# Outline

- DCC models
- Factor MIDAS model
- Data/Estimation/Results

# Conditional Correlation (CC) models

- General Setup
  - $r_t|\mathcal{F}_{t-1} \sim N(0, H_t)$; $H_t = D_t R_t D_t = \{\rho_{ij}\sqrt{h_{iit}h_{jjt}}\}$
  - $D_t = diag(h_{11t}^{1/2}...h_{NNt}^{1/2})$
  - $h_{iit}$ can be defined as any univariate GARCH model
  - $R_t = \{\rho_{ij,t}\}$ – symmetric p.d. matrix with $\rho_{ii} = 1$. $H_t$ is p.d. if $R_t$ is p.d.

- Constant CC Bollerslev (1990). $R_t = R$ is a constant matrix (the assumption is not very realistic for many many applications).

- Christodoulakis and Satchell (2002). Bivariate dynamic conditional correlation model. To ensure p.d. $R_t$, Fisher transformation $\rho_{12,t} = (e^{2v_t} - 1)/(e^{2v_t} + 1)$, where $v_t = \epsilon_{1t}\epsilon_{2t}/\sqrt{h_{11t}h_{22t}}$. $R_t$ is p.d. by construction.

# Tse and Tsui (2002) DCC model

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$$R_t = (1 - \theta_1 - \theta_2)R + \theta_1 \Psi_{t-1} + \theta_2 R_{t-1}$$

$\theta_1$ and $\theta_2$ are non-negative parameters and $\theta_1 + \theta_2 < 1$, $R$ is a symmetric $N \times N$ p.d. matrix with $\rho_{ii} = 1$, and $\Psi_{t-1}$ is the $N \times N$ correlation matrix of rolling-window "realized correlation", i.e.

$$\psi_{ij,t-1} = \frac{\sum_{m=1}^{M} u_{i,t-m} u_{j,t-m}}{\sqrt{(\sum_{m=1}^{M} u_{i,t-m}^2)(\sum_{m=1}^{M} u_{j,t-m}^2)}}$$

where $u_{it} = \epsilon_{it}/\sqrt{h_{iit}}$. $M > N$ is the necessary condition for $\Psi_{t-1}$ to be p. d. and therefore $R_t$ – p.d. The test for CCC will be $\theta_1 = \theta_2 = 0$.

# Engle (2002) DCC

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$$R_t = (diag(Q_t))^{-1/2} Q_t (diag(Q_t))^{-1/2}$$

where the $N \times N$ symmetric pd matrix $Q_t$ is given by

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha u_{t-1} u'_{t-1} + \beta Q_{t-1}$$

with $u_{it} = \epsilon_{it}/\sqrt{h_{iit}}$, $\bar{Q}$ is the $N \times N$ unconditional variance matrix of $u_t$ and $\alpha, \beta \geq 0$ are scalar parameters with $\alpha + \beta < 1$. The elements of $\bar{Q}$ can be estimated or set to their sample counterpart which will make the estimation even simpler.

- The difference between the two DCC models
  - Tse and Tsui (2002)

$$\rho_{ij,t} = (1-\theta_1-\theta_2)\rho_{ij,t-1} + \theta_2\rho_{ij,t-1} + \theta_1 \frac{\sum_{m=1}^{M} u_{i,t-m}u_{j,t-m}}{\sqrt{(\sum_{m=1}^{M} u_{i,t-m}^2)(\sum_{m=1}^{M} u_{j,t-m}^2)}}$$

  - Engle (2002)

$$\rho_{ij,t} = \frac{(1-\alpha-\beta)\bar{q}_{ij}+\alpha u_{i,t-1}u_{j,t-1}+\beta q_{ij,t-1}}{\sqrt{\left((1-\alpha-\beta)\bar{q}_{ii}+\alpha u_{i,t-1}^2+\beta q_{ii,t-1}\right)\left((1-\alpha-\beta)\bar{q}_{jj}+\alpha u_{j,t-1}^2+\beta q_{jj,t-1}\right)}}$$

# General dynamic covariance model, Kroner and Ng (1998)

$$H_t = D_t R_t D_t + \Phi \odot \Theta_t$$

where $\odot$ - Hadamard product,

$D_t = diag(\sqrt{\theta_{ii,t}})$

$\Theta_t = (\theta_{ij,t})$

$R_t$ is specified as Engle or Tsui correlation matrix,

$\Phi = (\phi_{ij})$, $\phi_{ii} = 0 \;\; \forall i$, $\phi_{ij} = \phi_{ji}$

$\theta_{ij,t} = \omega_{ij} + a_i' \epsilon_{t-1} \epsilon_{t-1}' a_j + g_i' H_{t-1} g_j \;\; \forall i, j$

$a_i, g_i, \quad i = 1, ..., N$ are $N \times 1$ vectors of parameters, and $\Omega = (\omega_{ij})$ is positive definite and symmetric.

