The Ensemble Kalman Filter: a state estimation method for hazardous weather prediction.

Sarah L Dance

Department of Mathematics and Department of Meteorology, University of Reading, UK

With thanks to Laura Baker, David Livings, Nancy Nichols and Ruth Petrie

RESEARCH COUNCILS UK

National Centre for Earth Observation

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Outline

Why worry about weather?

- During 2006 an estimated 130 million people worldwide were affected by storms, floods and droughts.
- Anticipated increase in severe storms due to anthropogenic climate change
- High resolution forecasts needed for fog and air quality

Univ Leeds

Boscastle storm 2004

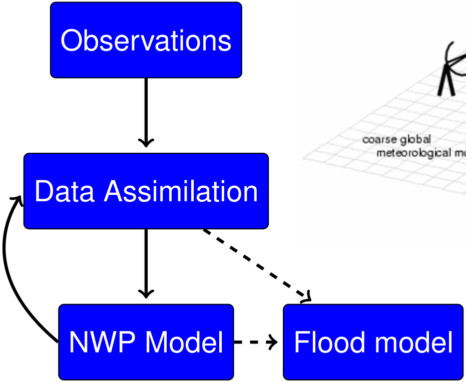
BBC

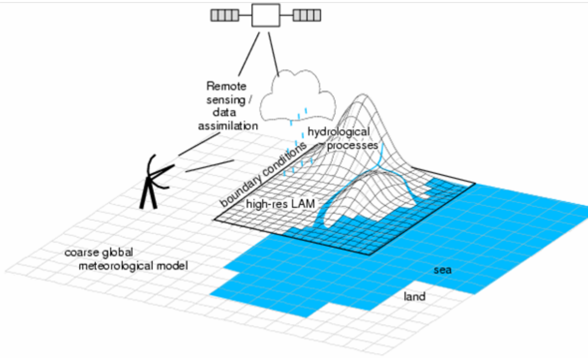
Birmingham tornado 2005

BBC

Heathrow fog Christmas 2006

Forecasting





Picture from Ross Bannister

NWP Forecasts

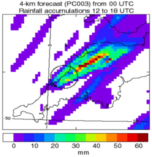
- Models are typically numerical solutions of PDES
- Global (60km) → Convective-scale (1-4km)
- Dynamics is very different on these scales
- Current models have length of state vector $O(10^7)$ variables
- Next UK model 1.5km with about a billion state variables.

Met Office model domains

Met Office 4km UM

hindcast Boscastle storm

2004



Observations

- About $O(10^6)$ observations currently assimilated every 3-6 hours.
- Observational coverage is heterogeneous
- Many observations are from remote sensing

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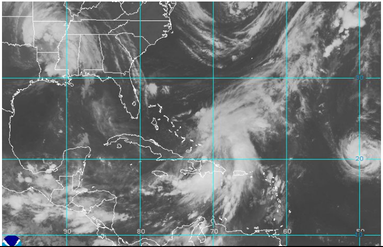

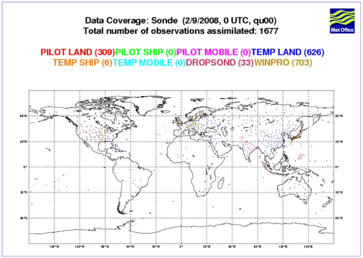

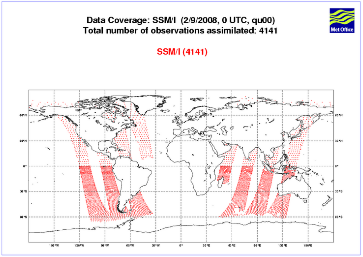

Issues

- System size

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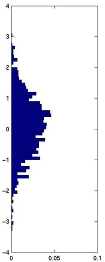
Current operational approach is variational assimilation. We will consider the Ensemble Kalman filter

The Ensemble Kalman filter (EnKF), Evensen (1994)

Idea:

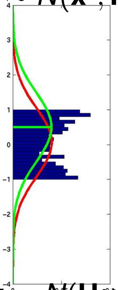
- Treat ensembles as Gaussian
- Carry out observation updates using approximate Kalman filter style equations.

