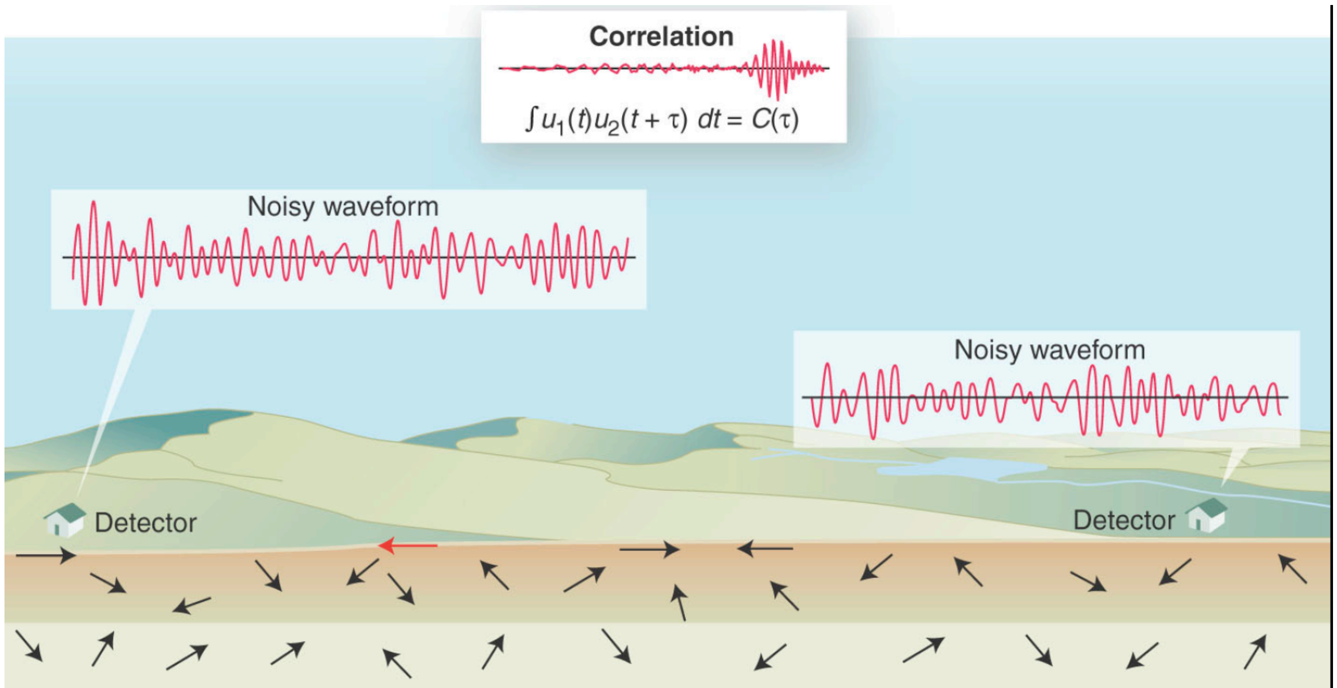
Perspectives on retrieval of Green's Function from correlations of noise

- RL Weaver, U Ullinois Physics



On retrieval of Green's Function from correlations of noise
. . . . the perspective of a Physicist and Experimentalist

Recent intense activity correlating ambient seismic noise at 10's of seconds has given us stunning maps of the earth's elasticity to depths of 10's of km or more. Theory in support of the methods has lagged. While it is quite clear that – under conditions of equipartition – such correlations ought to converge on the Green's function, it is less clear how much we should trust the results when the ambient waves do not satisfy the conditions.

I review a few of the proofs and laboratory demonstrations, paying particular attention to the degree of equipartition present in each case and estimates for the quality of convergence.

In many circumstances one can show that a laboratory or seismic correlation, even when the averaging is essentially converged, does not correspond to the Green's function.
The discrepancy is traceable to an insufficiently diffuse and equipartitioned noise field.
I discuss one way to finesse this failure of the conditions; a numerical example is offered.

Outline -

A few proofs

Convergence rates. Signal/Noise ?

Laboratory demonstrations -
- using fully diffuse, and almost fully diffuse, coda waves

Seismic applications
- using ambient not fully diffuse waves
- stunning results but how much can we trust them ?

A numerical experiment seeking to retrieve G from anisotropic noise using higher order averages.

Main Points

➞ Correlations of fields ψ, $C = \langle \psi\, \psi \rangle$

$\partial C/\partial t =$ Greens Function

except for

a) Fluctuations [insufficient averaging]

b) imperfectly diffuse fields ψ.

review of

proofs

laboratory demonstrations

theory for fluctuation levels

results with ambient seismic waves

➞ Towards a finesse of caveat (b)

Some numerical experiments

Proofs of correlation:

$$G(\vec{x},\vec{y};\tau) \sim \frac{\partial}{\partial \tau} < \psi(\vec{x},t)\psi(\vec{y},t+\tau) > ?$$

Obviously, it depends on the nature of ψ *and* what is meant by $< >$.
Original assertion G = dC/dτ *if* ψ is fully diffuse, i.e., *equipartitioned*

Lobkis, O.I., and R.L. Weaver, On the emergence of the Greens function in the correlations of a diffuse field, J. Acoust Soc Am, 110, 3011-3017, 2001

Plausibility arguments, sundry "proofs", laboratory study in a finite body.

Weaver, R.L., and O.I. Lobkis, Ultrasonics without a source: Thermal fluctuation correlations at MHz frequencies, Phys. Rev. Lett., 87 134301, 2001.

Proof for a thermally diffuse field, and
Lab demonstration of high quality retrieval from a thermally diffuse field

Snieder, R., 2004, Extracting the Green's function from the correlation of coda waves: A derivation based on stationary phase: Phys. Rev. E, **69**, 46610.

A "proof" applicable to open media

R L Weaver and O I Lobkis, "Diffuse waves in open systems and the emergence of the Greens' function," *J Acoust Soc Am* ***116***, 2731-4

A different proof, with different predicates, for an open medium

Wapenaar K 2004 Retrieving the elastodynamic Greens function of an arbitrary inhomogeneous medium by cross correlation, *Phys Rev Lett* **93** 254301

An entirely different relation, often cited as equivalent.

J. Acoust Soc Am, 110, 3011-3017, 2001

Plausibility arguments, sundry "proofs", and a laboratory study in a finite body

Three plausibility arguments / proofs were offered:

1) Randomly excited normal modes (in a <u>finite</u>) body

$$\phi(x,t) = \Re \Sigma_{n=1}^{\infty} a_n u_n(x) \exp\{i\omega_n t\}$$

$$< a_n a_m^* > = \delta_{nm} F(\omega_n)$$ *n.b: this follows from maximum entropy*

$$C = < \phi(x, t) \ \phi(y, t+\tau) > = \frac{1}{2} \Re \ \Sigma_{n=1}^{\infty} F(\omega_n) \ u_n(x) \ u_n(y) \exp\{-i\omega_n \tau\}$$

Compare with G . . .

$$G_{xy}(\tau) = \Sigma_{n=1}^{\infty} u_n(x) \ u_n(y) \ \frac{\sin \omega_n \tau}{\omega_n} \quad [\text{ for } \tau > 0 \text{ , 0 otherwise}]$$

So, $\partial C/\partial\tau = G - G^{\text{time reversed}}$, i.e, $G - G^*$ or Im G

2) Propagator Picture (applies in a bounded or unbounded medium)

$$\begin{Bmatrix}\phi(y)\\ \dot\phi(y)\end{Bmatrix}_{t+\tau} = \int_V \frac{1}{\rho(x)} \begin{bmatrix}\dot G_{yx}(\tau) & G_{yx}(\tau)\\ \ddot G_{yx}(\tau) & \dot G_{yx}(\tau)\end{bmatrix} \begin{Bmatrix}\phi(x)\\ \dot\phi(x)\end{Bmatrix}_{t} d^3x$$

From definition of G (exact)

and

$$< \phi(x,t)\ \phi(y,t) > = \delta^3(x-y)\ \Phi(x); \quad < \phi(x,t)\ \dot\phi(y,t) > = 0$$

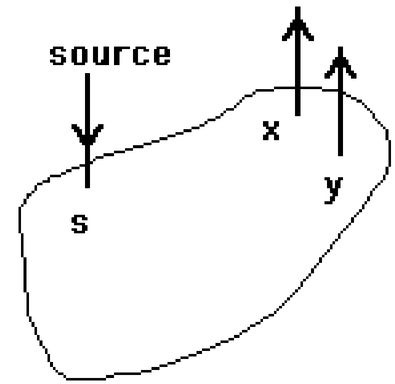
an assumption of no equal-time correlations

(It is not obvious what this condition means)

$$< \phi(y,t+\tau) \quad \phi(x,t) \quad > \quad = \Phi\ (x)/\rho(x) \qquad \dot G_{yx}(\tau)$$

We deduce the retrieval (for $\tau > 0$)

3) Exact/explicit normal mode treatment for a finite body with discrete random sources



$$V_x(t) = \Sigma_{n=1}^{\infty} u_n(x)\, u_n(s)\ \Im\, [\tilde{X}(\omega_n)\, \tilde{S}(\omega_n)\, \exp\{i\omega_n t\}/\omega_n]$$

$$V_y(t) = \Sigma_{n=1}^{\infty} u_n(y)\, u_n(s)\ \Im\, [\tilde{Y}(\omega_n)\, \tilde{S}(\omega_n)\, \exp\{i\omega_n t\}/\omega_n]$$

$$V_s(t) = \Sigma_{n=1}^{\infty} u_n(s)^2\ \Im\, [\tilde{S}(\omega_n)^2 \exp\{i\omega_n t\}/\omega_n]$$

Exact, normal mode expansion

$$C_{xy}(\tau) = \int_{t=-\Delta T/2}^{t=\Delta T/2} W(t)\ V_x(T+t)\, V_y(T+t+\tau)\, dt$$

(Definition of C)

$$C_{xy}(\tau) = \frac{1}{2}\Sigma_n \Sigma_m\, u_n(x)\, u_m(y)\quad [u_n(s) u_m(s)]\,/\omega_m \omega_n$$

$$\Re\ \tilde{X}(\omega_n)\, \tilde{S}(\omega_n)\, \tilde{Y}^*(\omega_m) \tilde{S}^*(\omega_m) \exp\{-i\omega_m \tau\}\, \tilde{W}(\omega_n - \omega_m)\, \exp\{iT(\omega_n - \omega_m)\}$$

exact

$$C_{xy}(\tau) = \frac{1}{2} \Sigma_n \Sigma_m u_n(x)\, u_m(y) \quad [u_n(s) u_m(s)] \,/ \omega_m \omega_n$$

$$\Re \; \tilde{X}(\omega_n)\, \tilde{S}(\omega_n)\, \tilde{Y}^*(\omega_m) \tilde{S}^*(\omega_m) \exp\{-i\omega_m \tau\}\, \tilde{W}(\omega_n - \omega_m)\, \exp\{iT(\omega_n - \omega_m)\}$$

Double sum reduces to diagonal terms . . . .

If W is long lasting enough

and/or *If* we average over many source positions s.

$$C_{xy}(\tau) \approx \tilde{W}(0)\, \Sigma_n \frac{[u_n(s)]^2 \,|\, \tilde{S}(\omega_n) \,|^2}{2\, \omega_n^2} \; \Re \; \tilde{X}(\omega_n) \tilde{Y}^*(\omega_n)\, u_n(x)\, u_n(y)\, \exp\{-i\omega_n \tau\}$$

An average over sources then gives our retrieval (almost . . . . . . caveats).

