

*Identification of periodic and  
cyclic fractional stable motions*

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**Fact:** for fixed  $H > 0$  and  $\alpha \in (0, 2)$ , there are infinitely many different,  $H$ -self-similar,  $\alpha$ -stable stationary increments processes

**Goal:** classify such processes

**Subclass:** self-similar, mixed moving averages

**Mixed moving average:**  $\alpha$ -stable, stationary increments process  $X_\alpha$  represented by

$$X_\alpha(t) \stackrel{d}{=} \int_X \int_{\mathbb{R}} (G(x, t+u) - G(x, u)) M_\alpha(dx, du),$$

where  $G$  is deterministic,  $X$  is some space and  $M_\alpha(dx, du)$  is an  $\alpha$ -stable, independently scattered random measure

**Self-similarity:** imposes additional assumptions on  $G$

**Example:** LFSM:  $X = \{1\}$ ,  $G(1, u) = au_+^{H-1/\alpha} + bu_-^{H-1/\alpha}$

**Connection to flows:** for a minimal  $G$ , any  $c > 0$ ,

$$G(x, cu) = \square \cdot G(\psi_c(x), u + \square) + \square,$$

where  $\psi_c(x)$  is a flow:  $\psi_{c_1 c_2}(x) = \psi_{c_1}(\psi_{c_2}(x))$

**Isomorphism property:** flows corresponding to two different minimal  $G$ 's of same process are isomorphic

**Types of flows:** dissipative + conservative; conservative = identity + cyclic + other conservative (identity + cyclic = periodic)

**Decomposition:** (minimal  $G$ , flows)

$$X_\alpha \stackrel{d}{=} \overbrace{X_\alpha^{(1)}}^{\text{dissip}} + \underbrace{\overbrace{X_\alpha^{(2)}}^{\text{ident}} + \overbrace{X_\alpha^{(3)}}^{\text{cyclic}} + \overbrace{X_\alpha^{(4)}}^{\text{other}}}_{\text{periodic}},$$

unique; disjoint, independent components

**Question:** what if  $G$  is non-minimal? Previously:  $X_\alpha^{(1)}$ ,  $X_\alpha^{(2)}$  were identified through non-minimal  $G$  (no flows)

### Main results:

- $X_\alpha$  is periodic iff

$$X = \{x \in X : \exists c > 0 : G(x, cu) = \\ = aG(x, u + b) + d \text{ for some } a, b, d\}$$

- There is analogous result for cyclic  $X_\alpha$
- Decomposition into 4 components can be obtained through non-minimal  $G$  (no flows)
- Examples of cyclic, periodic  $X_\alpha$  are studied
- Periodic  $X_\alpha$  have canonical representations