$x_i \sim N(\bar{\mathbf{x}}, \mathbf{P})$



nonlinear forecast $\longrightarrow$

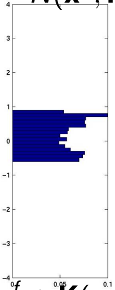
$x_i \sim N(\bar{\mathbf{x}}^f, \mathbf{P}^f)$



$\mathbf{y}|\mathbf{x} \sim N(\mathbf{H}x, \mathbf{R})$

$\longrightarrow$

$x_i \sim N(\bar{\mathbf{x}}^a, \mathbf{P}^a)$



$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{y} - \mathbf{H}\mathbf{x})$

The Ensemble Kalman Filter Equations

Let $\{\mathbf{x}_i\}$ $(i = 1, \ldots, m)$ be an m-member ensemble in $\mathbb{R}^n$. The ensemble mean is

$$\overline{\mathbf{x}} = \frac{1}{m} \sum_{i=1}^{m} \mathbf{x}_i.$$

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The ensemble perturbation matrix is the $n \times m$ matrix

$$\mathbf{X} = \frac{1}{\sqrt{m-1}}\begin{pmatrix} \mathbf{x}_1 - \overline{\mathbf{x}} & \mathbf{x}_2 - \overline{\mathbf{x}} & \ldots & \mathbf{x}_m - \overline{\mathbf{x}} \end{pmatrix}.$$

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The ensemble covariance matrix is the $n \times n$ matrix

$$\mathbf{P} = \mathbf{X}\mathbf{X}^T = \frac{1}{m-1} \sum_{i=1}^{m} (\mathbf{x}_i - \overline{\mathbf{x}})(\mathbf{x}_i - \overline{\mathbf{x}})^T.$$

Square root implementation

The analysis ensemble $\{\mathbf{x}_i\}$ is obtained as

$$\mathbf{x}_i = \widetilde{\mathbf{x}} + \mathbf{x}'_i,$$

for $i = 1, 2, \ldots, m$.

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for $i = 1, 2, \ldots, m$. The state-estimate, $\widetilde{\mathbf{x}}$, a column n-vector $\widetilde{\mathbf{x}}$ satisfies

$$\widetilde{\mathbf{x}} = \overline{\mathbf{x}^f} + \mathbf{K}(\mathbf{y} - \overline{\mathbf{H}(\mathbf{x}^f)}),$$

where

$$\mathbf{K} = \mathbf{X}^f(\mathbf{Y}^f)^T(\mathbf{Y}^f(\mathbf{Y}^f)^T + \mathbf{R})^{-1},$$

and

$$\mathbf{Y} = \frac{1}{\sqrt{m-1}} \begin{pmatrix} \mathbf{H}(\mathbf{x}_1) - \overline{\mathbf{H}(\mathbf{x})} & \mathbf{H}(\mathbf{x}_2) - \overline{\mathbf{H}(\mathbf{x})} & \ldots & \mathbf{H}(\mathbf{x}_m) - \overline{\mathbf{H}(\mathbf{x})} \end{pmatrix}.$$

Updating the perturbation matrix

Recall

$$\mathbf{x}_i = \widetilde{\mathbf{x}} + \mathbf{x}'_i,$$

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$$\mathbf{x}_i = \widetilde{\mathbf{x}} + \mathbf{x}'_i,$$

The column n-vector $\mathbf{x}'_i$ is the $i-$th column of the $n \times m$ matrix

$$\widetilde{\mathbf{X}} = \mathbf{X}^f \mathbf{T},$$

and $\mathbf{T}$ is an $m \times m$ matrix,

$$\mathbf{T}\mathbf{T}^T = \mathbf{I} - (\mathbf{Y}^f)^T \mathbf{D}^{-1} \mathbf{Y}^f.$$

[Chosen so that $\mathbf{P}^a$ satisfies $\mathbf{P}^a = (I - KH)\mathbf{P}^f$.]

