Explaining Exercise Patterns for Executive Stock Options

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Introduction I

- Executive stock options are options granted to executives and employees of a company as part of a compensation package
- In 1996, 39% of compensation of CEO's in the S&P500 was options (Murphy (1999))
- By 1999, this was 47%, and 94% of S&P500 companies granted options to executives (Hall and Murphy (2002))
- FASB (2004) and IASB 2 (2004) require companies to expense options

- It is observed empirically that executives:

(i) exercise stock options much earlier than expiry

Huddart and Lang (1996) find median fraction of option life elapsed upon exercise varied from 0.21 to 0.92. Carpenter (1998) reports an average time to exercise of 5.83 years for a sample of 10 year options

(ii) exercise stock options in a few large transactions or blocks

Huddart and Lang (1996) find median fraction of options exercised at one time varied from 0.13 to 1

- We will give a model of exercise behavior which is consistent with these features

Model Assumptions I

- Executive receives n call options, each with common strike K
- Options can be exercised at any time over infinite horizon
- Options are already vested or have no vesting period
- Company stock price V follows

$$\frac{dV}{V} = \nu dt + \eta dW$$

- Zero dividends, zero interest rates

Model Assumptions II

• Executive cannot trade the company stock V so cannot hedge options - incomplete market

• Executive also chooses holdings in a risky asset P where

$$\frac{dP}{P} = \mu dt + \sigma dB$$

and $dBdW = \rho dt$. Let $\lambda = \mu/\sigma$.

• Executive is risk averse with exponential utility

• Executive chooses exercise dates $\tau^n \leq ... \leq \tau^1$ where τ^j is exercise time when j options remain unexercised

• Upon exercising at τ^j, executive receives $(V_{\tau^j} - K)^+$

Literature

• Huddart (1994), Lambert et al (1991), Hall and Murphy (2002)...
• Detemple and Sundaresan (1999), Henderson (2005), Ingersoll (2006)...

• Binomial models of inter-temporal exercise: Jain and Subramanian (2004), Grasselli (2005)

The Executive's Problem: Optimal Exercise Policy and Optimal Portfolio Choice

- At time u, θ_u denotes the cash amount invested in P
- The executive's wealth at time s is given by

$$X_s = X_0 + \int_0^s \theta_u \frac{dP}{P} + \sum_{\tau_i \le s} (V_{\tau_i} - K)^+$$

- At time t, and with i options remaining, the executive solves

$$H^i(t,x,v) = \sup_{t \le \tau^i \le \ldots \le \tau^1} \sup_{(\theta_s)_{t \le s < \tau^1}} \mathbb{E}_t \left[\tilde{U}\left(\tau^1, X_{\tau^1}\right) \Big| X_t = x, V_t = v \right]$$

where

$$\tilde{U}(\tau, x) = -\frac{1}{\gamma} e^{-\gamma x} e^{\frac{1}{2}\lambda^2 \tau}$$

is *time consistent* exponential utility

We want $H^n(0,x,v)$

Time Consistent Utilities: Henderson (2004), Henderson and Hobson (2006)

• We are measuring utility at τ^1 but need to adjust for the fact that from τ^1 onward, cash can be invested into the optimal Merton portfolio resulting from risky asset P and bank account

• The term $e^{\frac{1}{2}\lambda^2\tau}$ is exactly that needed to compensate for opportunity cost of not exercising

• We want to avoid artificial incentives to exercise/wait based on the set-up of the portfolio choice problem

• Under the choice $\tilde{U}(\tau, x)$, the solution to

$$\sup_{(\theta_u);t\leq u\leq\tau} \mathbb{E}[\tilde{U}(\tau, X_\tau)|X_t = x]$$

where $dX = \theta dP/P$, does not depend on horizon τ

Let $\beta_\rho = 1 - \frac{2(\nu - \mu\rho\eta/\sigma)}{\eta^2}$ and suppose $\beta_\rho > 0$.
Define $\Gamma^1 = 0$ and for $j = 2, ..., n$

$$\Gamma^j = \left(\frac{1}{\tilde{V}^{j-1}}\right)^{\beta_\rho} \left(1 - e^{-\gamma(1-\rho^2)(\tilde{V}^{j-1}-K)^+}(1 - \Gamma^{j-1}(\tilde{V}^{j-1})^{\beta_\rho})\right)$$

and $\tilde{V}^j$ is the solution to

$$C_{\gamma(1-\rho^2),\beta_\rho,K,\Gamma^j}(\tilde{V}^j) = 0; \quad j = 1, ..., n$$

where

$$C_{g,\xi,\kappa,G}(x) = x - \kappa - \frac{1}{g}\ln\left[1 + \frac{g}{\xi}(1 - Gx^\xi)x\right]$$

