Stochastic problems in science and engineering

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Algebraic, differential, and integral equations are used in the applied sciences, engineering, economics, and the social sciences to characterize the current state of a physical, economic, or social system and forecast its evolution in time. Generally, the coefficients of and/or the input to these equations are not precisely known because of insufficient information, limited understanding of some underlying phenomena, and inherent randomness. For example, the orientation of the atomic lattice in the grains of a polycrystal varies randomly from grain to grain, the inclusion geometry in composite material is not known precisely, bone properties needed to develop reliable artificial joints vary significantly with individual and age, forces acting on a plane from takeoff to landing depend in a complex manner on the environmental conditions and flight pattern, and stock prices and their evolution in time depend on a large number of factors that cannot be described by deterministic models. Problems that can be defined by algebraic, differential, and integral equations with random coefficients and/or input are referred to as stochastic problems.

Four types of stochastic problems are considered. Some partial differential equations with deterministic coefficients and input are examined since they can be solved efficiently by probabilistic methods. A significant part of the presentation relates to random heterogeneous media, that is, stochastic problems defined by equations with random coefficients and deterministic input.