Approach underlines the need for a source average

time average from a single source does not suffice

It also lends itself to estimating variances

State of the proof ca 2001:

normal mode with maximum entropy statements about the a_n's
or normal mode representation with explicit sources s.

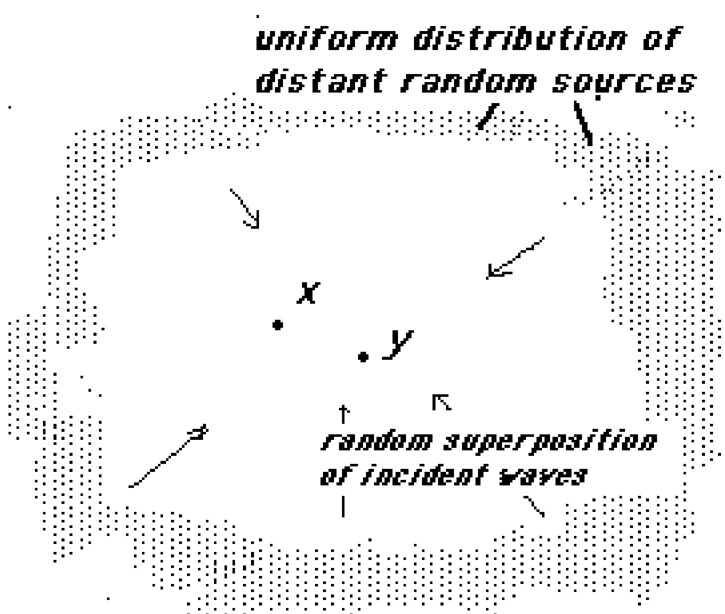
- inapplicable to unbounded media

Propagator proof

- meaning of assumption?

Then, Snieder, 2004,
"Extracting the Green's function from the correlation of coda waves: A derivation based on stationary phase" Phys. Rev. E, **69**, 46610

Showed that a distribution of distant random point sources in an unbounded (and homogeneous) medium



Gave a random field at x and y that correlated to give the (ballistic) Green's function G(x,y)

Reminiscent of the room acoustics (also Aki) proof that an incoherent superposition of plane waves from all directions gives a field-field correlation
$C = j_o(k|x-y|) = \mathrm{sinc}(k|x-y|) = \mathrm{Im}\, G$

A third argument:

Imagine an ensemble of sources s(**x**)f(t) distributed over all space. *cf Roux et al*

With <s> = 0; <s(**x**)s(**x**')> = δ^3(**x**-**x**').

Then

$$a_n = \int u^n(\boldsymbol{x}) s(\boldsymbol{x}) d^3\boldsymbol{x} \; f(\omega_n)/\omega_n$$

Satisfies modal-perspective definition of equipartition:

$$< a_n a_m^* >= \delta_{nm} <| f(\omega_n) |^2 >$$

Applicable to unbounded, heterogeneous, and even to lossy media, but requires
infinite number of sources, including sources near x and y.

aside . . .

The above proofs can be thought of as following in part from a (perhaps familiar?) identity . . .

$$\mathrm{Im}\tilde{G}(\boldsymbol{x},\boldsymbol{y},\omega) = -\omega\varepsilon \int \tilde{G}(\boldsymbol{x},\boldsymbol{u},\omega)\tilde{G}^*(\boldsymbol{u},\boldsymbol{y},\omega)d\boldsymbol{u}$$

a property of Greens functions,
relating the square of the operator G to its Imaginary part.

. . . other forms of this "Ward" identity entail

integrals over bounding surfaces

integrals over the dissipative parts of the volume

versions without the *

e.g.
$$\tilde{G}(\omega+\Omega/2)-\tilde{G}(\omega-\Omega/2)$$
$$= -2\omega\Omega \int \tilde{G}(\omega+\Omega/2)\tilde{G}(\omega-\Omega/2)$$

Problems with above arguments for $G = \partial C/\partial \tau$

➨ The first (Weaver) requires modal Equipartition

$$< a_n a_m^* > \;= \delta_{nm} F(\omega_n)$$

Manifest nonsense in an open medium

➨ The second (Snieder) requires plane waves to be solutions of the wave equation (and smoothly randomly distributed in incident direction)

Manifestly incorrect in
heterogeneous media
or near boundaries

➨ The third (Roux) asks for sources over all ∞;

is this reasonable? Is it necessary?

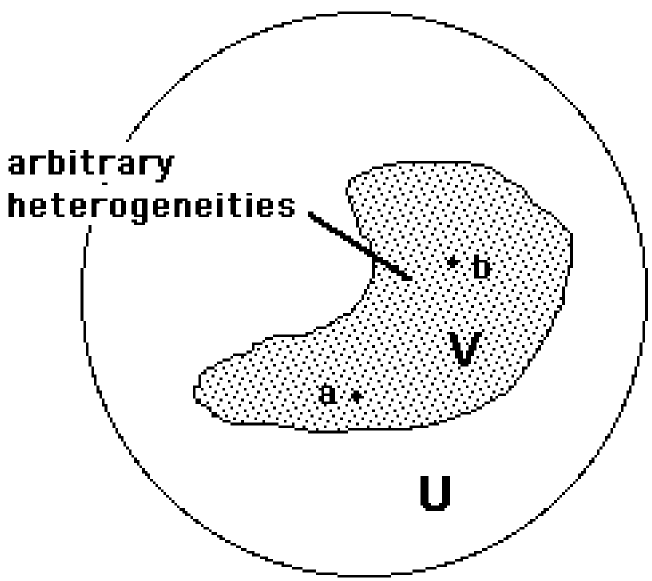
Also : what about *incompletely* diffuse wave fields ?

Needed:

An argument for $\partial C(a,b)/\partial\tau \sim G(a,b)$ based on a more useful notion of a diffuse field.

R L Weaver and O I Lobkis, "Diffuse waves in open systems and the emergence of the Greens' function," *J Acoust Soc Am* ***116***, 2731-4 (2004)

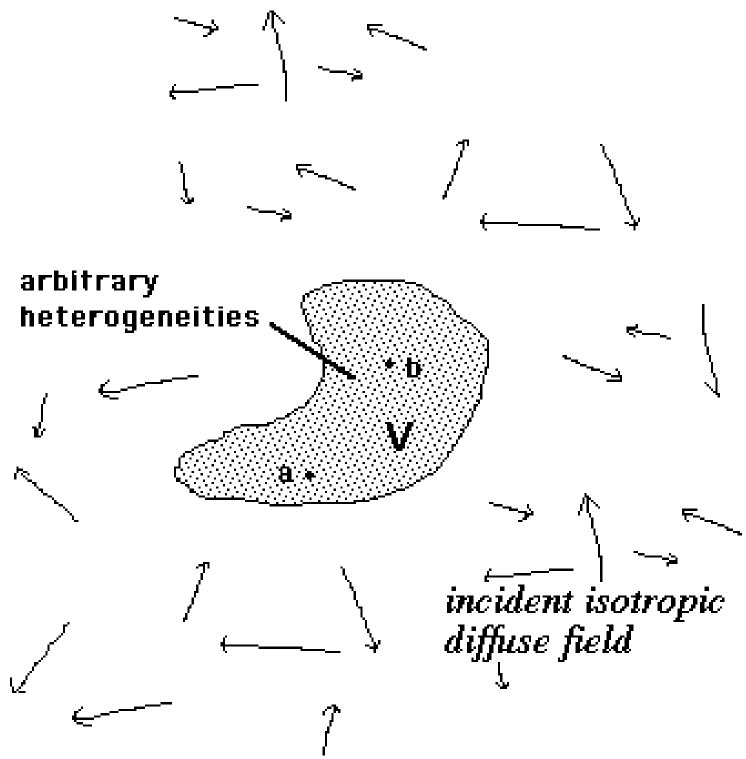
Presented a proof, for a heterogeneous open medium using a notion of a diffuse field which is either



➨ Due to uncorrelated sources s(x)f(t) over entire volume *external* to U+V

or

➨ Due to an *incident* field in U that satisfies room-acoustics notion of a diffuse field = incoherent superposition of incident plane waves.



$$\mathrm{Im}\, G_{ab} \sim \omega < \psi(\boldsymbol{a})\, \psi^{*}(\boldsymbol{b}) >$$

➨ Retrieved G includes scatterings, not just ballistic waves.

➨ It suffices for the *incident* field to be diffuse by the ordinary local definition

But real sources are not distributed so uniformly

So why, by the way, ought we expect a real diffuse field to be equipartitioned?

What is meant by that?

Equipartition:

What is it?
Why/when does it happen?
What is it good for?

R. Weaver, "On Diffuse Waves in Solid Media," *J. Acoust. Soc. Am.,* **71**, 1608-1609 (1982)
R. Weaver, "Diffuse Elastic Waves at a Free Surface," *J. Acoust. Soc. Am.*, **78**, 131-136 (1985)
Renaud Hennino, Nicolas Trégourès, Nicolaï M Shapiro, Ludovic Margerin, Michel Campillo, Bart A van Tiggelen and R L Weaver, "Observation of equipartition of seismic waves," *Phys Rev Lett* **86** 3447-50 (2001)
Also, the idea of *detailed balance* : R. Weaver, *J Sound & Vibr* (1984)

Essential concept: After enough multiple scattering,
(each scattering or reflection generating *mode conversion*)

A wave field achieves a dynamic balance amongst
different wave types and
different propagation directions

First(?) occurrence of the concept,

for elastic waves in thermal equilibrium:

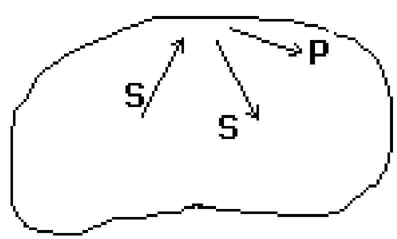
P. Debye (1914) Theory of Specific Heat of (insulating) Solids
(see, e.g. C Kittel : Intro to Solid State Physics)

Then, for more general diffuse elastic waves:

D Egle 1981 Diffuse waves in solid media *J Acoust Soc Am* **70**, 476

Egle argued that multiply reflected elastic waves in a finite reverberant body
would mode convert and achieve a steady state balance between P and S waves,
Calculated by him using Monte Carlo methods and
known mode conversion reflection coefficients
and an assumption that rays are incident at uniformly random angles.

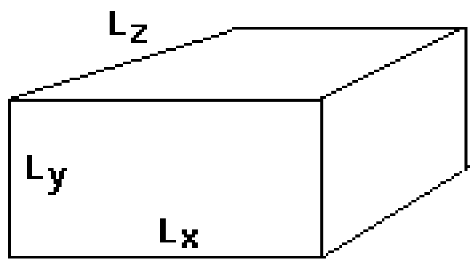
Egle found, for $\nu = 0.30$,
a ratio of *12.9*



Then,

R. Weaver, "On Diffuse Waves in Solid Media,"
J. Acoust. Soc. Am., **71**, 1608 (1982)
claimed, that, like the thermally equilibrated waves of Debye,
a fully randomized elastic wave field should partition its energy in proportion to the number of modes of each type at a given frequency in a given volume.

