Some Research Directions of RMT in Physics

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Before I Start

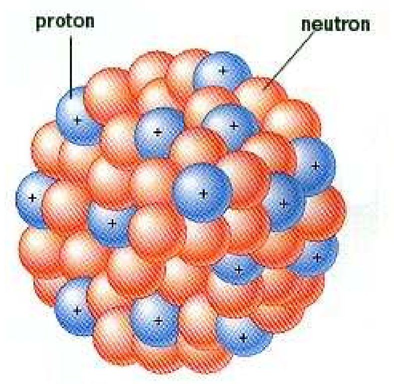
this is my personal list of particularly interesting topics

$\longrightarrow$ other physicists have their own list

I do not give references, I do not describe the history

$\longrightarrow$ ask me if you need more information

Embedded Random Matrices



system with A particles

A particle states, $n = 1, \ldots, N$

assume l body interaction

l particle states, $\alpha = 1, \ldots, N_G$

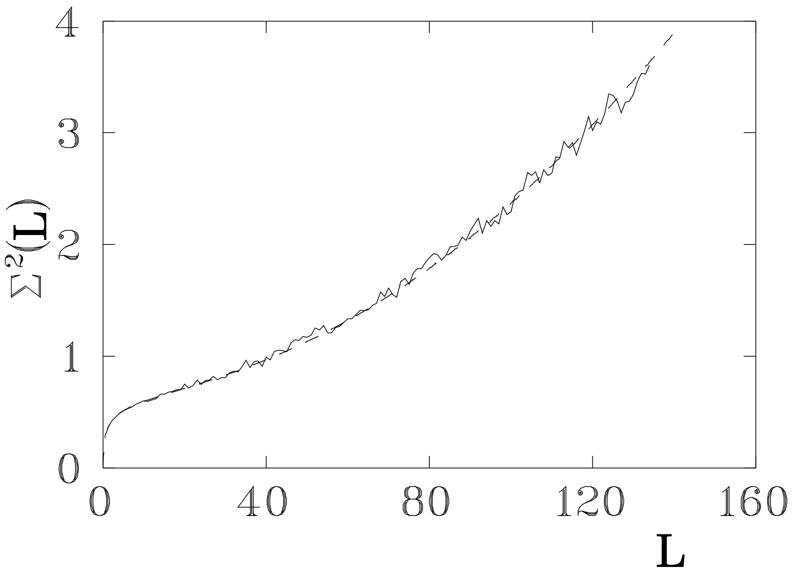
creation and annihilation operator $c_\alpha^\dagger$ and c_α

l body Hamiltonian: $$\mathcal{H} = \sum_{\alpha,\beta} V_{\alpha\beta} c_\alpha c_\beta^\dagger$$

assume Gaussian distributed $V_{\alpha\beta}$

statistical properties of H with entries $H_{nm} = \langle n|\mathcal{H}|m\rangle$???

Ensembles with “External Field”



non–invariant model

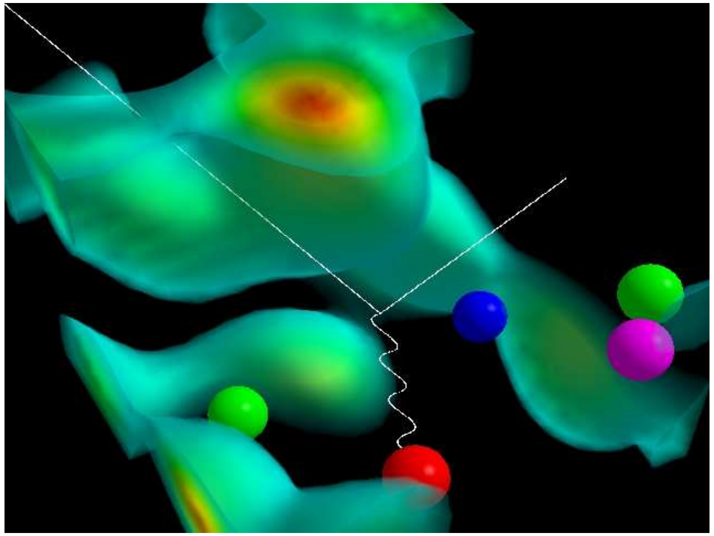
$$H \;=\; H_0 + \alpha H_{\mathrm{RMT}}$$

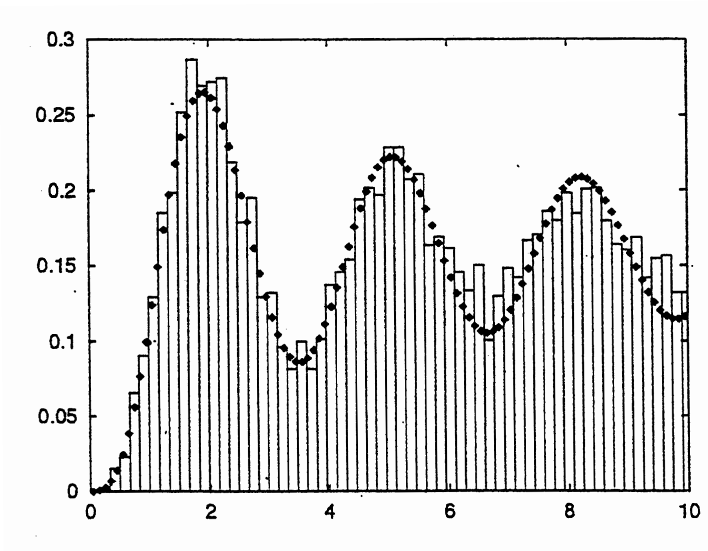
leads always to group integrals: $\displaystyle\int d\mu(U)\exp(i\,\mathrm{tr}\,xU^{-1}kU)$

