A Geometric Approach to Distinguish Between a New Source and Random Fluctuations

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Abstract

One of the fundamental problems in the analysis of experimental data is determining the statistical significance of a putative signal that may be only a small perturbation in a noisy experimental background. For example, searching for events arising from new particle resonances in the presence of background is a problem of interest in particle physics, and distinguishing a faint spectral line or a new source from a random fluctuation in the data arises in the context of astrophysics. These are challenging statistical problems that defy the standard analytically feasible solutions. The problem of distinguishing a signal from background or noise is formulated in terms of classical hypothesis testing, where a null hypothesis describes the background and an alternative hypothesis characterizes the signal as a perturbation of the background. Often a chi-square goodness-of-fit or a likelihood ratio test (LRT) statistic is employed for this testing problem. However, the former does not yield good power in detecting the signal and the latter has lacked an analytically tractable reference distribution required to calibrate a test statistic. We present the recently developed new test statistic based on a score process (Pilla and Loader; arXiv: math.ST/0511503) to detect the presence of a signal. We review its reference distribution, which has an elegant geometrical interpretation and broad applicability, while noting the connection with the LRT. We illustrate the technique in the context of a model problem from particle physics: the search for a new particle resonance. Extensions of the relevant theory for applications to astrophysics and astronomy will be discussed. More information is available at `http://stat.case.edu/~pillar/PRL/PRL.htm`.