Proposition 1 *If* $\gamma(1-\rho^2) > 0$, *the constants* $\tilde{V}^j, \quad j = 1, ..., n$ *satisfy*

$$\tilde{V}^n < \tilde{V}^{n-1} ... < \tilde{V}^1$$

Proposition 2 *The exercise times* $\tau^n \leq ... \leq \tau^1$ *are the first passage times of stock price* V *to constant thresholds* $\tilde{V}^j; j = 1, .., n$*:*

$$\tau^j = \inf\{t : V_t \geq \tilde{V}^j\}; \quad j = 1, ..., n$$

Case 1: *Suppose* $\beta_\rho > 0$*. The thresholds* $\tilde{V}^j; \quad j = 1, ..., n$ *are as stated and for* $j = 1, ..., n$, $H^j(0, x, v) = G^j(x, v) =$

$$-\frac{1}{\gamma} e^{-\gamma x} \left[1 - \left(1 - e^{-\gamma(1-\rho^2)(\tilde{V}^j - K)^+} (1 - \Gamma^j (\tilde{V}^j)^{\beta_\rho}) \right) \left(\frac{v}{\tilde{V}^j} \right)^{\beta_\rho} \right]^{\frac{1}{1-\rho^2}}$$

Case 2: *If* $\beta_\rho \leq 0$, $\tilde{V}^j = \infty$ *for* $j = 1, ..., n$ *and the executive waits indefinitely.*

Corollary 3 Perfect Hedging Case. *Assume* $\rho^2 = 1$. *Let* $\beta_1 = 1 - \frac{2(\nu - \mu\eta/\sigma)}{\eta^2}$.

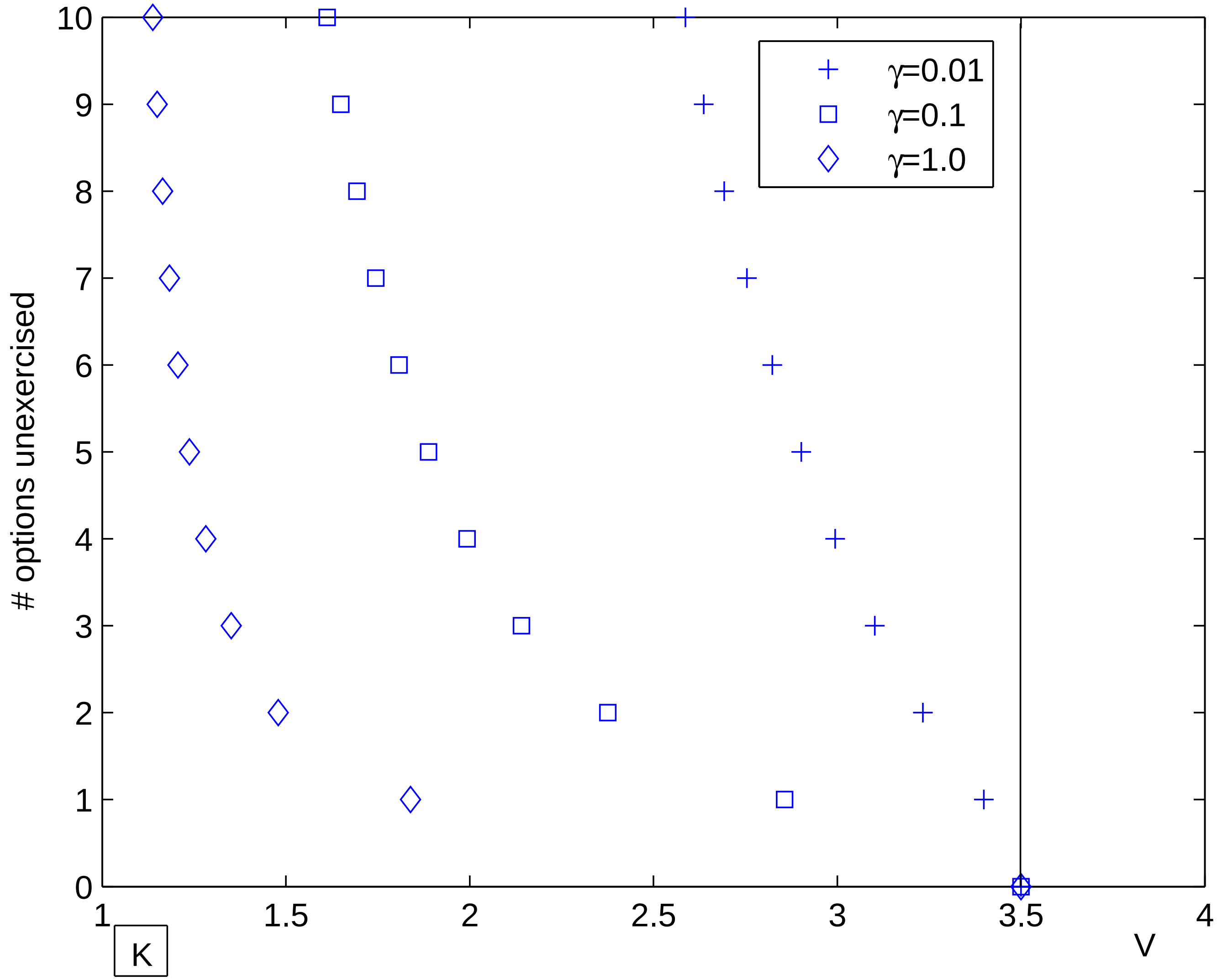
Case 1: *If* $\beta_1 > 1$, *the constant exercise thresholds satisfy*

