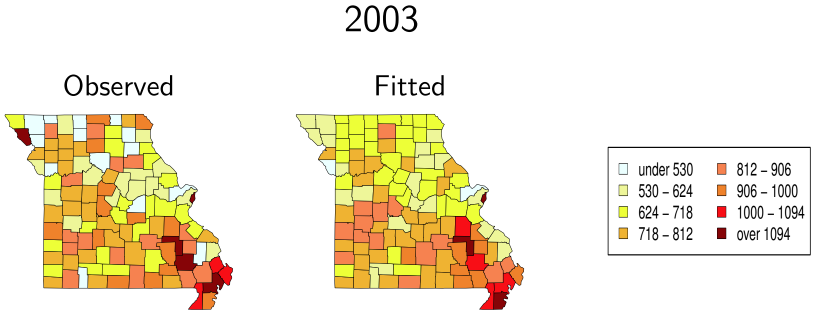
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



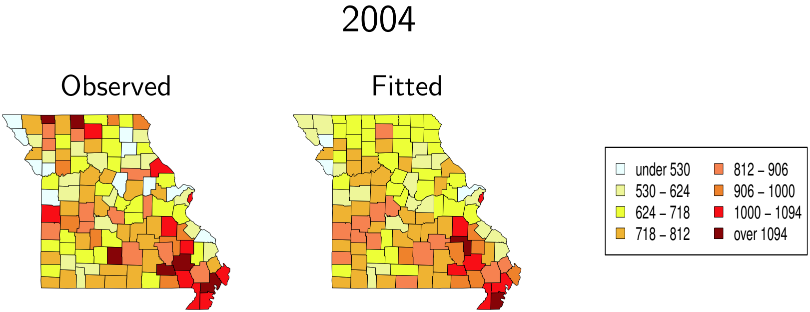
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



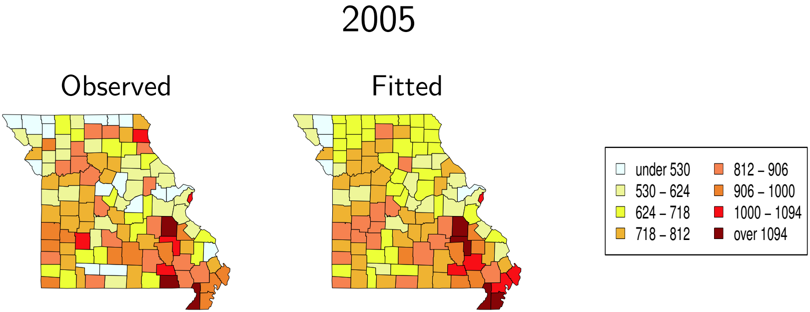
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



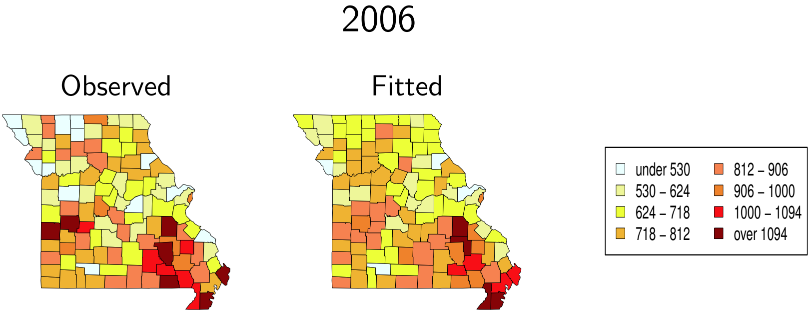
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



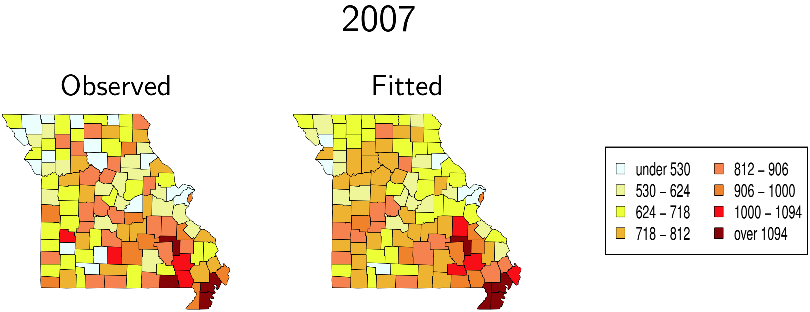
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



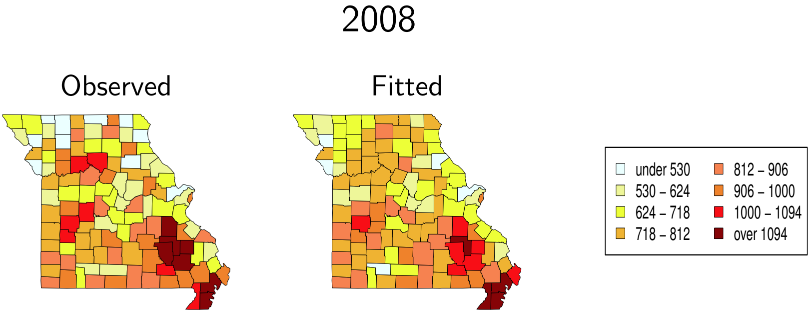
Missouri: standardized mortality ratio per 100,000 inhabitants for 45 to 64 years old age group



