

Credit Ratings

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Abstract

The systems of credit ratings maintained by agencies such as Moody's and Standard & Poor's play an important role in evaluating credit risk. The agencies' historical data are valuable in interpreting the rating of a given obligor. Two issues that arise in using those historical data are:

- how to describe the dependence among rating changes for several obligors;
- how to relate those changes to the broader economy.

Both issues can be addressed by using an appropriate copula.

Standard & Poor's has published an analysis of historical data on cumulative default rates and rating transitions for the years 1981–2002.

Summaries include:

- Cumulative Average Default Rates;
- Average One-Year Transition Rates.

Cumulative Average Default Rates (%)

Jan. 1 rating	Years after static pool formation														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AAA	0.00	0.00	0.03	0.06	0.10	0.17	0.25	0.38	0.43	0.48	0.48	0.48	0.48	0.56	0.67
AA	0.01	0.03	0.08	0.16	0.27	0.39	0.53	0.65	0.75	0.85	0.95	1.06	1.15	1.22	1.30
A	0.05	0.15	0.28	0.44	0.62	0.81	1.03	1.25	1.52	1.82	2.06	2.26	2.43	2.61	2.88
BBB	0.37	0.94	1.52	2.34	3.20	4.02	4.74	5.40	5.99	6.68	7.40	7.97	8.55	9.10	9.77
BB	1.38	4.07	7.16	9.96	12.34	14.65	16.46	18.02	19.60	20.82	21.98	22.79	23.58	23.99	24.51
B	6.20	13.27	19.07	23.45	26.59	29.08	31.41	33.27	34.58	35.87	36.98	37.97	38.95	39.96	41.09
CCC	27.87	36.02	41.79	46.26	50.46	52.17	53.60	54.36	56.16	57.21	58.15	58.95	59.59	60.70	60.70
Invest. grade	0.13	0.34	0.57	0.87	1.20	1.52	1.83	2.13	2.41	2.72	3.02	3.26	3.50	3.73	4.03
Spec. grade	5.17	10.27	14.81	18.46	21.31	23.67	25.71	27.36	28.83	30.07	31.20	32.09	32.97	33.72	34.52
All ratings	1.67	3.36	4.86	6.12	7.14	8.02	8.80	9.47	10.07	10.64	11.17	11.60	12.02	12.40	12.85

Average One-Year Transition Rates (%)

Rating	Rating at year-end								
	AAA	AA	A	BBB	BB	B	CCC	D	N.R.
AAA	89.37	6.04	0.44	0.14	0.05	0.00	0.00	0.00	3.97
AA	0.57	87.76	7.30	0.59	0.06	0.11	0.02	0.01	3.58
A	0.05	2.01	87.62	5.37	0.45	0.18	0.04	0.05	4.22
BBB	0.03	0.21	4.15	84.44	4.39	0.89	0.26	0.37	5.26
BB	0.03	0.08	0.40	5.50	76.44	7.14	1.11	1.38	7.92
B	0.00	0.07	0.26	0.36	4.74	74.12	4.37	6.20	9.87
CCC	0.09	0.00	0.28	0.56	1.39	8.80	49.72	27.87	11.30

An obligor's credit rating gives useful information about the probability that it will default on its obligations, based on the historical frequencies for that rating.

Other information, such as *rating momentum* upward or downward is also useful in predicting rating transitions, but is often ignored.

These historical summaries give a quantitative interpretation to a rating. They are also used to measure risk, such as in testing the adequacy of capital.

Moody's has described a Capital Model intended as a benchmark for capital modeling in structured finance, such as for DPCs or SIVs. It provides a complete Monte Carlo simulation of the future of a vehicle.

The results include estimates of the probability of default and the expected losses due to default, which measure the robustness of the vehicle, and can be used to justify a desired rating such as triple-A.

Part of the process is simulating credit rating migrations. Such simulations begin with a Markov chain defined by a matrix of transition probabilities (one-year or one-month, or an infinitesimal generator).

Ratings for multiple obligors cannot be simulated independently—that fails to capture the risk of a cluster of defaults. Early models used independent chains, but based on a *stressed* transition matrix to compensate.

The stressing was deliberately conservative, leading to overstatement of both risk and capital. More realistic simulation with dependent transitions can be less conservative.

A *Gaussian copula* can be used to introduce dependence.

Consider n obligors. Let $r_{i,t}$ be the rating of the i th obligor at time t . We want $P(r_{i,t+1} = r' | r_{i,t} = r)$ to be, on average, some target probability. We:

- break the unit interval into subintervals with lengths equal to the target probabilities;
- construct a random variable $U_{i,t}$ on that interval;
- assign the obligor's new rating according to the subinterval that $U_{i,t}$ falls in.

If $U_{i,t}$ is marginally uniformly distributed, then the goal is met.