- General problems
  - Sampling errors
  - Model errors
  - Dealing properly with nonlinearity
  - What to do about BCs in limited area models?
- Problems for specific implementations
  - Algorithm bias
  - Ensemble collapse

Noisy correlations

Picture from Petrie (2008)

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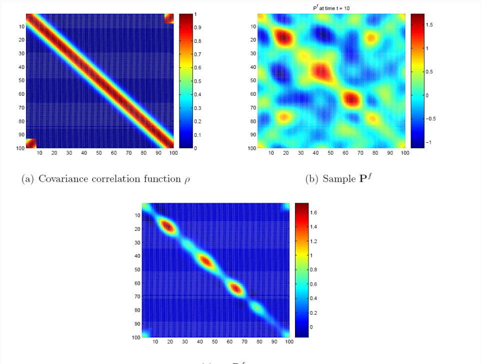


(a) Covariance correlation function ρ

(b) Sample $\mathbf{P}^f$

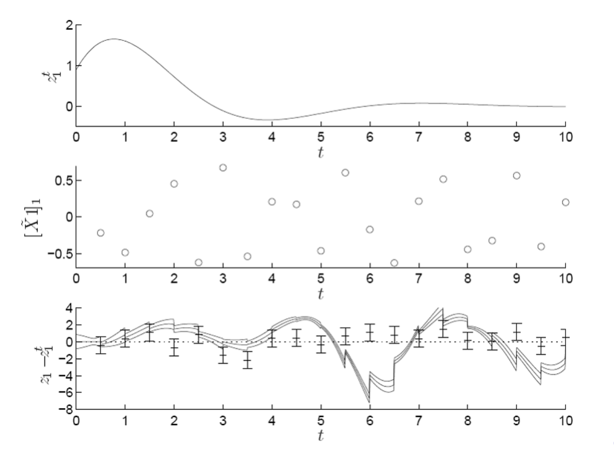
(c) $\rho \circ \mathbf{P}^f$

- Adds extra degrees of freedom (Hamill et al 2001)
- But introduces imbalance (Mitchell et al 2002, Lorenc 2003)
- Imposes lengthscales in the analysis?

Algorithm Bias

For consistency, we want $\widetilde{\mathbf{x}} = \mathbf{x}^a, \Rightarrow \widetilde{\mathbf{X}}\mathbf{1} = \mathbf{0}$.

We showed this does not necessarily hold (Livings Et Al, 2008).



Bias cont

This results in $\overline{\mathbf{x}^a} \neq \widetilde{\mathbf{x}}$.

Furthermore,

$$\mathbf{P}^a = \widetilde{\mathbf{X}}\widetilde{\mathbf{X}}^T - \frac{m}{m-1}\overline{\mathbf{x}'}\,\overline{\mathbf{x}'}^T,$$

Ensemble standard deviation will be too small

$\Rightarrow$ filter divergence?

To avoid the problem, need some extra conditions on $\mathbf{T}$ such that

$$\widetilde{\mathbf{X}} = \mathbf{X}^f\mathbf{T}.$$

This one is the most practically useful...

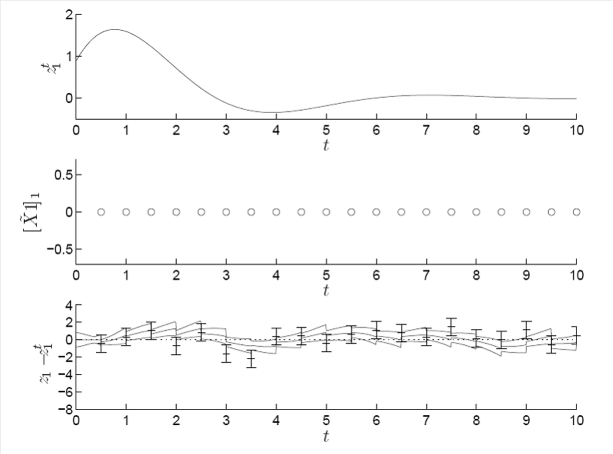
Theorem (Livings et al, 2008)

If $\mathbf{T}$ satisfies

$$\mathbf{T}\mathbf{T}^T = \mathbf{I} - (\mathbf{Y}^f)^T\mathbf{D}^{-1}\mathbf{Y}^f.$$

and is a symmetric matrix, then the resulting SRF is unbiased in the sense that $\widetilde{\mathbf{x}} = \overline{\mathbf{x}^a}$.

ETKF with symmetric **T** (Livings et al, 2008)



Summary

- The Ensemble Kalman filter has potential advantages over other methods for some applications
- There are several implementations
- Care is needed in choice of implementation to avoid bias and ensemble collapse
- Can we make basis of algorithm more rigorous and remove ad-hoc fixes?