Standard argument for counting modes:



In a 3-d rectangular region,
(with Dirichlet Boundary Conditions)

There is a mode at any wavevector $\vec{k}$

$$\vec{k} = k_x\hat{\mathbf{e}}_x + k_y\hat{\mathbf{e}}_y + k_z\hat{\mathbf{e}}_z, \quad \omega = c\,|\,\vec{k}\,|$$

where $k_x = n\pi/L_x$, $k_y = m\pi/L_y$, $k_z = j\pi/L_z$ (n, m, j > 0)

This allows us to count the modes with frequencies less than ω:
$N = L_xL_yL_z\,\omega^3/6c^3\pi^2$ – scales with volume, and inversely with c^3.
(In 2-d it is proportional to area and inversely proportional to c^2)

$N = L_x L_y L_z\, \omega^3/6c^3\pi^2$ for each wave type implies

$N_{shear} = 2 * \text{Volume} \times (\omega^3/6\pi^2) / c_S^3$

$N_P = \text{Volume} \times (\omega^3/6\pi^2) / c_P^3$

Ratio =

$N_{shear}/N_P = 2c_P^3/c_S^3 \sim$

typically 10 or more, depends on ν.

Numerical value at $\nu = .3$ (13.1)

~agreed with Egle's calculation (12.9)

also calculated ratio of

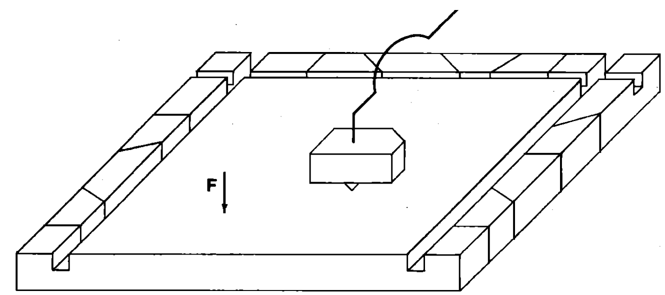
Rayleigh wave surface energy density

to volumetric density of bulk waves

The agreement between the calculations was not a numerical co-incidence, but rather,

a consequence of *reciprocity* and *energy conservation*.(*)

Discussion thereon is found in
R. Weaver, "Diffuse Waves in Finite Plates,"
J. Sound and Vibr., **94**, 319-335 (1984)
(see also experiment follow up:*J.A.SA*., **79**, 1986)



A thick plate with a diffuse field

in which the steady state balance amongst the several guided *Lamb* waves of a plate was calculated

(and it was noted that equipartition factor c^3 is really $v_{group}c_{phase}^2$)

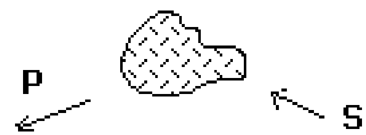
And where it was shown that this partition is the only way to have a steady-state balance of energy amongst the different wave types.

(* or maximum entropy)

Reciprocity and Energy conservation

establish that

The Rate at which
incident S energy (from direction n_S)
converts to P energy (in direction n_P)

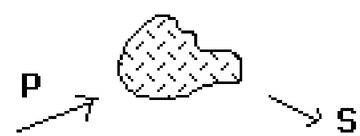


slow

is proportional to phase space volume for P ~ c_p^{-3} = small

and

The Rate at which
incident P energy (from direction n_P)
converts to S energy (in direction n_s)



fast

is proportional to phase space volume for S ~ c_S^{-3} = big

With the <u>same</u> Proportionality factor!
Equilibrium balance therefore requires much more S energy density

R. Weaver, "Diffuse Elastic Waves at a Free Surface,"
J. Acoust. Soc. Am., **78**, 131-136 (1985)

Discussed the consequences of equipartition for elastic waves at a surface

Showed: A picture of a diffuse field as an incoherent mixture of plane waves incident upon the free surface from the bulk + their reflections + incident Rayleigh wave

Gave the same predictions for $< \psi^2(x) >$ as Im[Greens function(x,x)]

Also showed that the Rayleigh waves are responsible for about 70% of the vertical surface motion in a diffuse field.

15 years later . . .

Renaud Hennino, Nicolas Trégourès, Nicolaï M Shapiro,
Ludovic Margerin, Michel Campillo, Bart A van Tiggelen and
Richard Weaver, "Observation of equipartition of seismic waves,"
Phys Rev Lett **86** 3447-50 (2001)

Applied these ideas to measured coda at the earth's surface.
and showed that those Mexican codas had

- the expected equipartitions ($\mathcal{E}_S$ / $\mathcal{E}_P$ ~ 7 at surface of half space)
- and values of H/V and H/$\mathcal{E}$
- and values of various mean square strains

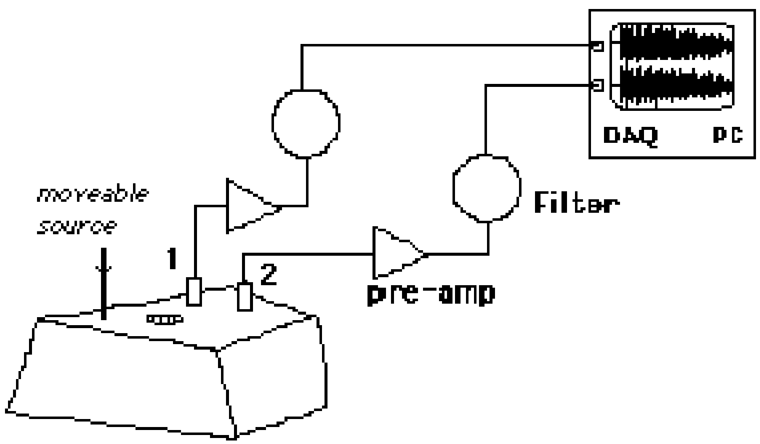
. . . indicating that the coda was

i) equipartitioned

ii) therefore thoroughly multiply scattered . . ?

Story from Cargese . . .

A laboratory confirmation (in a finite specimen). . .

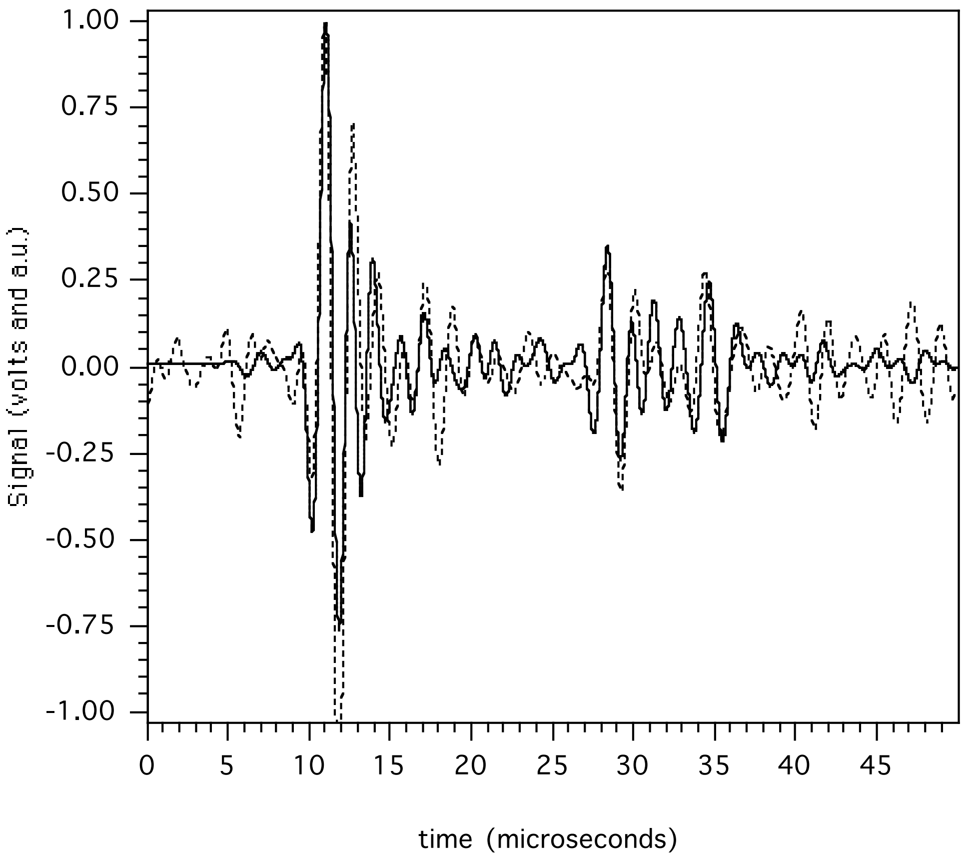


Capture each source's coda at each of two receiver positions

Correlate from each source and average . .

Compare with direct signal 1 $\rightarrow$ 2

Results . . .



Comparison of direct "pitch-catch" signal

———

and

correlation

An average over
7 sources
and long codas

Chief observation: C~G, but there are fluctuations -

how to *apriori* estimate confidence?

how to know what's C and what's fluctuation?

Theories for the fluctuations

how to estimate how much averaging is needed ?
and to estimate whether a feature seen in C is real

1) In a Finite Body

RL Weaver and O I Lobkis, "The mean and variance of diffuse field correlations in finite bodies", *The Journal of the Acoustical Society of America* , **118**, 3447 (2005),

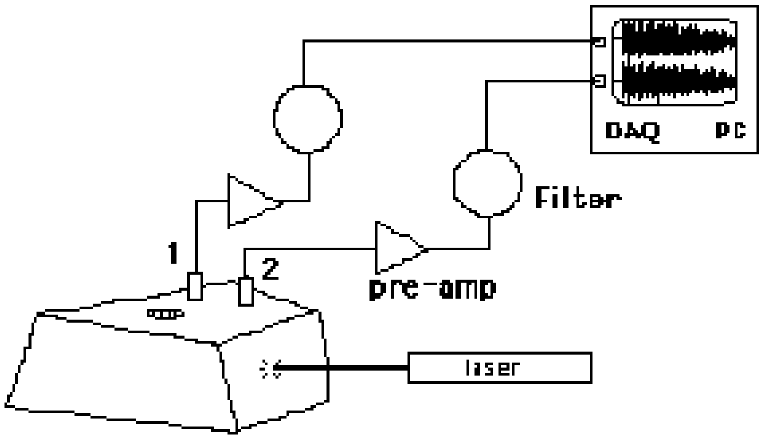
$$\overline{\operatorname{var} R}^{ab} / \overline{< R >}^2$$

$$= [2 + (2\,or\,3)d(\tau) + \frac{1}{2}(\frac{T_H}{T})^2(1 - \exp\{-2T / T_H\})]$$

per source

where T_H = "Heisenberg time" = modal density = $2\pi\ \partial N(\omega)/\partial\omega$