Element by element

$$h_{ii,t} = \theta_{ii,t} \quad \forall i$$

$$h_{ij,t} = \rho_{ij,t}\sqrt{\theta_{ii,t}\theta_{jj,t}} + \phi_{ij}\theta_{ij,t}$$

This model can be reduced to Engle or Tse Tsui DCC model if the following restrictions are imposed:

– $\Phi = 0$

– $a_i = \alpha_i l_i, \; g_i = \beta_i l_i, \quad \forall i$, where $l_i$ is the $i^{th}$ column of an $N \times N$ identity matrix., and $\alpha_i$ and $\beta_i$, $i = 1, ..., N$ are scalars.

# Summary

The advantages of the dynamic correlation models are

- Easiness of estimation.
- Has small number of parameters $O(k)$ (analysis of large covariance matrices)
- The 2-step procedure produces consistent results.

Therefore they can be used in the large-scale estimation. However, the DCC assumption that the whole correlation matrix is driven by a small number of parameters is not reasonable one for the purposes of the large-scale estimation.

# Diagonal Factor MIDAS model

$$r_{t+h,t}|\mathcal{F}_{t-1} \sim N(0, H_{t+h,t})$$

$$r_{t+h,t} = \Lambda f_{t+h,t} + \epsilon_{t+h,t}$$

$$H_{t+h,t} = \Lambda F_{t+h,t}\Lambda' + \Sigma$$

$r_{t+h,t}$ — $n \times 1$ vector of asset returns

$f_{t+h,t}$ — $m \times 1$ vector of orthogonal factors measurable w.r.t. $\mathcal{F}_{t+h}$, $m < n$

$\Sigma$ — $n \times n$ diagonal matrix of the idiosyncratic noise covariance

$\Lambda$ — $n \times m$ factor loading matrix

$F_{t+h,t}$ — diagonal matrix of conditional factor covariance

# Diagonal Factor MIDAS model (cont.)

$$\{F_{t+h,t}\}_{kk}|\mathcal{F}_t = \mu_k^h + \phi_k^h \sum_{j=1}^{j_{max}} b(j, \theta_k^h)\{F_{t-j+1,t-j}\}_{kk}$$

where $\{F_{t-j+1,t-j}\}_{kk}$ — one period realized volatility of the $k^{th}$ factor. $\{F_{t-j+1,t-j}\}_k = \sum_{s=1}^{l} f_{kt-j+s/l}^2$ Factor Construction:

$$f_{kt+j/l} = w_k' r_{t+j/l}$$

The first factor is the "market", i.e. $w_1 = \iota/n$. All others are constructed using factor analysis from the residuals of the linear projection of individual stocks on the first factor.

# Asymmetric specification

$$\{F^{asy}_{t+h,t}\}_k = \left[\mu_k^{h+} + \phi_k^{h+} \sum_{j=1}^{j_{max}} b(j, \theta_k^{+}) Q_{kt-j+1,t-j}\right] I_{\{f_{1t,t-1}>0\}} +$$

$$\left[\mu_k^{h-} + \phi_k^{h-} \sum_{j=1}^{j_{max}} b(j, \theta_k^{-}) Q_{kt-j+1,t-j}\right] I_{\{f_{1t,t-1}\leq 0\}}$$