$$\tilde{V}^n = ... = \tilde{V}^1 = \tilde{V} = \left(\frac{\beta_1}{\beta_1 - 1}\right) K$$

and the value of the n options at time 0 is given by

$$G^n(x, v) = x + n(\tilde{V} - K)\left(\frac{v}{\tilde{V}}\right)^{\beta_1}$$

Case 2: *If* $\beta_1 \leq 1$, $\tilde{V}^n = ... = \tilde{V}^1 = \tilde{V} = \infty$ *and the executive waits indefinitely.*



10
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γ=0.01
γ=0.1
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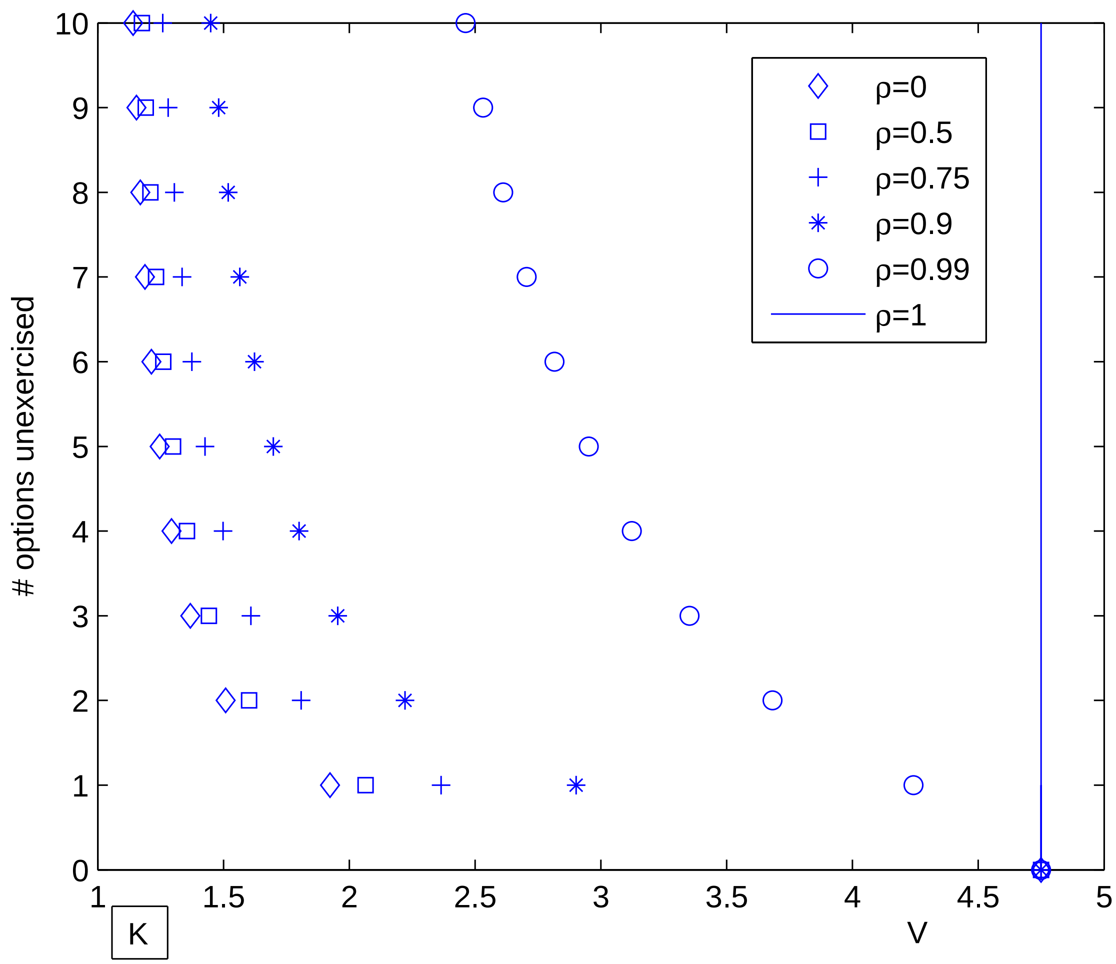
a