Experiments

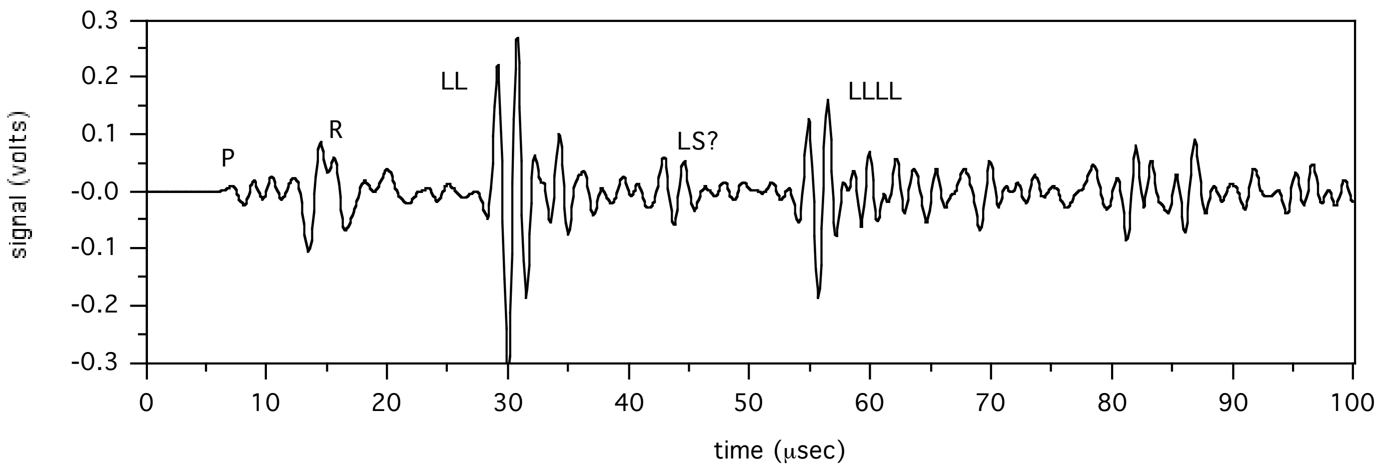


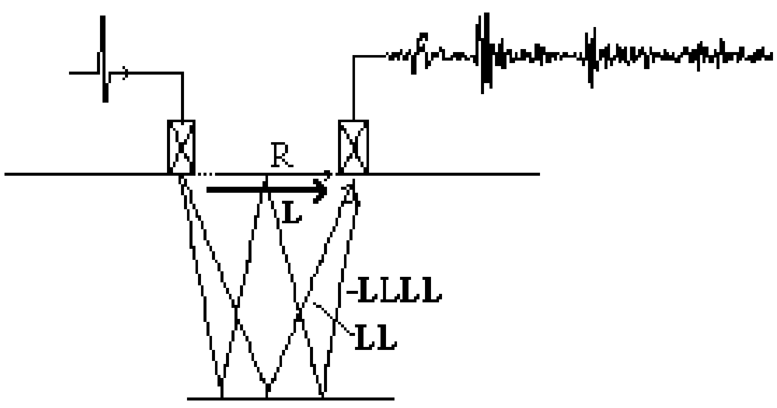
Thermo-elastic surface excitations from pulsed laser, ~132 different positions

A Q-switched Nd:YAG laser excites elastic waves in an irregular aluminum block of nominal dimension 15 cm.

The signals from two piezoelectric transducers are amplified and lo-pass filtered before being digitized by a PC.

Direct Signal from one transducer to the other:





Observe ray arrivals corresponding to

Surface Rayleigh wave

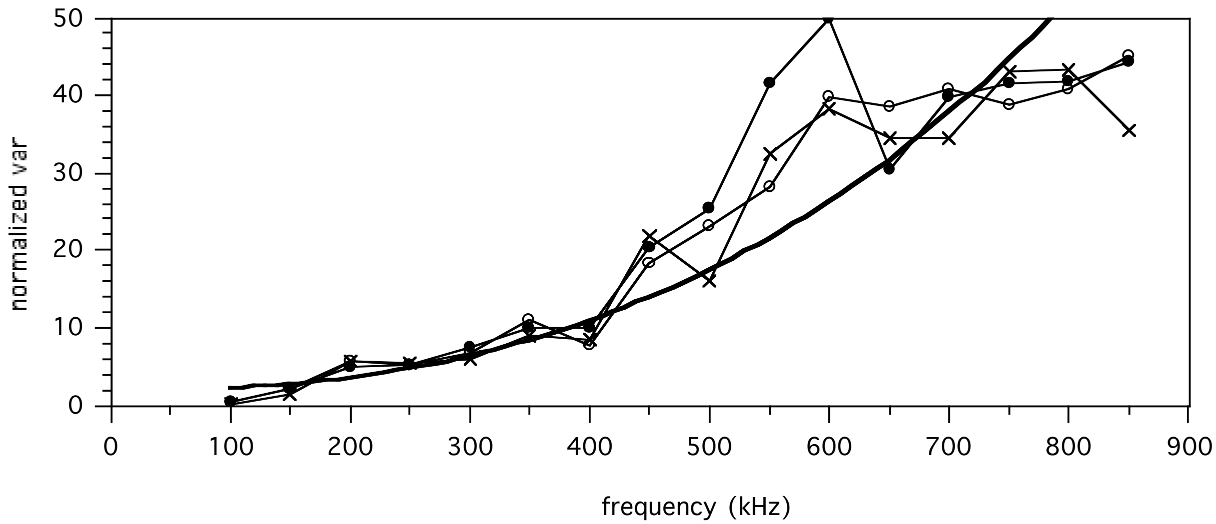
Surface skimming P wave (P)

Bottom reflected P wave (LL)

P wave reflected twice off bottom(LLLl

Observed
Normalized
Variances

compared
w theory.



Theory of normalized variance (assuming randomly distributed sources and a data capture duration T) :

$$[2+(2\,or\,3)d(t)+\frac{1}{2}(\frac{T_H}{T})^2(1-\exp\{-2T/T_H\})]$$

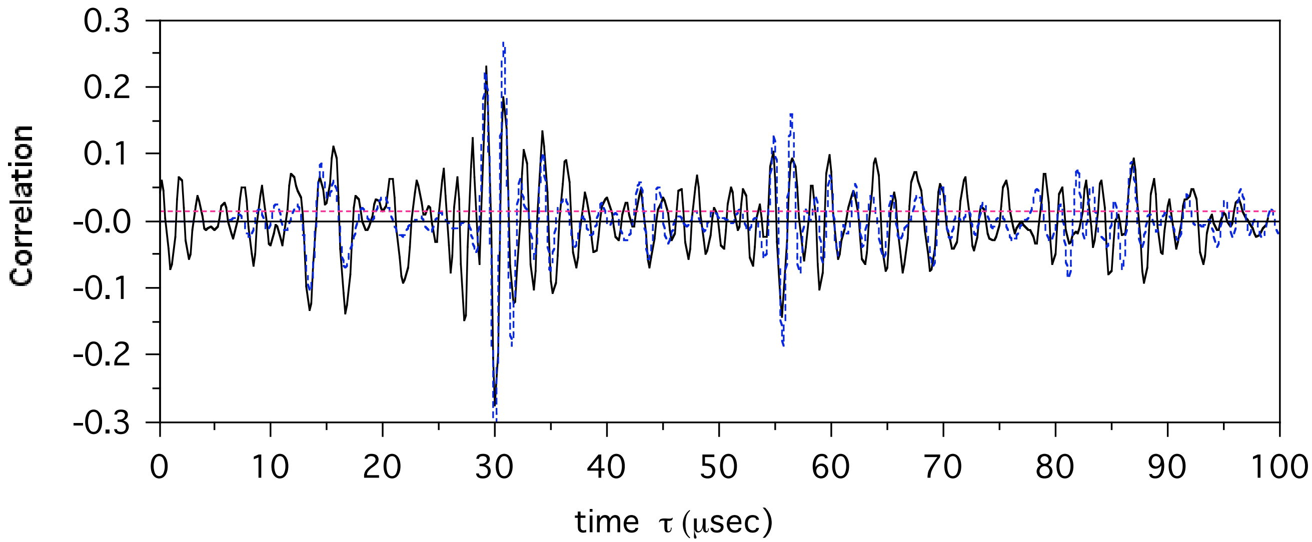
Comparison of

a) Direct signal between transducers

b) Adjusted Correlation

- adjusted for source time function
- and transducer response functions

c) observed (var / {numberofsources N})$^{1/2}$

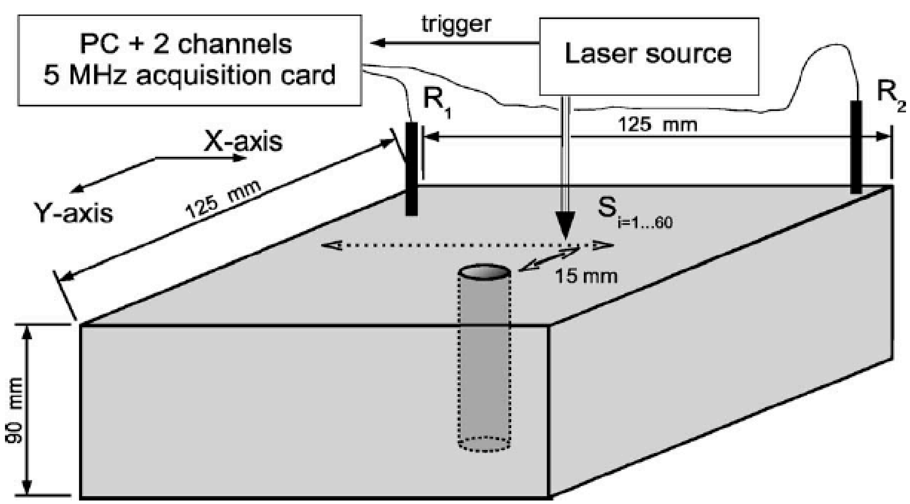


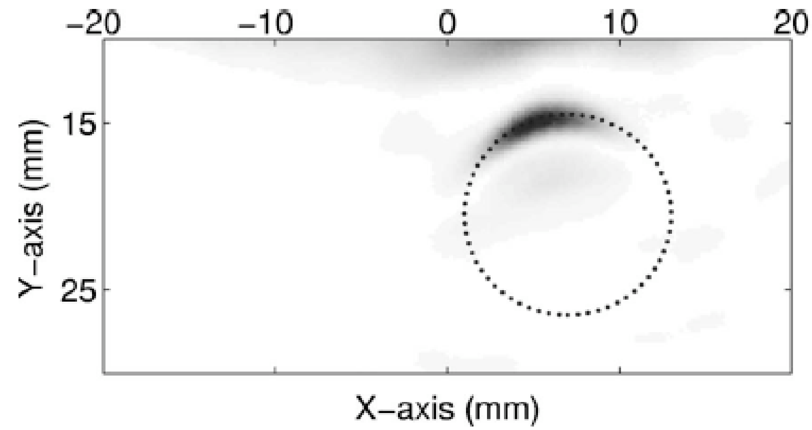
Strength of fluctuations agrees with theory

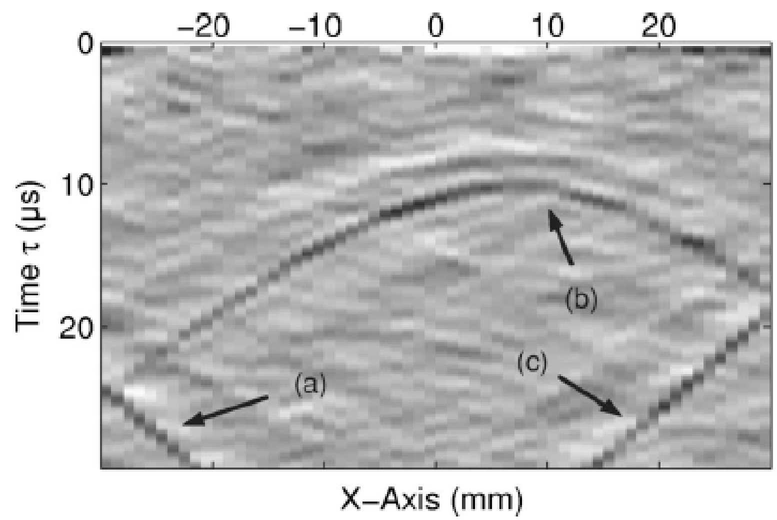
But deviation (G-C essentially) is in excess of expected error.

. . . . . why ?

Imaging with coda correlations . .