$\mathrm{U}(N)$ ok, but $\mathrm{O}(N)$ and $\mathrm{USp}(2N)$ big problem

Harish–Chandra’s versus Gelfand’s spherical functions

Quantum Chromo Dynamics — Chiral RMT





Dirac operator describes motion of quarks in gauge fields, has chiral symmetry

$$i\not{D} = \begin{bmatrix} 0 & i\not{D}^c \\ (i\not{D}^c)^\dagger & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & W \\ W^\dagger & 0 \end{bmatrix},$$

W random matrix

Supersymmetric Representation of RMT

Gaussian ensemble ($\beta = 1, 2, 4$) of $N \times N$ random matrices H

k–level correlations $$R_k^{(\beta)}(x_1, \ldots, x_k) = \frac{\partial^k}{\prod_{p=1}^k \partial J_p} Z_k^{(\beta)}(x + J)\Big|_{J=0}$$

generating function obeys the identity (yes, this is exact!)

$$Z_k^{(\beta)}(x + J) = \int d[H] \exp(-\mathrm{tr}\, H^2) \prod_{p=1}^{k} \frac{\det(H - x_p - J_p)}{\det(H - x_p + J_p)}$$

$$= \int d[\sigma] \exp(-\mathrm{trg}\, \sigma^2) \mathrm{detg}^{-N}(\sigma - x - J)$$

where σ is a $2k \times 2k$ or $4k \times 4k$ supermatrix

$\longrightarrow$ drastic reduction of dimensions

Built–in Structures of Supersymmetry

Hermitean $N \times N$ ordinary matrix $H = UxU^\dagger$

$$d[H] = \Delta_N^2(x) d[x] d\mu(U) \quad \text{where} \quad \Delta_N(x) = \prod_{n<m} (x_n - x_m)$$

$\longrightarrow$ level repulsion

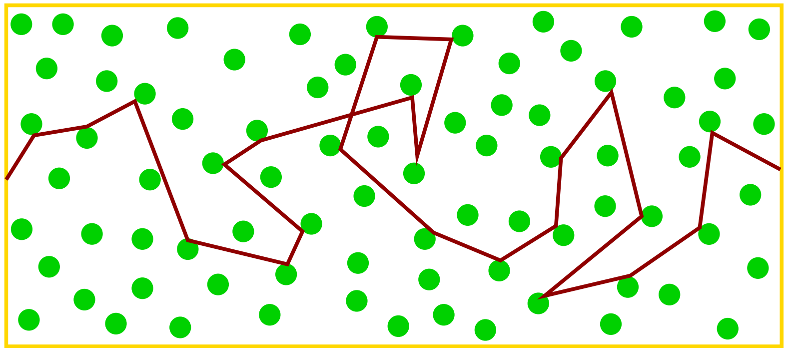
Hermitean $2k \times 2k$ supermatrix $\sigma = usu^\dagger$

$$d[\sigma] = B_k^2(s) d[s] d\mu(u) \quad \text{where} \quad B_k(s) = \det\left[\frac{1}{s_{p1} - is_{q2}}\right]_{p,q=1,\ldots,k}$$

$\longrightarrow$ determinantal processes, double integral form of kernels

Supersymmetry and Disordered Systems

electron moves diffusively in a probe with scatterers



d dimensions

random disorder potential

$$\langle V(\vec{r})V(\vec{r}\,')\rangle \sim \delta^{(d)}(\vec{r}-\vec{r}\,')$$

field theory: supersymmetric non–linear σ model with action

$$S[Q] = \int d^d r \left(\mathcal{D}\partial_i Q(\vec{r})\partial_i Q(\vec{r}) - i\omega\Lambda Q(\vec{r}) \right)$$

where $Q = Q(\vec{r})$ is a supermatrix field in a coset space

Mathematical Problems in Supersymmetry

mathematicians have so far focused on superalgebras

this is because of supersymmetry in high–energy physics

reason for supersymmetry in RMT is completely different

$\longrightarrow$ different questions

more theory of supergroups is needed,

particularly harmonic analysis and representation theory

in this context also Rothstein boundary terms, etc

Recovery of the Replica Trick

in many statistical mechanics systems, one has to average the free energy $\ln Z$ where Z is the partition function

it is extremely hard to average $\ln Z$, but easy to average $Z^n, \ n = 1, 2, 3, \ldots$ (n replicas), one tries the trick

$$\langle \ln Z \rangle \ = \ \lim_{n \to 0} \frac{\langle Z^n \rangle - 1}{n}$$

mathematically ill–defined, because n is integer

in principle, this is a powerful alternative to supersymmetry

one thought that replica trick is bound to fail in RMT !

recently, one made it work in RMT and retrieved exact results !!!