ρ=0
ρ=0.5
ρ=0.75
ρ=0.9
ρ=0.99
ρ=1
options unexercised
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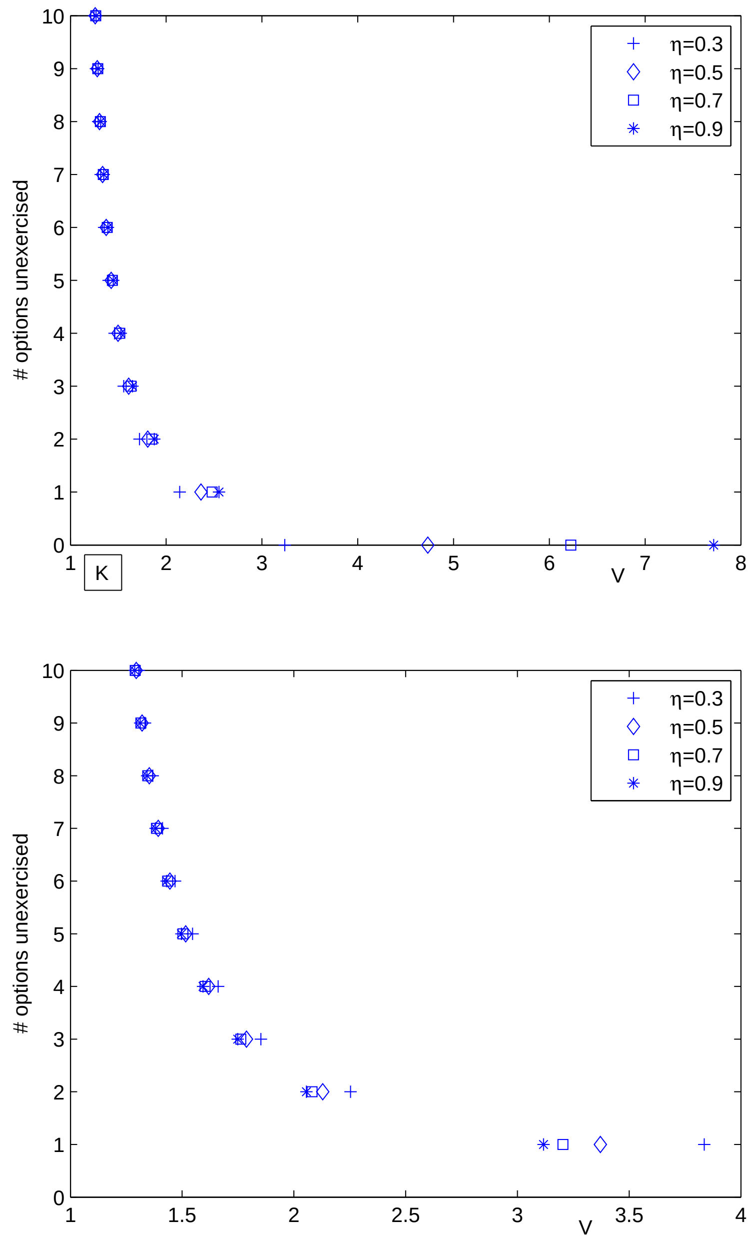
b



options unexercised
η=0.3
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options unexercised
η=0.3
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η=0.9
V