To make the U s dependent, they are drawn from a *copula*. The simplest is the Gaussian copula:

$$U_{i,t} = \Phi(Z_{i,t}),$$

where the Z s are correlated standard Gaussian deviates. Now

$$\begin{aligned} P(r_{i,t+1} \leq r' | r_{i,t} = r) &= P(Z_{i,t} \leq \zeta_{r,r'}) \\ &= \Phi(\zeta_{r,r'}). \end{aligned}$$

for appropriate cut-points $\zeta_{r,r'}$.

Moody's uses

$$Z_{i,t} = \lambda_1 Z_{G,t} + \lambda_2 Z_{R_i,t} + \lambda_3 Z_{I_i,t} + \lambda_4 Z_{F,i,t}$$

where $\sum \lambda_j^2 = 1$.

Here:

- $Z_{G,t}$ is global;
- $Z_{R_i,t}$ is for the obligor's region, R_i ;
- $Z_{I_i,t}$ is for the obligor's industry, I_i ;
- $Z_{F,i,t}$ is firm-specific;

and these factors are all independent $N(0, 1)$.

The λ s are chosen to give specified *asset value correlations*, which are in turn extracted from rating histories.

Moody's model produces transitions with complex dependence at each time t . All transitions at one time step are still independent of transitions at any other time.

With a one-month time step, the model will not give extended periods of a year or more with higher downgrade rates. It also shows no rating momentum.

It could be extended by adding correlation over time.

Recall:

$$Z_{i,t} = \lambda_1 Z_{G,t} + \lambda_2 Z_{R_i,t} + \lambda_3 Z_{I_i,t} + \lambda_4 Z_{F,i,t}$$

If the firm-specific factor $Z_{F,i,t}$ is positively serially correlated, a large value of either sign is likely to be followed by another of the same sign, so rating changes, in the same direction, are clustered in time: rating momentum.

If the global factor $Z_{G,t}$ is serially correlated and cyclic, clusters of migrations in the same direction are likely to occur in corresponding cycles: the credit cycle.

The added realism should further reduce the need for conservative approximations such as stressing.

The model's credit cycle would still not be linked to the business cycle. Other parts of Moody's Capital Model produce:

- interest rates;
- exchange rates;
- credit spreads;
- recovery rates.

All or part of the regional factor $Z_{R_i,t}$ could be replaced with something driven by, for instance, interest rates.

We then have

$$Z_{i,t} = \mathbf{x}_t^T \boldsymbol{\beta} + \epsilon_{i,t}$$

where the ϵ s have a similar factor correlation structure. This implies that

$$\begin{aligned} P(r_{i,t+1} \leq r' | r_{i,t} = r) &= P(Z_{i,t} \leq \zeta_{r,r'}) \\ &= \Phi \left(\frac{\zeta_{r,r'} - \mathbf{x}_t^T \boldsymbol{\beta}}{\sigma_\epsilon} \right), \end{aligned}$$

which is related to the *cumulative probit* model,

$$P(r_{i,t+1} \leq r' | r_{i,t} = r) = \Phi(\mu_{r,r'} + \mathbf{x}_t^T \boldsymbol{\gamma}_r).$$

This is a model for an ordered categorical response variable, which we can fit separately for each initial rating r .

Standard software can be used to fit the cumulative probit model and the similar *cumulative logit* model,

$$P(r_{i,t+1} \leq r' | r_{i,t} = r) = \Psi(\mu_{r,r'} + \mathbf{x}_t^T \gamma_r)$$

where

$$\Psi(x) = \frac{e^x}{1 + e^x}$$

is the inverse of the *logistic* function

$$\text{logit}(\psi) = \ln\left(\frac{\psi}{1 - \psi}\right).$$

But standard software can handle only a single factor, and cannot fit a common β across initial ratings r .

Cumulative logistic fit, no correlations, to Standard & Poor's annual static pools:

Initial Rating	GDP Defl. Coef.	GDP Defl. SE.	GDP Y/Y Coef.	GDP Y/Y SE.	1-Yr Rate Coef.	1-Yr Rate SE.	5-Yr Rate Coef.	5-Yr Rate SE.
AAA	87.36	39.68	-14.61	5.22	0.35	0.11	-0.53	0.11
AA	-48.53	22.81	6.08	3.02	0.04	0.06	-0.02	0.06
A	-34.65	16.85	7.33	2.26	-0.03	0.04	0.02	0.04
BBB	-24.73	18.46	11.44	2.65	-0.06	0.05	0.04	0.05
BB	-130.80	17.46	7.11	2.72	-0.14	0.05	0.21	0.05
B	-85.02	20.49	13.26	2.91	-0.10	0.05	0.18	0.05
CCC	-54.57	48.20	16.12	6.59	-0.38	0.11	0.43	0.12