

Statistical Issues in Engineering Contexts

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Problem 1: Optimization

- Output(s) $y_M(x, w)$
- Inputs: x **controllable**, w **uncontrollable**
manufacturing variations with d.f. F
- What to optimize:
Mean? Variance? Mean + 3 sigma?
Mean/sigma?

- **Multiple criteria**
 - Optimize y_1 subject to constraint on y_2 ?
- **Heterogeneous space**
 - Cut down/up to homogeneous piece(s)?
- **Sequential design in stages**
 - One-at-a-time experiments infeasible

Aslett, Buck, Duvall, Sacks, Welch (1998). “Circuit optimization via sequential computer experiments.”
Appl Statist

- Electronic Circuit Simulation of an Output-Buffer Circuit
- Several important outputs (among many):
 - 2 primary delay times TL, TH
 - 4 voltage spikes VSL,VSH,VCL,VCH
- 36 inputs:
 - 20 controllable transistor widths
 - 16 uncontrollable manufacturing process variables (multivariate normally distributed)

Objective

Optimize the controllable inputs so that all outputs are small, over the distribution of the uncontrollable inputs -- **Taguchi robust parameter design**

Objective (cont'd)

Formally

- \mathbf{x} = vector of controllable inputs
- \mathbf{w} = vector of uncontrollable inputs with d.f F
- Constraints:
 - (cTL) $TL \leq t_{max}$
 - (cTH) $TH \leq t_{max}$
- Objective:
 - $\min_{\mathbf{x}} \max \{VSL, VSH, VCL, VCH\}$
subject to (cTL), (cTH)
 - do for multiple choices of t_{max}

Objective (cont'd)

This does not take account for variability in \mathbf{w} . To do so:

- Define $TL^+(\mathbf{x}) = E(TL(\mathbf{x}, \bullet)) + 3 \text{ sd}(TL(\mathbf{x}, \bullet))$ and define $TH^+(\mathbf{x}), VSL^+(\mathbf{x}), \dots$ similarly
- Constraints:
 - (c TL⁺): $TL^+ \leq t_{max}$
 - (c TH⁺): $TH^+ \leq t_{max}$
- Objective:
 - $\min_x \max \{VSL^+, VSH^+, VCL^+, VCH^+\}$
subject to (cTL⁺), (cTH⁺)

Strategy

- Replace outputs by approximations (GaSPs) from computer experiments

{Note: Evaluating TL^+ ...will require evaluating integrals of TL, \dots }

- Optimizing by an approximation can be unstable unless approximation is “tight”
- Unlikely that a one-stage experiment is effective
- Initial experimental stage(s) leads to both dimension reduction and narrowing the ranges of inputs enabling better approximation

Issues

- Ad hoc strategies – can they be made “coherent”
- Prescription for cutting down (or up)?

Problem 2: Calibration/Validation

- Engineering system with components subject to manufacturing variation

$$y_F(x_{nom} + \delta_{man}; t) = y_M(x_{nom} + \delta_{man}, u_{calib}; t) + b(x_{nom}; t) + \varepsilon(t)$$

- **Variations δ** – many unknowns
- **Replicates** – (same δ) how much help?

Example: Automobile Suspension System

Inputs and Data

- Two u 's; seven x 's; δ 's unknown and, within a single vehicle, have the same impact as a calibration parameter
- Field Data: 7 runs of the same vehicle over a course (therefore, all at the same calibration and unmeasured variations)
- If k vehicles there would be $7k$ δ 's to worry about
- Model data: 65 runs