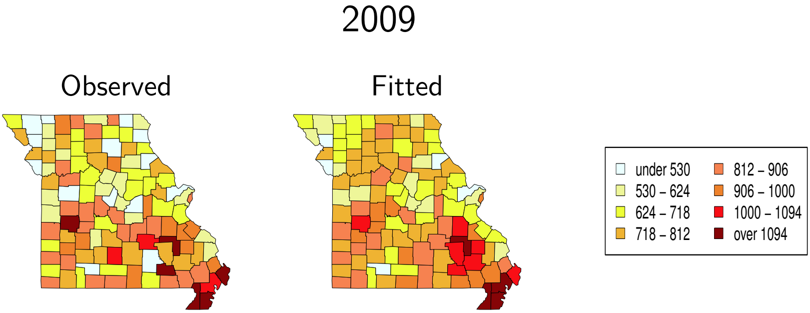
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



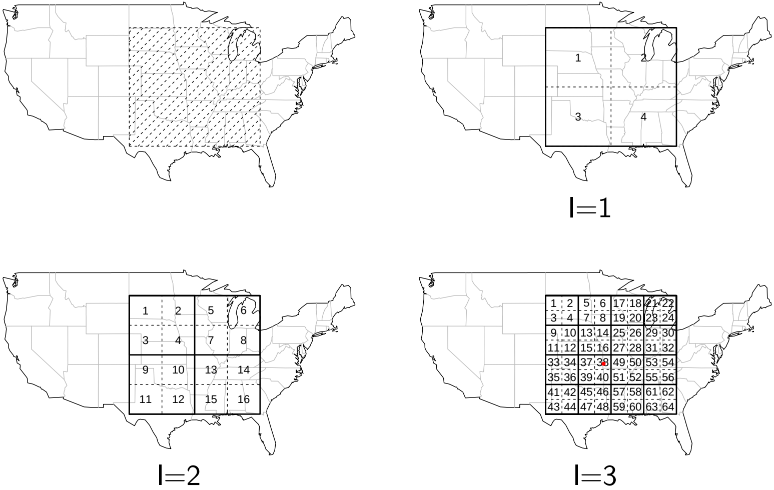
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



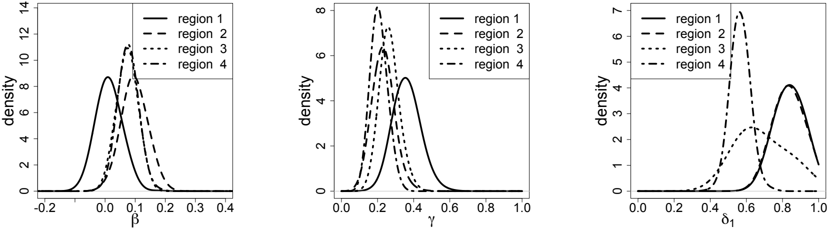
Missouri: standardized mortality ratio per $100,000$ inhabitants for 45 to 64 years old age group



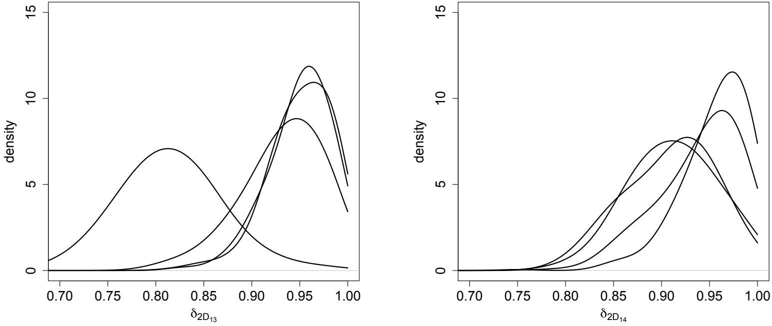
Tornados in the American Midwest



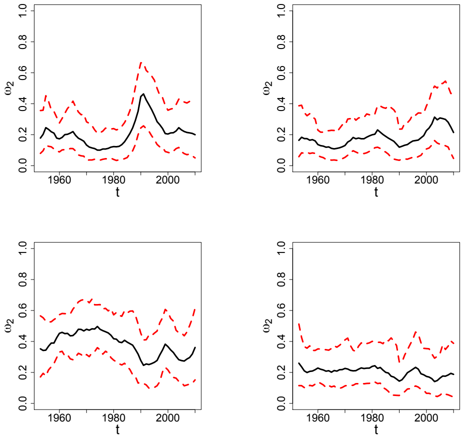
Posterior densities for β_j, γ_j, and δ_{1j}



Posterior densities for the discount factors δ_{2j} corresponding to the descendants of subregion (1,3) and subregion (1,4)



Posterior median (solid line) and 95% credible interval (dashed line) for each element of $\boldsymbol{\omega}_{t2,10}$



Model comparison

Competing models:

- Model I: Multiscale spatiotemporal model for Poisson data.
- Model II: Model that assumes that each finest level subregion has its own temporal evolution according to the Poisson state-space model.
- Model III: Spatiotemporal model: $y_{tj}|\mu_{tj} \sim Poisson(\mu_{tj})$, with $log(\mu_{tj}) = log(e_{tj}) + \alpha_0 + a_j + b_t$, $b_t = \phi b_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, W)$, and the a_j's are spatial random effects that follow a CAR specification.

Model comparison

Log conditional Bayes factor of Model I against Models II and III

	Model II	Model III
Missouri data	283.9	215.1
Tornado data	254.8	139.8

Outline

Conclusions

- Novel multiscale spatiotemporal model for Poisson data.
- Modeling strategy naturally respects nonsmooth transitions between geographic subregions.
- Analysis of the discount factors provides a powerful way to identify regions with spatiotemporal dynamics that warrant further investigation.
- Divide-and-conquer modeling strategy leads to computational procedures that are parallelizable, scalable, and fast.