Controlled Stochastic Networks in Heavy Traffic

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Controlled Networks: Routing, Sequencing, Input Control...

Under "Heavy Traffic Conditions" one can formally approximate by:

Brownian Control Problem.

Form of the controlled process:

$$Q^z(t) \doteq z + bt + B(t) + RY(t),$$

$B(\cdot)$ is a Brownian motion. $z \in \mathbb{R}_+^m$ is the initial inventory vector.

The process $Y(\cdot)$ is a control which is a n-dimensional RCLL, adapted process.

It is admissible if: $Q^z(t) \geq 0$, for all $t \geq 0$ and $I(t) \doteq KY(t)$ is non decreasing and nonnegative.

b is a vector and R, K are matrices given by the problem data.

Cost Criterion.

$$\mathbb{E}\left(\int_0^\infty e^{-\beta s}(h(Q^z(s))ds + cdI(s))\right)$$

$h : \mathbb{R}^m \to \mathbb{R}$ is a measurable function and $c, \beta \in (0, \infty)$. Let $V(z)$ denote the value function.

Not covered by classical stochastic control theory:
$V(z)$ may not be achieved, no dynamic programming principle or HJB theory.

Workload Formulation (Singular Control with State Constraints).

$(R, K) \to (M, G)$.

A basic relationship: $MR = GK$.

State Space: $S \doteq \{My : y \in \mathbb{R}_+^m\}$.

Controlled process:

$$W^w(t) \doteq w + \tilde{b}t + \tilde{B}(t) + GI(t).$$

where $w \doteq Mz$, $\tilde{b} \doteq Mb$ and $\tilde{B}(\cdot) \doteq MB(\cdot)$.

$I(\cdot)$ is a control which is a RCLL, nondecreasing, nonnegative, adapted process.

It is admissible if: $W^w(t) \in S$, for all $t \geq 0$.

Cost Function. Let for $w \in S$,

$f(w) \doteq \inf\{h(z) : Mz = w; z \geq 0\}$.

$$\mathbb{E}\left(\int_0^\infty e^{-\beta t}(f(W^w(s))ds + cdI(t))\right).$$

Under broad conditions the two control problems are "equivalent" (Harrison -Van Mieghem).

Frequently the dimensional reduction is significant.

Basic Strategy.

1. Solve the SCSC problem.

2. Use the equivalence between control problem to obtain the solution of the BCP (Deterministic nonlinear optimization).

3. Interpret the solution to obtain control for the network control problem.

4. (Asymptotic) optimality of the control policy.

Key Step: SCSC problem.

1. HJB theory (ongoing work with R. Atar)
2. Explicit solutions (ongoing work with A. Ghosh)