Larose et al *JASA* 119 (2006)

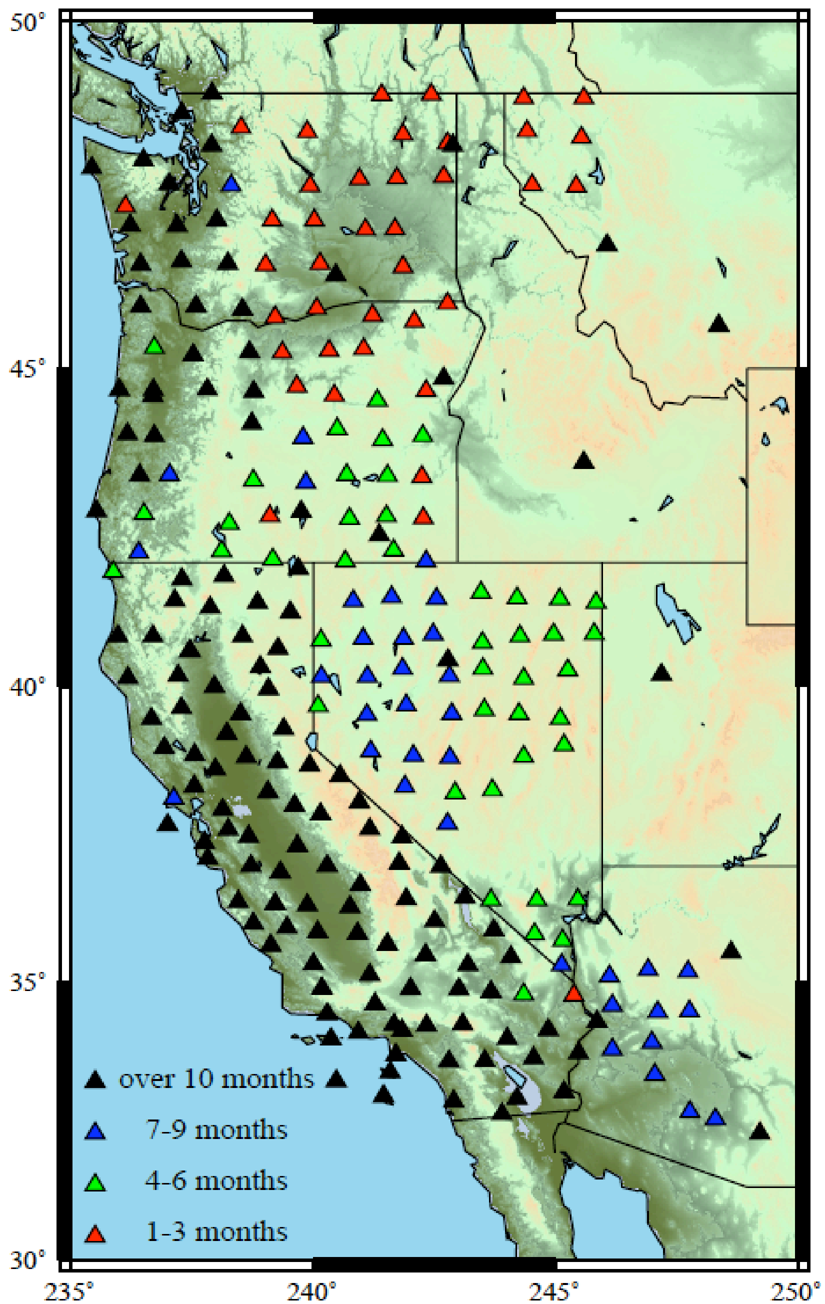
2) theory for fluctuations in an open medium . . .

$$\frac{\text{Retrieved ray arrival peak energy}}{\text{energy of background fluctuations}} =$$

$$\frac{ray\ peak\ height^2}{\text{var}} = \frac{1}{2\sqrt{\pi}} \frac{NT}{\Delta} \frac{c^2}{r^2 \overline{\omega}^2} \qquad \text{(in 3-d)}$$

NT = number of sources * duration of each coda,
Δ = inverse bandwidth,
so NT/Δ = amount of information collected.

So, good resolution requires:
amount of information collected >>
[source-receiver separation in units of $\lambda/2\pi$]$^{d-1}$

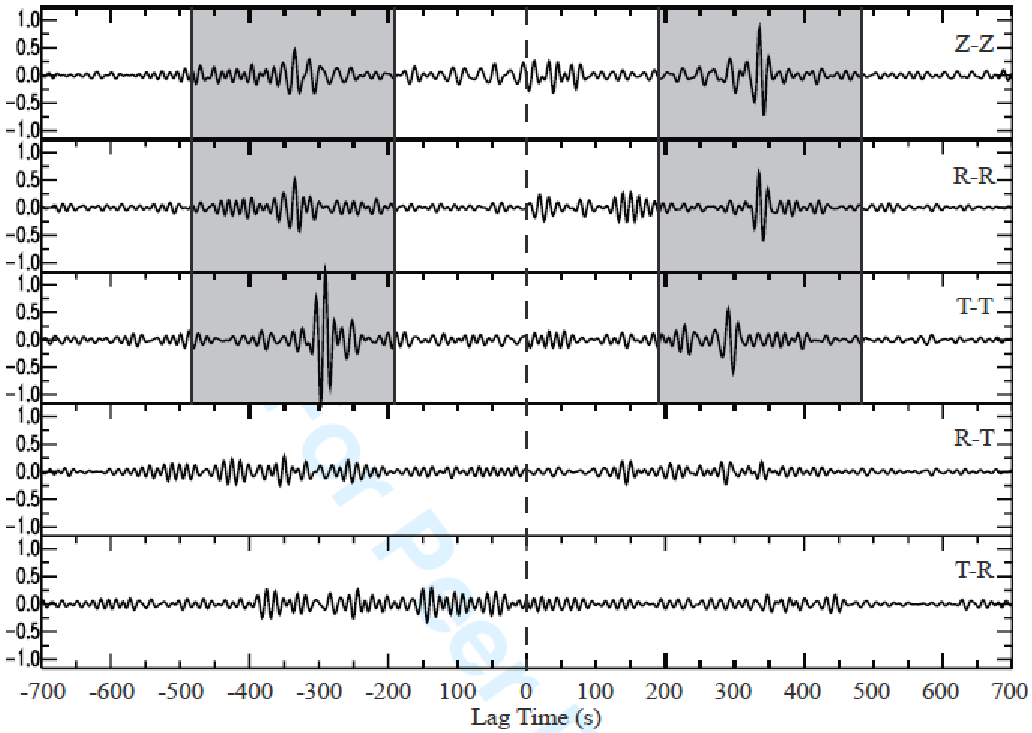


Ambient ~10 sec
Seismicity in western US

(generated mostly
by ocean storms?)

Detected on an array of
Wide-band stations.

A sample correlation (Ritzwoller's group) of ambient ~10 second period seismic noise in CA, with a year of data collecting.



Rayleigh wave

Love wave

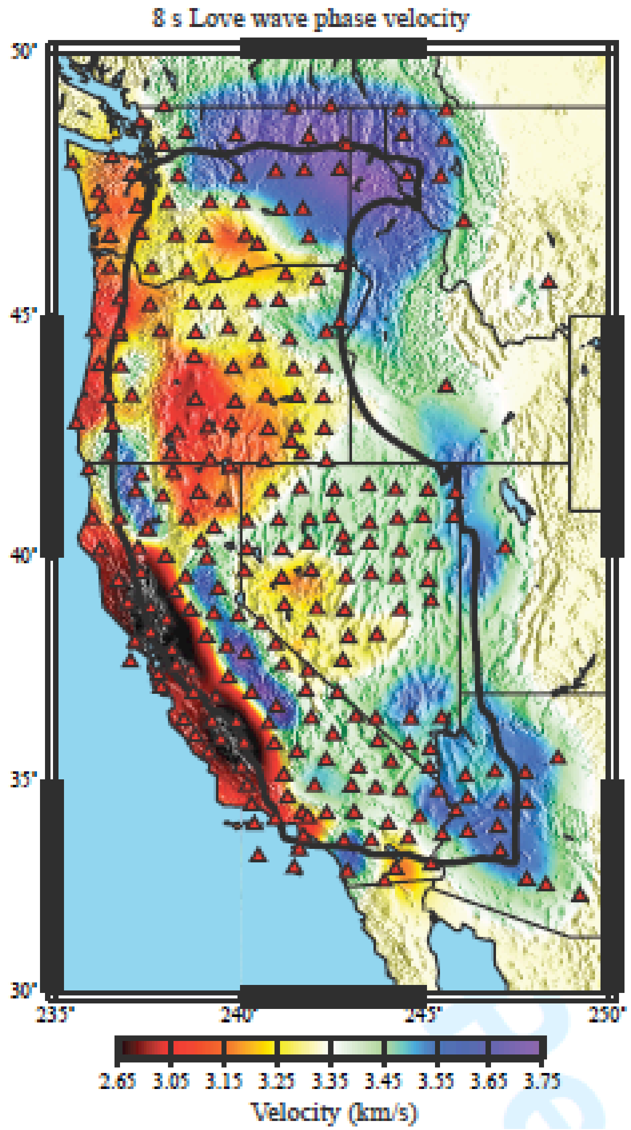
Shows a) failure of symmetry in τ

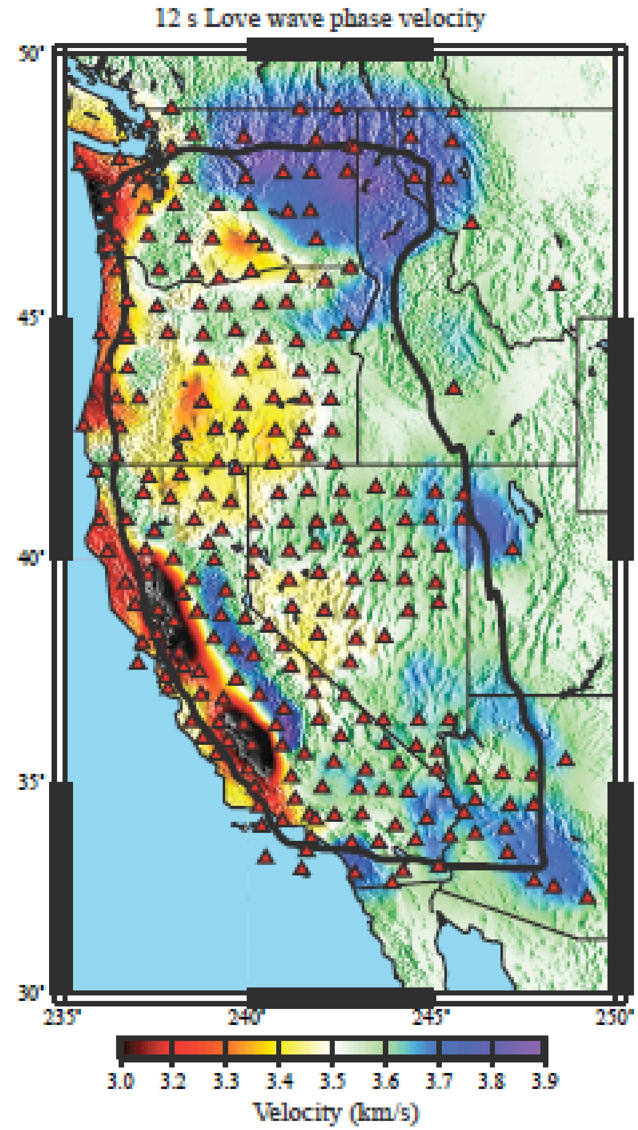
b) some (even noncausal!) deviations from simple ray arrivals

