

Is that what you call the "simplest" case? :)
No wonder we astronomers are running into so many problems!

Some Typical Upper Limit Scenarios.
-Vinay Kashyap

In all the cases, I assume that there is a source region of area A_{src} which contains almost 100% of the source counts, and a disjoint background region of area A_{bkg} , and that we observe N_{src} and N_{bkg} counts in each area:

case 1: moderate source in high background

$$A_{src}=1 \text{ \& } A_{bkg}=10 \text{ \& } N_{src}=38 \text{ \& } N_{bkg}=150$$

when we adopt the standard $S/N=3$ detection criterion, and further assume the Gehrels approximation for the statistical error, this results in a non-detection ($S/N=2.8$) and it requires $N_{src}=40$ for a detection at $S/N=3$ (implying an upper limit of 25).

if we set the threshold based on the probability of not being able to obtain a certain number of counts as a fluctuation from the background, then a source would be considered detected at "3-sigma" if $N_{src}>27$, thus setting an upper limit of 12.5 on the source intensity. (note - this does not take into account ψ^* as in your step A.4, and I am not sure it should, because the detection threshold does not care about counts due to the source.) so in this case, the source would be detected, and we can calculate the posterior distribution for the source intensity, and that results in a 99.7% credible range of (0,43.5) (for an interval containing the mode) or (7.3,45.2) (for an equal-tail interval) for the source intensity.

case 2: weak source in low background

$$A_{src}=1 \text{ \& } A_{bkg}=100 \text{ \& } N_{src}=8 \text{ \& } N_{bkg}=150$$

$S/N=3$ case -- no detection because $S/N=1.6$, $N_{src}=18$ for a detection, and upper limit = 16.5

background threshold case -- $N_{src}=6$ for detection at 99.7% confidence, hence source would be detected; no upper limit, but a *confidence bound* to the source intensity of 17.9 (smallest interval including the mode) or 18.75 (equal-tail interval).

case 3: no source in low background

$$A_{src}=1 \text{ \& } A_{bkg}=100 \text{ \& } N_{src}=2 \text{ \& } N_{bkg}=150$$

$S/N=3$ case -- as above, same upper limit, 16.5

background threshold case -- $N_{src}=6$ for detection at 99.7% confidence, hence no detection, and hence the upper limit would be 4.5