C

Impact of Stock Volatility

- Stock volatility can increase or decrease exercise threshold. Arises from trade-off between two effects: (i) convexity and (ii) risk aversion (higher idiosyncratic risk)
- By taking $n > 1$ options, show all thresholds move together with volatility; and
- more pronounced effect when fewer options remaining
- Bettis et al (2005), Hemmer et al (1996) and Huddart and Lang (1996) all find empirically that executives exercise earlier the greater the volatility of stock price

Restricted Exercise

• What if the executive is forced to exercise all $i \leq n$ options at a single time of his choosing ?

• At an intermediate time t, and with i options remaining, the executive's optimization problem is to find

$$H_r^i(t, x, v) = \sup_{\tau_r^i} \sup_{\theta} \mathbb{E}_t \left[\tilde{U} \left(\tau_r^i, X_{\tau_r^i -} + i(V_{\tau_r^i} - K)^+ \right) \middle| X_t = x, V_t = v \right]$$

We want $H_r^i(0, x, v)$.

Proposition 4 *The restricted exercise time of i options, τ_r^i, is characterized as the first passage time of V to constant threshold $\tilde{V}_r^i$ such that*

$$\tau_r^i = \inf\{t : V_t \geq \tilde{V}_r^i\}$$

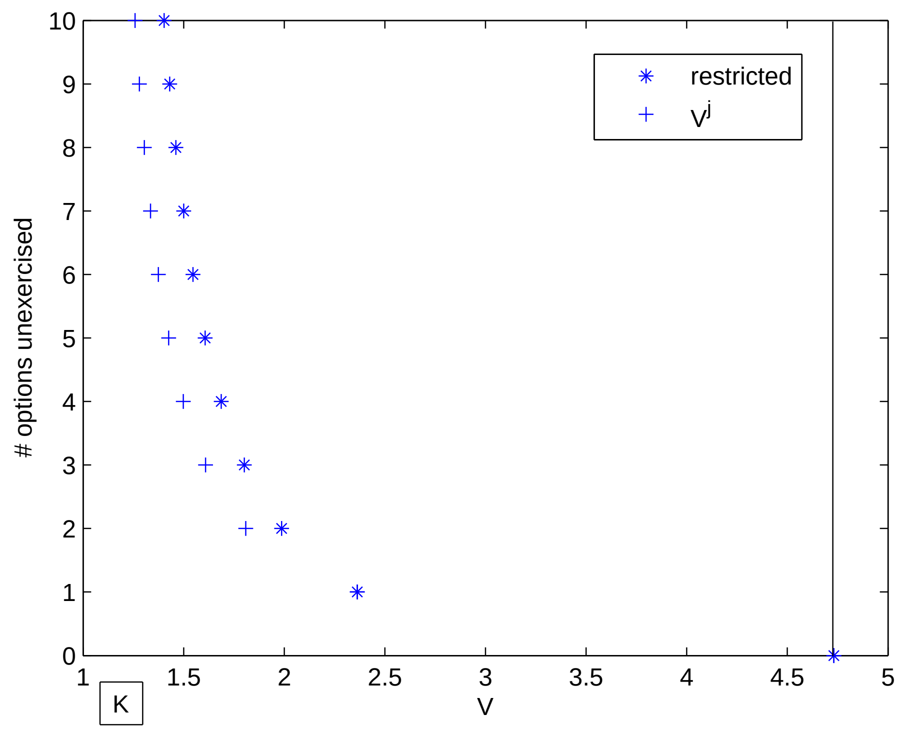
Case 1: *If $\beta_\rho > 0$, $\tilde{V}_r^i$ solves*

$$C_{i\gamma(1-\rho^2),\beta_\rho,K,0}(\tilde{V}_r^i) = 0$$

and

$$G_r^i(x,v) = -\frac{1}{\gamma}e^{-\gamma x}\left[1 - \left(1 - e^{-i\gamma(1-\rho^2)(\tilde{V}_r^i - K)^+}\right)\left(\frac{v}{\tilde{V}_r^i}\right)^{\beta_\rho}\right]^{\frac{1}{1-\rho^2}}$$

Case 2: *If $\beta_\rho \leq 0$, $\tilde{V}_r^i = \infty$ and the executive waits indefinitely*



restricted
V^j
options unexercised
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d

Costly Exercise

• The executive expends effort to exercise: he must inform his company or broker to transact, he may spend time researching before making a decision, ...