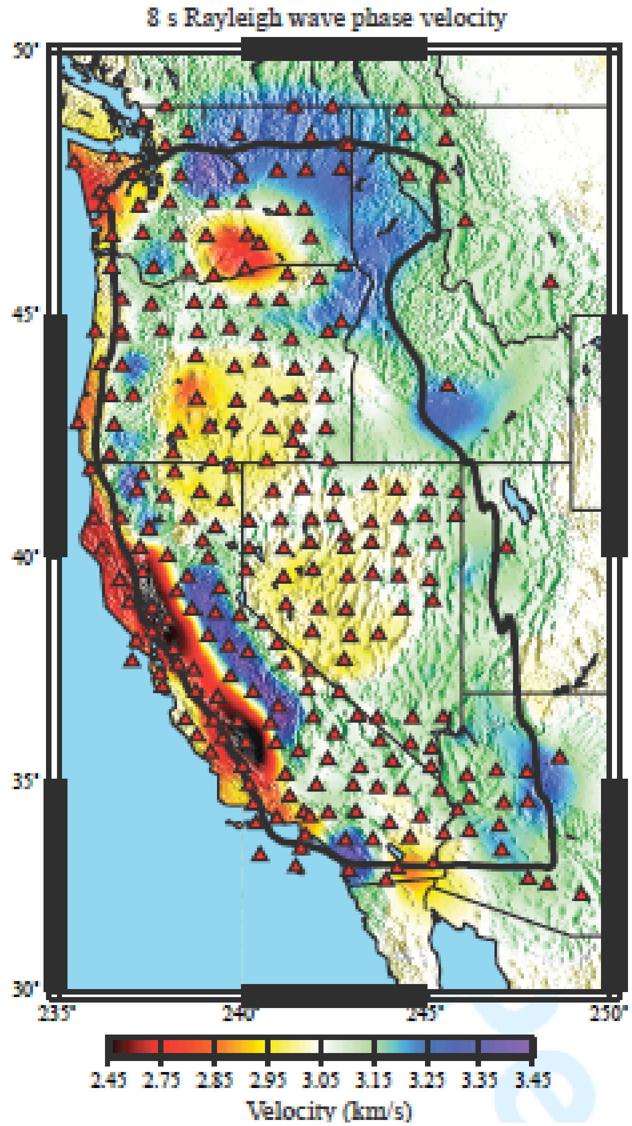
Theory says fluctuations/error ought to be *tiny*. (~1%)

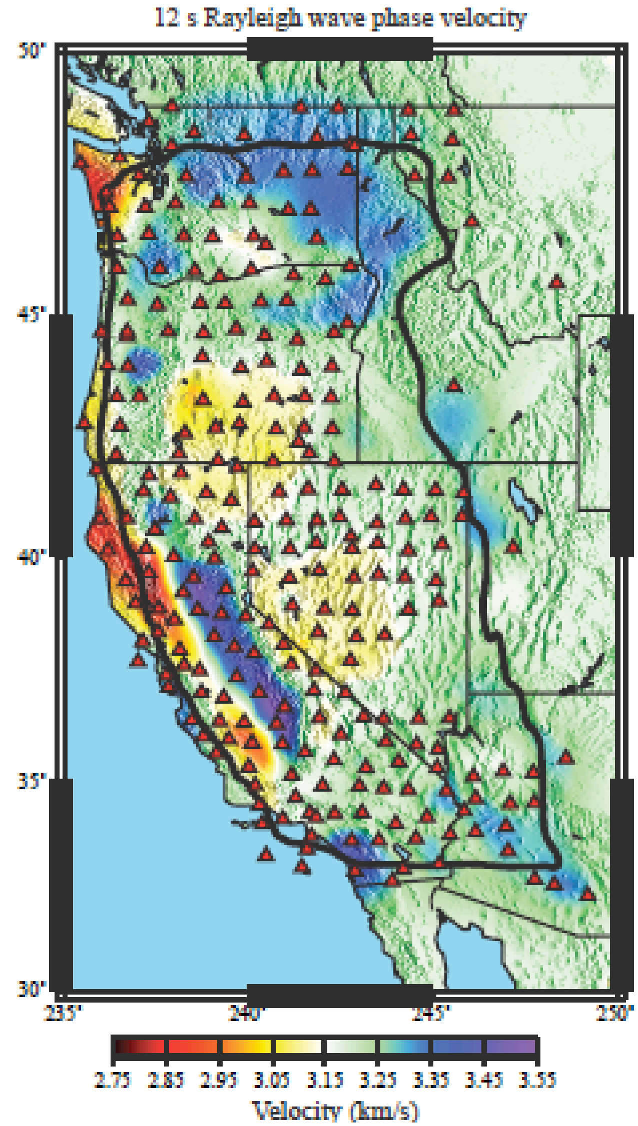
So, the anomalous features above are real, i.e. $C = \partial_\tau < \psi\, \psi > \neq \; G$

Nevertheless maps of wavespeed are constructed by tomography . .



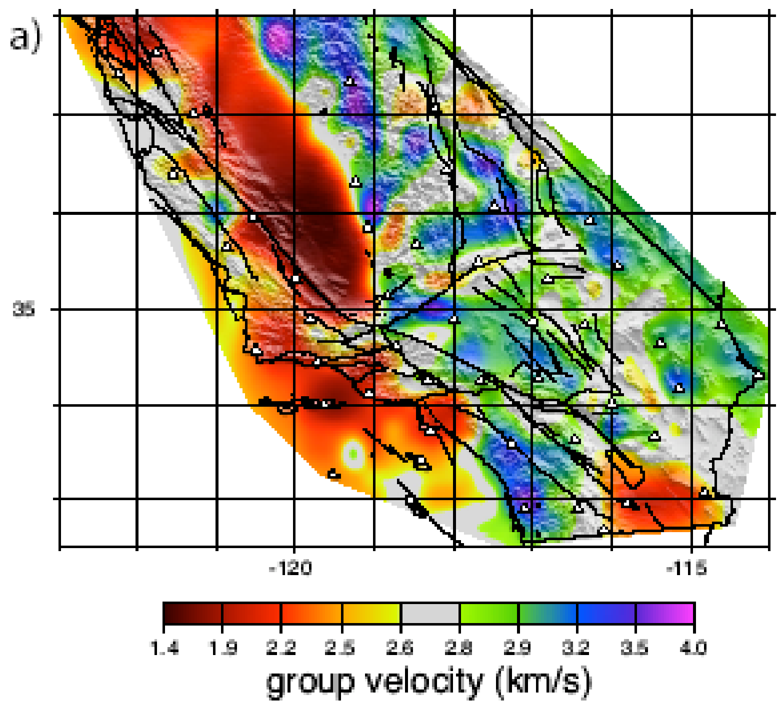






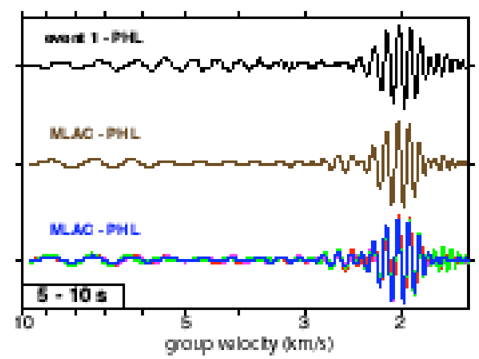
Conclude - even though ambient waves are not fully diffuse, maps are still constructable.

Shapiro, N.M., M. Campillo, L. Stehly, and M.H. Ritzwoller, 2005, High-Resolution Surface-Wave Tomography from Ambient Seismic Noise: Science **307**:1615-1618



A map of Surface-wave Velocity in California

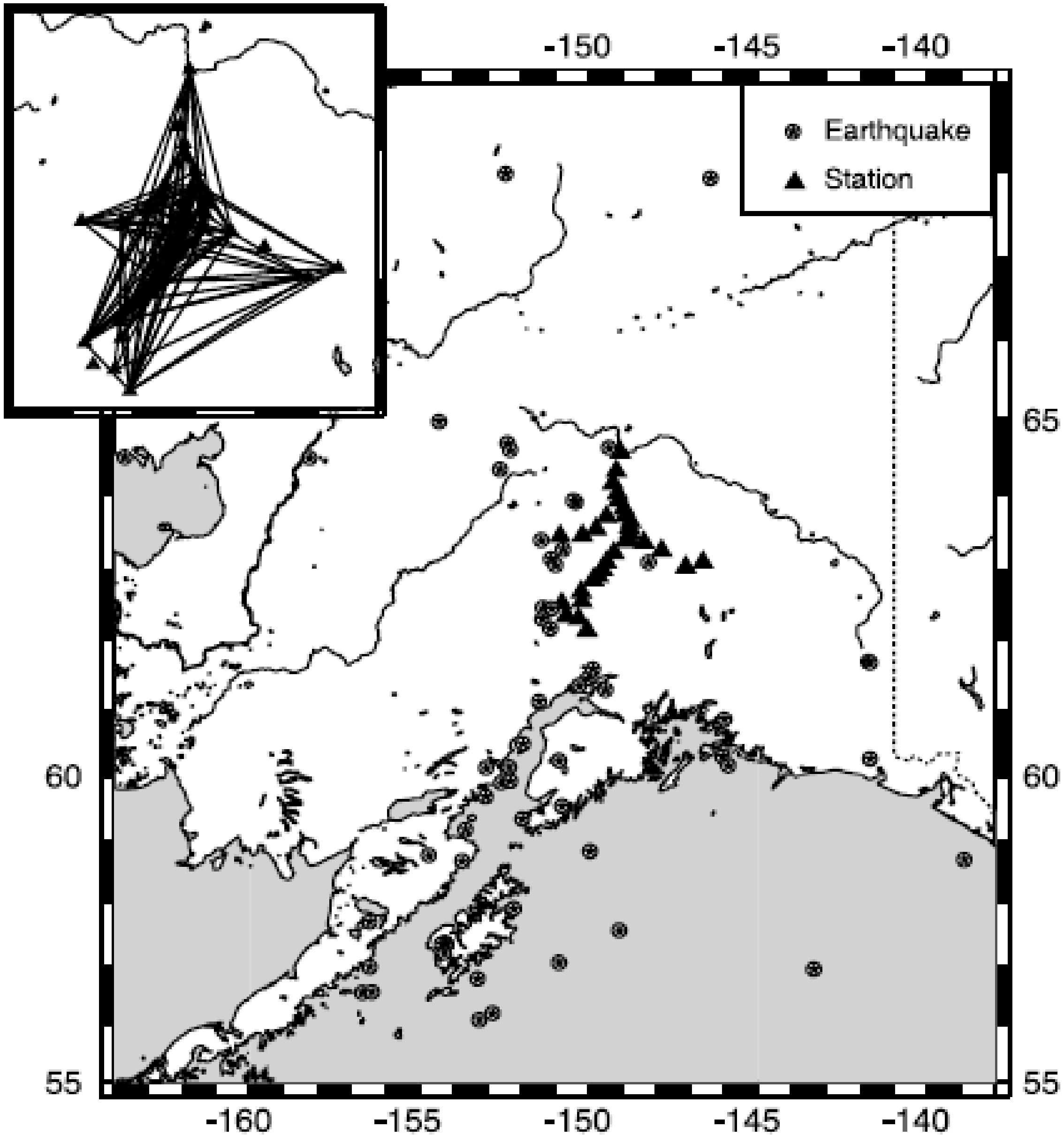
Obtained from correlating 30 days of seismic noise



