

Beta ensembles

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The beta ensembles with pdf

$$p(x_1, \dots, x_n) = C_{n,\beta} \prod_{j < k} |x_j - x_k|^2 \cdot \prod_{k=1}^n e^{-x_k^2}$$

generalize the classical random matrix eigenvalues (that correspond to $\beta = 1, 2, 4$). Dumitriu and Edelman (1) discovered that p is the eigenvalue density of a tridiagonal random matrix with independent entries. There are several open questions regarding these and the group will begin by reading the papers referred to below. Some aspects/questions about beta-ensembles:

- Tridiagonal random matrix ensembles: As said above. Note that this also simplifies the problem of simulating from the pdf p .
- If we take a general potential V , i.e., consider the pdf

$$p(x_1, \dots, x_n) = C_{n,\beta} \prod_{j < k} |x_j - x_k|^2 \cdot \prod_{k=1}^n V(x_k).$$

Are there matrix models whose eigenvalue pdf is this? An alternate proof in the case $V(x) = x^2$ would be a good starting point.

- Bulk scaling results for general β (even for $V(x) = x^2$)? Following conjectures/heuristics due to Edelman and Sutton (2), recently Ramirez, Rider and Virág (3) have shown the correct

scaling *in the edge* of the spectrum and identified the analogues of Tracy-Widom distributions for general β in terms of the probability of explosions of certain diffusions.

- Going beyond existence of bulk scaling limit-how to describe the limiting distribution? For $\beta = 2$ we get a determinantal point process in the limit. Here it may be relevant to see the work of Deift and Gioev (?).
- What can be said about very large or very small β ?

References

- [1] Dumitriu, Ioana, Edelman, Alan. (2002). Matrix models for beta ensembles. *J. Math. Phys.* **43** (2002), no. 11, 5830–5847.
- [2] Edelman, Alan. and Sutton, Brian. From Random Matrices to Stochastic Operators arXiv:math-ph/0607038
- [3] Ramirez, J., Rider, B. and Virág, B. Beta ensembles, stochastic Airy spectrum, and a diffusion. arXiv:math.PR/0607331