where $I_{\{f_{1t,t-1}\leq 0\}}$ is an indicator function, and

$$I_{\{f_{1t,t-1}\leq 0\}} + I_{\{f_{1t,t-1}>0\}} \equiv 1$$

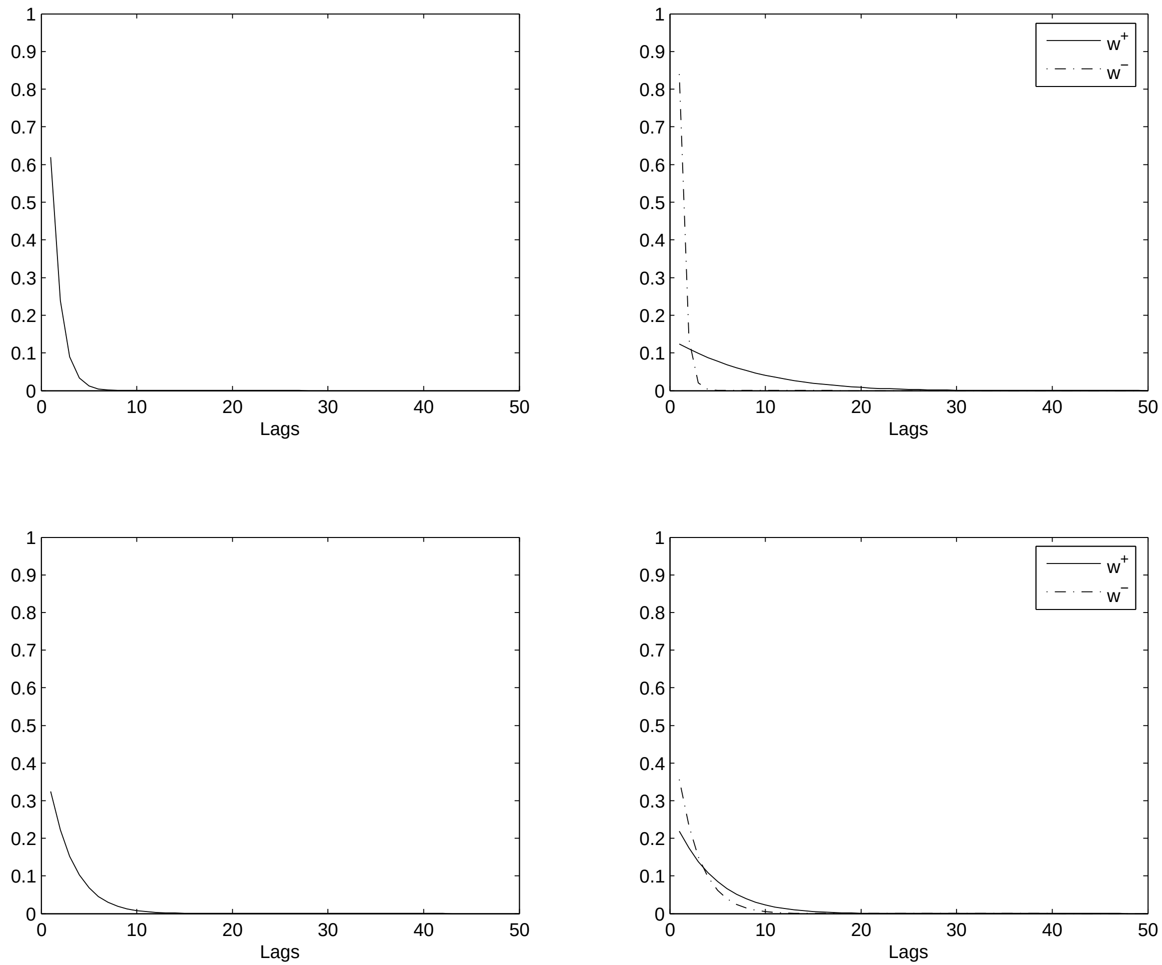


Figure 1: MIDAS weights of the estimated factor volatility. $w^+$ corresponds to conditioning on the positive market returns, $w^-$ corresponds to conditioning on the negative.

# Advantages of the Factor MIDAS model

- Has small number of parameters $O(k)$
- Number of lags can be increased at no cost
- Uses information available in high-frequency data
- Uses realized volatility as a measure of variance instead of squared returns
- The 2-step procedure produces consistent results.

# Estimation and Testing

- Estimation
  1. Construct high-frequency factors
  2. Construct daily realized volatility of the factors
  3. Estimate the variance of the factors by univariate MIDAS and construct the variance-covariance matrix

- Estimate Diagonal Factor MIDAS models for
  - Symmetric and Asymmetric specification
  - Number of factors $1, 2, 3$
  - Number of stocks $5, 10, 15, 22$

- Evaluate performance using three portfolios
  - Testing standardized portfolio returns
  - Constructing $HIT$ auxiliary regression
  - Comparing with DCC model

  The considered portfolios are:

1. Minimum Variance portfolio: $w_{t+h,t}^{(1)} = \frac{\hat{H}_{t+h,t}^{-1}}{\iota' \hat{H}_{t+h,t}^{-1} \iota}$
2. Value Weighed portfolio: $w_{t+h,t}^{(2)} = \frac{w_{t,t-h} \odot (1+r_{t+h,t})}{w'_{t,t-h}(1+r_{t+h,t})}$
3. Equally Weighted portfolio: $w_{t+h,h}^{(3)} = \iota / n$

# Performance Evaluation. Standardized Residuals

- Under $H_0$ (correctly specified conditional variance-covariance matrix),

  $(\lfloor T/h \rfloor - 1)\hat{s}_l{}^2 = \sum_{t=0}^{\lfloor T/h \rfloor} \frac{(r'_{(t+1)h,th} w^l_{(t+1)h,th})^2}{s^2_{(t+1)h,th}} \sim \chi^2(\lfloor T/h \rfloor - 1)$,
  where $s^2_{lt+h,t} = (w^l_{t+h,t})' H_{t+h,t} w^l_{t+h,t}$ — estimated conditional variance of the portfolios $l = \{1, 2, 3\}$.