earthquake

1 year of correlations

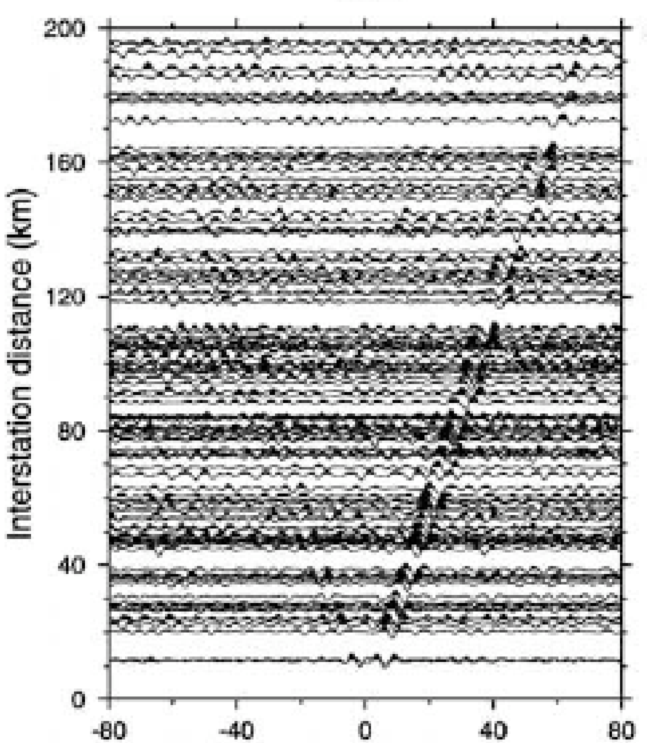
4 one-month correlations



Paul, Campillo, Margerin,
and Larose
correlated the codas
of several Alaskan earthquakes

Correlations of coda

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 110, (2005)



Time (sec)

From a set of 100 regional earthquakes, each with a coda of about 300 sec duration.

NT = 30,000sec
Δ= 20 sec
λ = 20km

At 40 km distance, $r\omega/c$ =13
pk height/rmsbackground= 10%

NB. Asymmetry reduced if raw data is confined to later coda

Virtues of using ambient noise

there is lots of it: many years x many stations

Downside

it is imperfectly diffuse

Virtues of using earthquake coda

If late enough it is more closely equipartitioned

Downside

There is not much of it, especially of the very late variety

Need many strong earthquakes with different sources

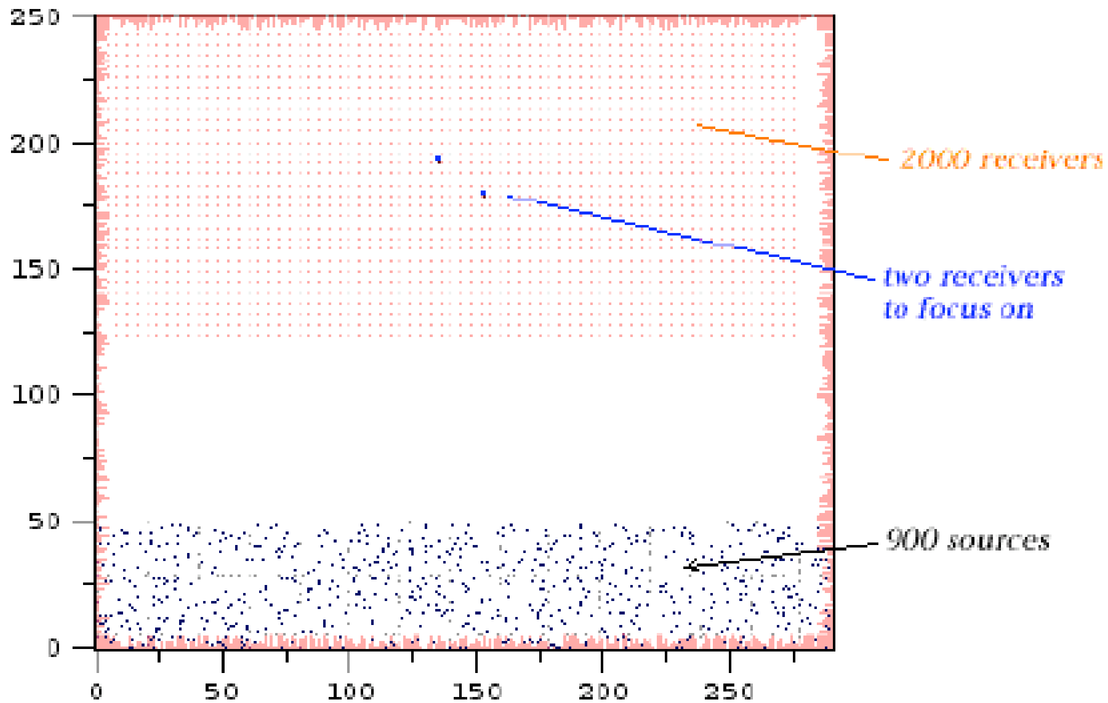
Speculation that we can combine the virtues:(?)

Examine the coda of ambient noise correlations

. . . . and correlate that.

Numerical experiment

A 251 x 291 rectangular domain with
- dissipation
- inhomogeneous distribution of sources
- net wave flux



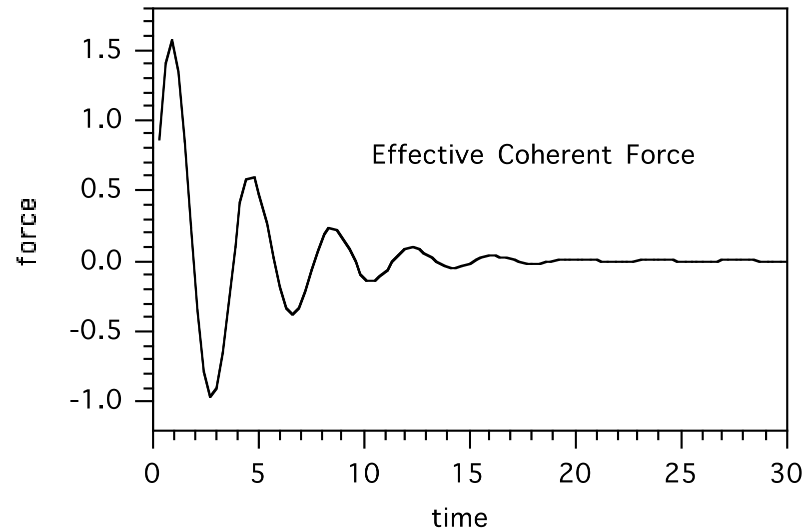
We shall focus on the responses at two particular sites

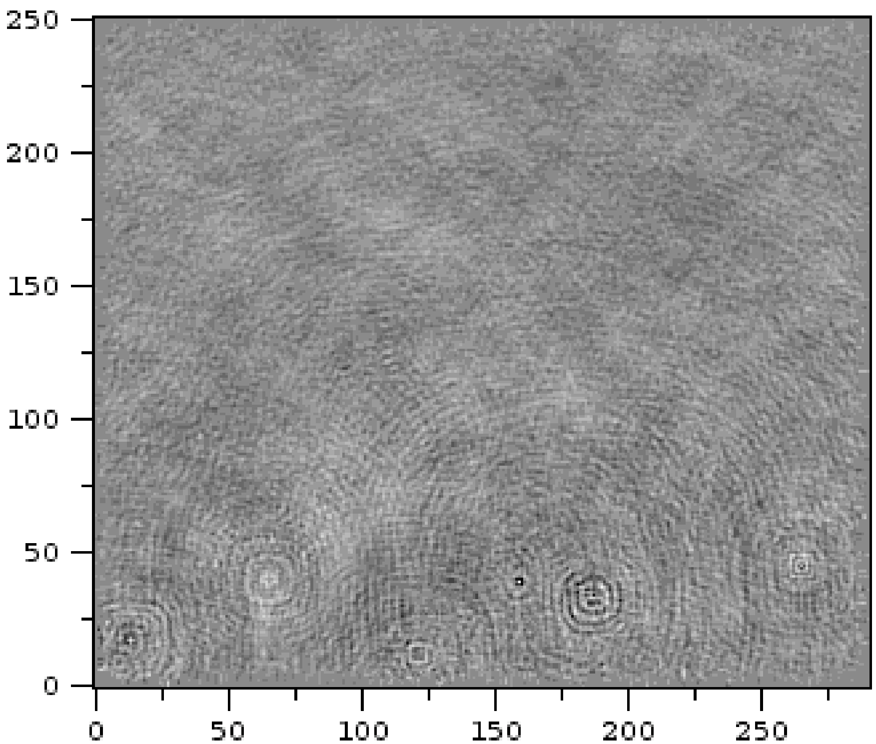
DNS run for 20,000 transit times.
Signals recorded on each of 2000 receivers.

Wavespeed ~1 (plus dispersion)
Dissipation (one e-fold of energy loss per transit)

Each source applies a forcing which is white noise convolved with

A band limitation centered on a frequency of 0.17 Hz



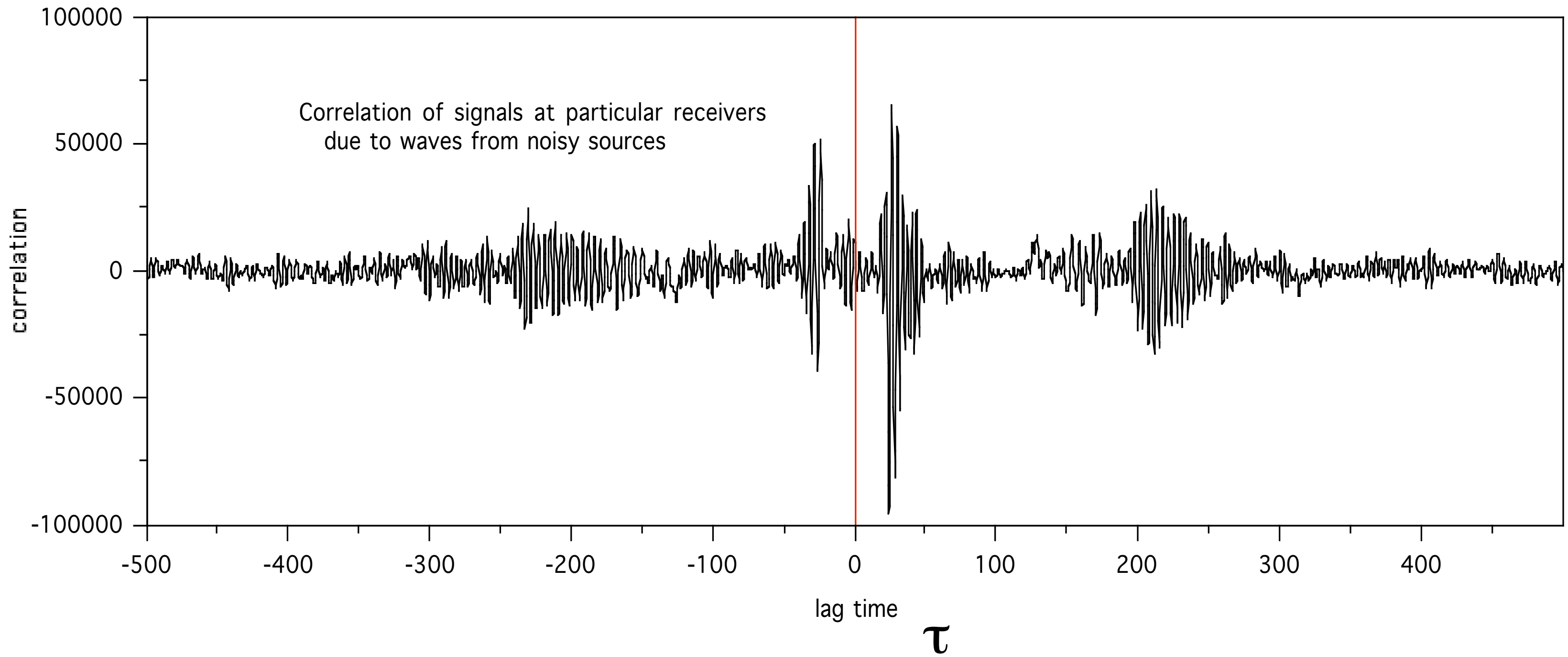


Snapshot of wave field 8000 seconds after sources began (only 6 sources here)

Field is imperfectly diffuse.