• We represent this by a cost c which is lost each time the executive exercises options

• Exercise strategy $q = (q_1, ..., q_k), \quad 1 \leq k \leq n$ where $\sum_{j=1}^{k} q_j = n$ and $q_j \geq 1; 1 \leq j \leq k$. The size of the jth block exercised is q_j and the strategy q represents the sequence of block sizes.

• Exercise strategies q are associated with a set of thresholds $\tilde{V}_c^q$. In general, $q_j; \quad j = 1, ..., k$ options are exercised at threshold $\tilde{V}_c^{q_j, q_{j+1}, \ldots, q_k}$. Eg. If $n = 3$ options, we may have $q = (1, 1, 1)$ with associated thresholds $\tilde{V}_c^{1,1,1}$ at which the first of three options is exercised, $\tilde{V}_c^{1,1}$ when the second is exercised and $\tilde{V}_c^{1}$ where the final option is exercised. Other strategies are $q = (1, 2)$, $q = (2, 1)$ or $q = (3)$.

• Denote $q = (q_1, ..., q_k) = (q_1, p) = (l, p)$ where $p = ((q_2, ..., q_k))$

• Solve for value at time zero under any strategy q, $G^q(x, v)$, and define optimal exercise strategy to be that q which maximizes $G^q(x, v)$

• Consider $k = 1$. Cost c paid when $q_1 = l$ options exercised at some τ_c^1, reducing effective payoff to $l(V_{\tau_c^1} - (K + c/l))^+$. ie. the per-option strike is increased from K to $K + c/l$

• Apply results of restricted exercise with modified strike

Proposition 5 Costly Exercise *The k exercise times $\tau_c^k \leq ... \leq \tau_c^1$ are characterized as the first passage times of V to constant thresholds $\tilde{V}_c^q$.*

Case 1: *If $\beta_\rho > 0$, $\tilde{V}_c^q$ solve*

$$C_{l\gamma(1-\rho^2),\beta_\rho,K+c/l,\Xi^p}(\tilde{V}_c^q) = 0$$

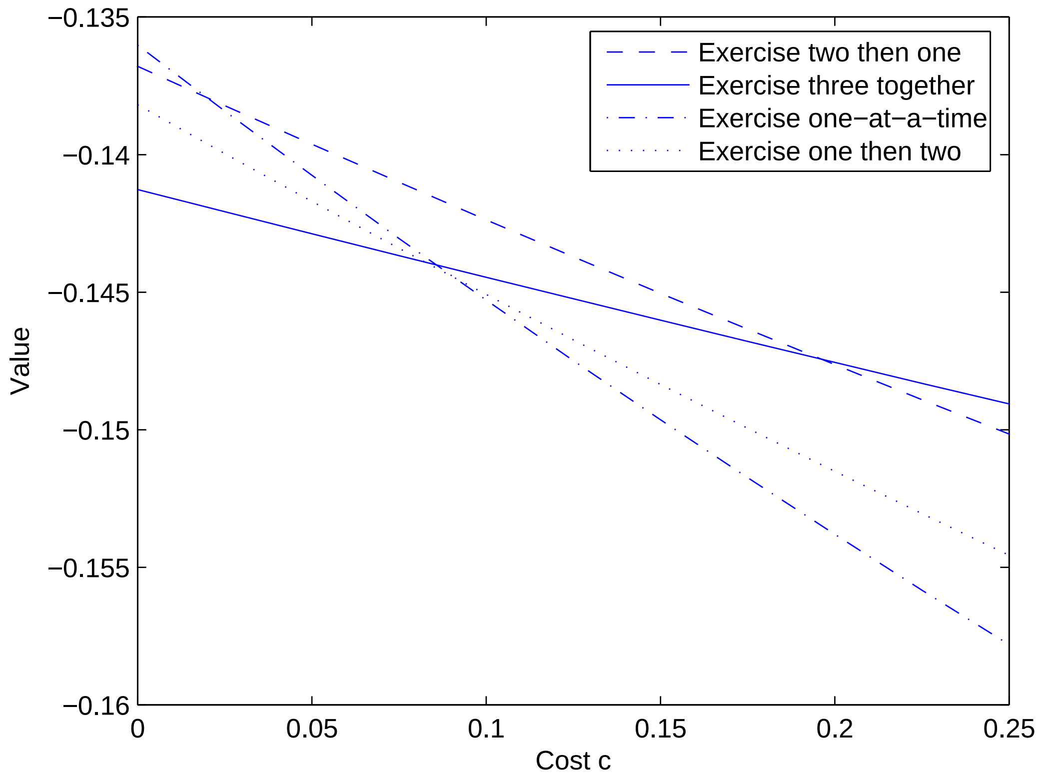
where constants Ξ^q are given by $\Xi^0 = 0$ and