- Accept $H_0$ with $\alpha = .05$ if

  $\frac{\chi^2_{\alpha/2}}{(\lfloor T/h \rfloor - 1)}(\lfloor T/h \rfloor - 1) < \hat{s}^2 < \frac{\chi^2_{1-\alpha/2}(\lfloor T/h \rfloor - 1)}{(\lfloor T/h \rfloor - 1)}$

# Results

Standard deviations of the different portfolios (minimum variance, value weighted and equally weighted) using 5 and 22 stocks and five day interval.

| Model | Factors | MinVar | Value | Equal | MinVar | Value | Equal |
|---|---|---|---|---|---|---|---|
| | | | Five stocks | | | Twenty-two stocks | |
| DCC | – | 1.046 | 0.975 | 0.963 | $1.170^*$ | $0.877^*$ | 0.981 |
| asym | 1 | 1.020 | 0.960 | 0.992 | 1.123 | 0.899 | 1.014 |
| sym | 1 | 1.010 | 0.963 | 0.986 | $1.247^*$ | $0.830^*$ | 1.024 |
| asym | 2 | 1.025 | 0.967 | 1.011 | 1.071 | 0.945 | 1.002 |
| sym | 2 | 1.022 | 0.963 | 1.003 | 1.066 | 0.955 | 1.005 |
| asym | 3 | 0.992 | 0.955 | 0.973 | 1.032 | 0.948 | 1.000 |
| sym | 3 | 1.008 | 0.962 | 0.994 | 1.045 | 0.940 | 0.985 |

# Performance Evaluation. Conditional Autoregressive Value-at-Risk, Engle and Manganelli (2000)

- Portfolio's $l$ return in period $\{t+h,t\}$ is $R^l_{t+h,t} = {w^l_{t+h,t}}' r_{t+h,t}$
- Define a binary variable $HIT^l_{t+h,t} = I_{\{R^l_{t+h,t} < VaR(q)\}}$, $q$ – quantile of interest.
- Under the null of correct specification and known $q$, $E(HIT^l_{t+h,t} | \mathcal{F}_{t-1}) = q$
- $H_{00}$: $\delta_i = 0, \forall i$ in

$$HIT_{t+h,t} - q = \delta_0 + \sum_{i=1}^{r} \delta_i HIT_{t-(i-1)h,t-ih} + \delta_{r+1} VaR_{t+h,t} + \nu_t$$

# Performance Evaluation. Conditional Autoregressive Value-at-Risk, Engle and Manganelli (2000) (cont)

- Under the null of correct specification and unknown $q$,

- $H_0$: $\delta_i = 0, \forall i > 0$ in

$$HIT_{t+h,t} - q = \delta_0 + \sum_{i=1}^{r} \delta_i HIT_{t-(i-1)h,t-ih} + \delta_{r+1} VaR_{t+h,t} + \nu_t$$

- Example: If $R^l_{t+h,t}$ is normal, and $q = .05$

$$VaR_{t+h,t}(.05) = -1.65\hat{\sigma}_{t+h,t}$$

# HIT regression results for the 5% quantile of the minimum variance and equally weighted portfolios using 5 and 22 DJ assets. Symmetric and asymmetric models.

| | 5 days horizon | | | | 10 days horizon | | | |
|---|---|---|---|---|---|---|---|---|
| | Min. Variance | | Equally weighted | | Min. Variance | | Equally weighted | |
| Model | MIDAS | DCC | MIDAS | DCC | MIDAS | DCC | MIDAS | DCC |
| asym$_{5,1}$ | 0.254 | 0.056 | 0.334 | 0.436 | 0.669 | 0.285 | 0.693 | 0.011 |
| sym$_{5,1}$ | 0.804 | 0.056 | 0.174 | 0.436 | 0.779 | 0.285 | 0.012 | 0.011 |
| asym$_{5,3}$ | 0.322 | 0.056 | 0.223 | 0.436 | 0.779 | 0.285 | 0.074 | 0.011 |
| sym$_{5,3}$ | 0.248 | 0.156 | 0.024 | 0.436 | 0.501 | 0.285 | 0.052 | 0.011 |
| asym$_{22,1}$ | 0.081 | 0.004 | 0.680 | 0.015 | 0.072 | 0.000 | 0.071 | 0.000 |
| sym$_{22,1}$ | 0.052 | 0.004 | 0.234 | 0.015 | 0.669 | 0.000 | 0.038 | 0.000 |
| asym$_{22,3}$ | 0.164 | 0.004 | 0.130 | 0.015 | 0.532 | 0.000 | 0.051 | 0.000 |
| sym$_{22,3}$ | 0.531 | 0.004 | 0.998 | 0.015 | 0.084 | 0.000 | 0.367 | 0.000 |