On correlating 660,000 seconds of signals at the two special receivers,

. . . . $\partial < \psi_1(t+\tau)\psi_2(t) > / \partial\tau =$



NB. Note the time-non symmetry as with ambient seismic correlations

Note the noncausal amplitude near τ=0 as with ambient seismic correlations

Can't be ImG.

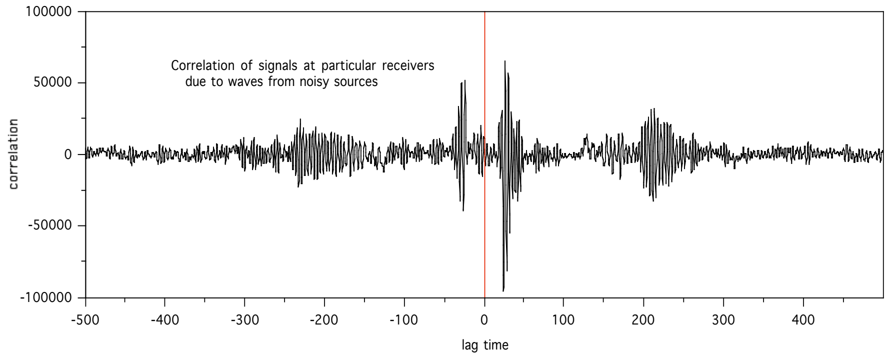
NB. This correlation has converged.

because the expected error (< 1% of peak) is below 'signal' out to beyond τ = 1000. Thus this is all real!

Finesse . . ?

A correlation like

$C_{12}(\tau)$



Is constructed for each receiver r = 3,4 . . 2000

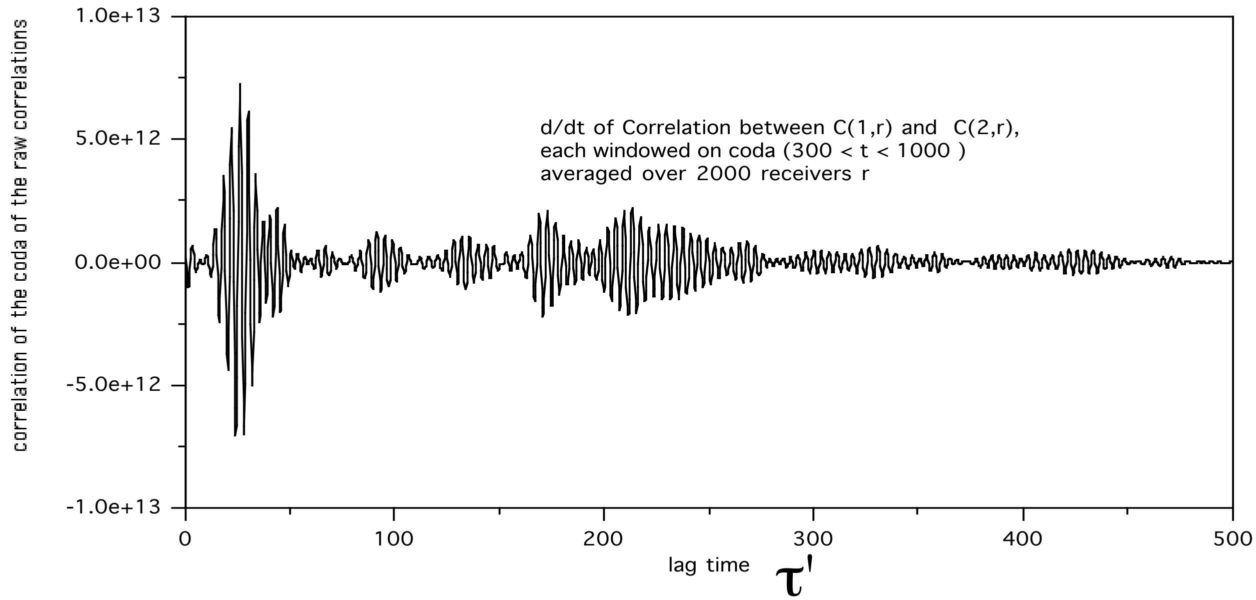
$C_{1r}(\tau)$ and $C_{2r}(\tau)$,

each windowed into a reliable coda regime $300 < \tau < 1000$

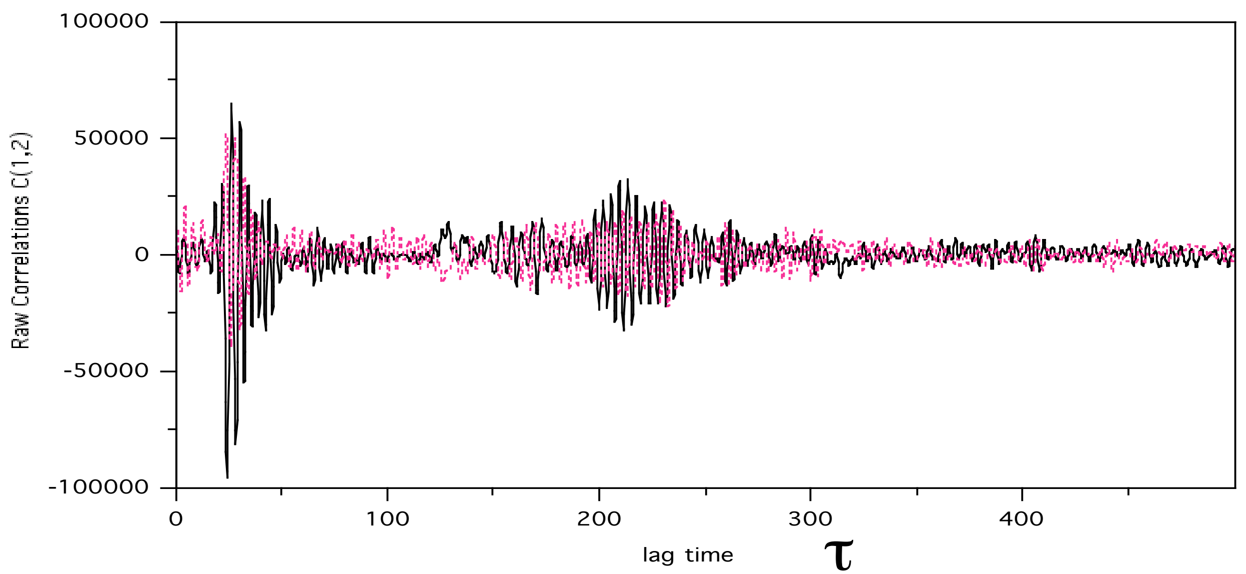
Then the meta-correlation is constructed

$$X(\tau') \equiv \frac{\partial}{\partial \tau'} \sum_r < C_{1r}(\tau) C_{2r}(\tau + \tau') >_\tau$$

$$X(\tau') \equiv \frac{\partial}{\partial \tau'} \sum_r < C_{1r}(\tau) C_{2r}(\tau + \tau') >_\tau$$



Compare raw C_{12}



Conclude -

with <u>enough</u> correlations (e.g. 2000 sites)
from <u>enough</u> of a time record (e.g. 100,000 cycles)

A decent Im G can be constructed

<u>even if the raw fields are not equipartitioned.</u>

In Sum . . . .

Simple proofs of $G \sim \partial C/\partial t$ are common now

Many lend themselves to estimates of variance

Most seismic records are averaged more than sufficiently

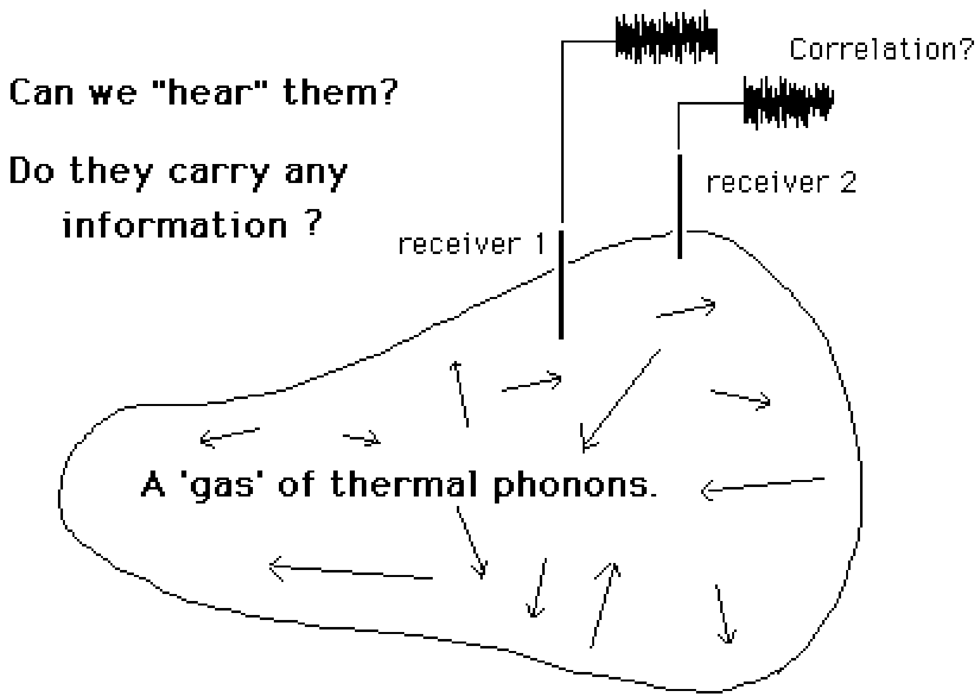
The chief caveat, and reason for $G \neq \partial C/\partial t$, is
The raw fields are insufficiently equipartitioned

Can be fixed by focusing on multiply scattered coda
available - if there is sufficient S/N -
at late times from distinct sources
or
at late times in ambient noise correlations

Even when $G \neq \partial C/\partial t$, C's are permitting good images.

The best diffuse field is that provided by thermal fluctuations of elastic waves

A gas of phonons as it were . . . .



The strength of a thermal ultrasonic field at MHz frequencies

1) Classical Thermal Fluctuation analysis tells us;

Each mode has small energy $kT \approx 4.2 \times 10^{-21}$ joules

For typical solids,

with mode counts below 1 MHz of ~ 300 modes / cm^3

We have energy densities of ~ 10^{-12} Joules / m^3

and rms strain amplitudes of ~ 3×10^{-12}

and rms displacement amplitudes of ~ 10^{-15} meter

< radius of electron!

2) How difficult is it to detect such weak signals?

We'll see . . . .

3) Why should we do so?

Answer:

They are perfectly diffuse,

and carry ultrasonic information

Comparison of a Direct Pulse-Echo Signal, and a Thermal Noise Correlation