$$\Xi^q \equiv \Xi^{l,p} = \left(\frac{1}{\tilde{V}_c^q}\right)^{\beta_\rho} (1 - e^{-l\gamma(1-\rho^2)(\tilde{V}_c^q-(K+c/l))^+}(1 - \Xi^p(\tilde{V}_c^q)^{\beta_\rho}))$$

The value to the executive at time zero, $G_c^q(x, v)$, given follows strategy q is

$$-\frac{1}{\gamma}e^{-\gamma x}\left[1 - (1 - e^{-l\gamma(1-\rho^2)(\tilde{V}_c^q-(K+c/l))^+}(1 - \Xi^p(\tilde{V}_c^q)^{\beta_\rho}))\left(\frac{v}{\tilde{V}_c^q}\right)^{\beta_\rho}\right]^{\frac{1}{1-\rho^2}}$$

Case 2: *If $\beta_\rho \leq 0$, $\tilde{V}_c^q = \infty$ and the executive waits indefinitely.*



−0.135
−0.14
−0.145
−0.15
−0.155
−0.16
Value
0
0.05
0.1
0.15
0.2
0.25
Cost c
Exercise two then one
Exercise three together
Exercise one−at−a−time
Exercise one then two

e

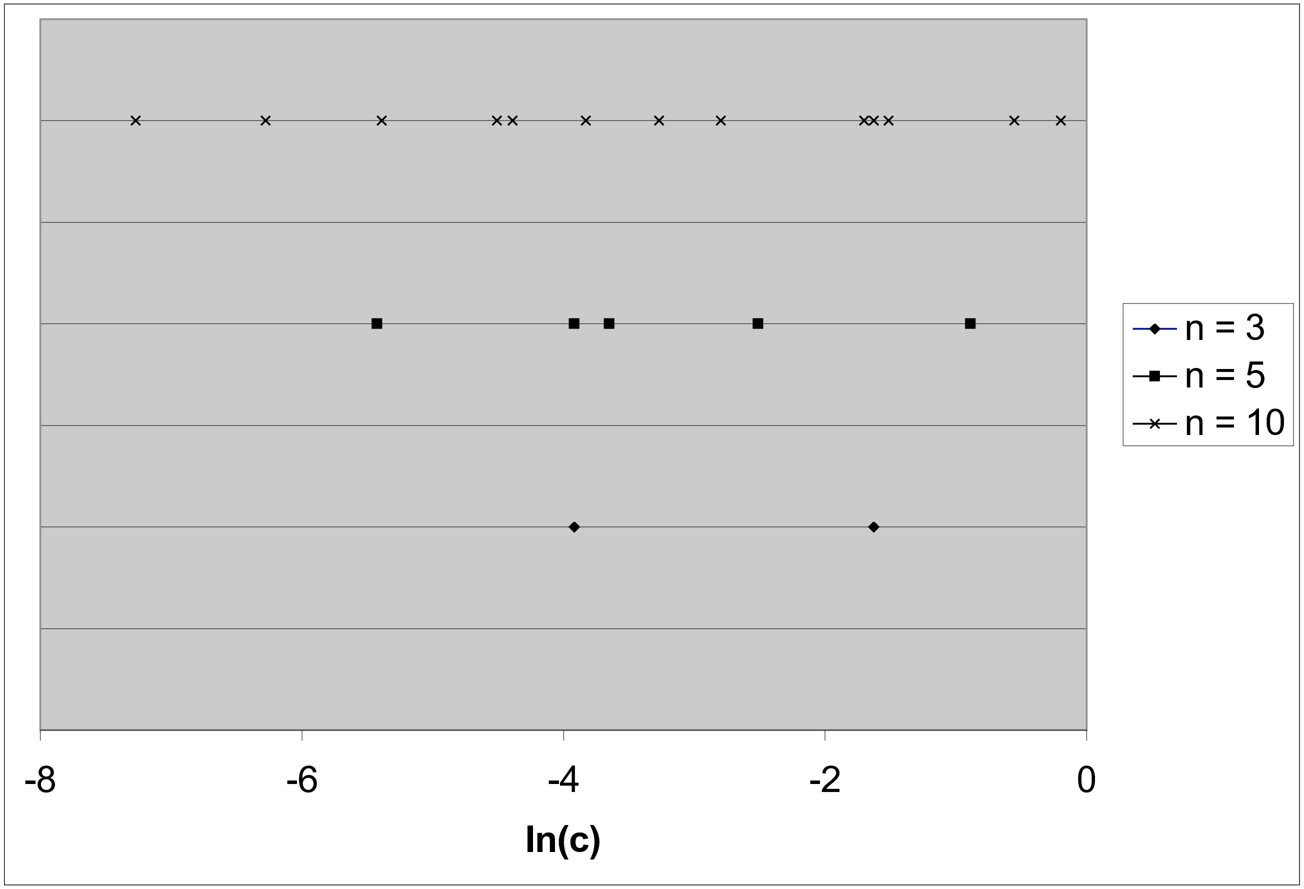


Figure 1: As $\ln(c)$ increases : for $n = 3$, switch from $q = (1, 1, 1)$ to $q = (2, 1)$ to $q = (3)$; for $n = 5$, switch from $q = (1, 1, 1, 1, 1)$, $q = (2, 1, 1, 1)$, $q = (2, 2, 1)$, $q = (3, 1, 1)$, $q = (4, 1)$ to $q = (5)$; for $n = 10$, switch from $q = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$, $q = (2, 1, 1, 1, 1, 1, 1, 1, 1)$, $q = (2, 2, 2, 1, 1, 1, 1)$, $q = (3, 2, 2, 1, 1, 1)$, $q = (3, 3, 2, 1, 1)$, $q = (4, 2, 2, 1, 1)$, $q = (4, 3, 2, 1)$, $q = (5, 3, 1, 1)$, $q = (6, 3, 1)$, $q = (7, 2, 1)$, $q = (7, 3)$, $q = (8, 2)$, $q = (9, 1)$, to $q = (10)$.

Observations

• Executives exercise in blocks once c is large enough - trade-off between risk aversion and costly exercise
Consistent with empirical evidence: Huddart and Lang (1996) find median fraction exercised varied between 0.13 to 1

• Number of blocks decreases with c

• Largest block size increases with c

