Summary of the Low-dimensional behavior group activities

7 April, 2005

1 Goals and obstacles

Inspired and intrigued by the work of Kalnay, Ott, *et al.*, the goal of this group is to understand how very large dimensional systems exhibit behavior that can be effectively "captured" (or "explained") by low-dimensional "phenomena." Indeed, one of the goals is to really pin down or define what is meant by "low-dimensional behavior." In that sense, the group started off with an undefined or ill-defined or ill-understood term and hoped to make it well-defined or better-understood! This is also the major obstacle we are trying to overcome. The hope is that an understanding of various aspects of such low-dimensional behavior could lead to a more efficient scheme for data assimilation. One aspect of our goal was to critically assess the previous work (e.g. by Ott *et al.* and by others).

2 Activities

The first few meetings focused on the dynamical systems motivated understanding of different ensemble generation schemes - singular value decomposition (SVD), bred vectors, Lyapunov vectors, etc. Juan started us off with an introduction (flavored by the exposure in Carl Wunsch's book) to SVD in a very general setting. Amit related that (using the articles by Legras and Vautard and by others) to the specific linearized dynamics used in ensemble generation. After having covered these basics, we discussed the relation between and shortcomings and advantages of these different schemes.

A concurrent related discussion focused on the important (growing) directions and how these are captured by the ensembles. This naturally lead to the work on local ensemble Kalman filter, which is the focus for the past couple of weeks.

3 Some Conclusions and future directions

We managed to identify that there are (at least) three different "low-dimensional behaviors" which sometimes get mixed up.

- Phenomena that are local in the state space
- Phenomena that are local in the physical space (spatial localization, e.g. in local ensemble KF). This is simply a restricted class of the above.
- Low-dimensionality "in the covariance matrix." These are two different subcategories of this aspect:
  - Low rank (say $k \ll D$, where D is the dimension of the state space) of the "true" covariance matrix
  - Low rank (say k') of the covariance matrix obtained from the ensemble of size N

If we know that $k' > k$ (of course, $k' < N$), then at least there is a hope that the ensemble of size N will be "good enough" for capturing the growing directions. No satisfactory empirical evidence or theoretical justification for such a statement has been obtained yet and we are actively discussing this in current meetings. Furthermore, it does not seems at all clear that, even if $k' > k$, the k growing directions are typically "contained" within the k' growing directions indicated by the ensemble covariance matrix. Some exploratory numerical studies clarifying these issues could be quite useful.