A dream CLT for unitary matrices
(Diaconis+Evans, following earlier work by Diaconis+Shashahani, Wieland, also Johansson, Hugh, Keating, Oconnell).

λ_i- eigenvalues of random N-by-N Unitary matrix.

$f : \mathbb{T} \to \mathbb{R}$, $f(e^{i\theta}) = \sum \widehat{f}_j e^{ij\theta}$ $f_N = \sum_{i=1}^N f(\lambda_i)$

Theorem: a) If $V = \sum |j|\,|\widehat{f}_j|^2 < \infty$ then $(f_N - Ef_N)/\sqrt{V}$ converges in distribution to standard normal.

b) If $V_k = \sum_{-k}^k |j|\,|\widehat{f}_j|^2$ is slowly varying then $(f_N - Ef_N)/\sqrt{V_N}$ converges in distribution to standard normal.

Includes CLT's for smooth functions, for indicators of arcs, . . .

Would like:
Similar statement for GUE

Other ensembles? (e.g. β-ensembles, $\beta \in \mathbb{R}$)

Best available result so far: Lipschitz f (by concentration methods) (Anderson, Dumitriu, Z.)

Indicators of intervals/compact sets.

Maybe "dream" theorem can be obtained by determinantal methods in the GUE case? (Krishnapur)

Different comment: universality of spacing in the bulk: Johansson (for entries which are arbitrary r.v.+independent Gaussian.)