• Block size decreases across series of exercise dates
Testable implication of the model

Cost to Shareholders

• Shareholders are risk neutral. Cost is risk neutral value of options, given optimal exercise behavior of executive

Proposition 6 *Assuming risk averse executives granted n options with strike K exercise according to the costly exercise model of Proposition 5, the cost to shareholders is*

$$\sum_{j=1}^{k} q_j \mathbb{E}^{\mathbb{Q}}(V_{\tau_c^j} - K)^+ = \sum_{j=1}^{k} q_j (\tilde{V}^{q_j,q_{j+1},\ldots,q_k} - K)\left(\frac{v}{\tilde{V}^{q_j,q_{j+1},\ldots,q_k}}\right)$$

where under $\mathbb{Q}$, $dV/V = \eta dW^{\mathbb{Q}}$.

Implications

• Recent models of Hull and White (2004), Cvitanic et al (2005) assume executives exercise at some single exogenous threshold level to obtain a tractable cost calculation

• In contrast, in our model, both the number of thresholds and their level will depend on characteristics of executives (γ, c, hedging capabilities) and the company stock

• What if we restricted executive to exercise at a single threshold ? Cost is typically *underestimated* versus optimal exercise. Reverse can occur for low volatility.

Conclusions and Further Extensions

- Closed form model for exercise behavior of risk averse executives subject to costly exercise
- Risk aversion causes executives to exercise options one-at-a-time at an increasing set of stock price thresholds, all lower than the single complete market threshold
- Risk aversion together with costly exercise results in block exercise behavior
- Including vesting would further emphasize block exercise
- Implications for cost to shareholders